Physics 412 - Homework 2

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1 Problem 1

1.1

$$\begin{aligned}
\nabla(fg) &= f\nabla g + g\nabla f \\
&= (\frac{\partial(fg)}{\partial x} + \frac{\partial(fg)}{\partial y} + \frac{\partial(fg)}{\partial z})
\end{aligned}$$

You use product rule for each of these partials

$$\begin{aligned}
&= (\frac{\partial(fg)}{\partial x} + \frac{\partial(fg)}{\partial y} + \frac{\partial(fg)}{\partial z}) \\
&= ([\frac{f\partial g}{\partial x} + \frac{g\partial f}{\partial x}] + [\frac{f\partial g}{\partial y} + \frac{g\partial f}{\partial y}] + [\frac{g\partial f}{\partial z} + \frac{f\partial g}{\partial z}]) \\
&= ([\frac{f\partial g}{\partial x} + \frac{f\partial g}{\partial y}\frac{f\partial g}{\partial z}] + [\frac{g\partial f}{\partial x} + \frac{g\partial f}{\partial y} + \frac{g\partial f}{\partial z}]) \\
&= f\nabla g + g\nabla f
\end{aligned} \tag{1}$$

1.2

$$\begin{aligned}
\nabla.(fA) &= f\nabla.A + A.\nabla f \\
&= (\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f)}{\partial z}).(P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}) \\
&= (\frac{\partial f}{\partial x}(P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}) + \frac{\partial f}{\partial y}(P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}) + \frac{\partial f)}{\partial z}(P(x,y,z)\hat{i} + \\
&= (\frac{\partial f}{\partial x}P(x,y,z)\hat{i} + \frac{\partial f}{\partial y}P(x,y,z)\hat{i} + \frac{\partial f)}{\partial z}P(x,y,z)\hat{i}) + (\frac{\partial f}{\partial x}Q(x,y,z)\hat{j} + \frac{\partial f}{\partial y}Q(x,y,z)\hat{j} + \frac{\partial f)}{\partial z}Q(x,y,z)\hat{j}) + (\frac{\partial f}{\partial x} \\
&= f\nabla.A + A.\nabla f
\end{aligned} \tag{2}$$

1.3

$$\nabla \times (fA) = f(\nabla \times A) - A \times \nabla f \tag{3}$$

2 Problem 2

$$
\begin{aligned}
R &= \frac{r_1 - r_2}{|r_1 - r_2|^3} \\
&= \frac{3\hat{i} + \hat{j} - 6\hat{k}}{\sqrt{3^2 + 1 + 6^2}} * \frac{1}{(r_1 - r_2)} \\
&= \frac{3\hat{i} + \hat{j} - 6\hat{k}}{\sqrt{46} * (3\hat{i} + \hat{j} - 6\hat{k})} \qquad (4) \\
\frac{3\hat{i} + \hat{j} - 6\hat{k}}{(3\sqrt{46}\hat{i} + \sqrt{46}\hat{j} - 6\sqrt{46}\hat{k})} \\
&= \frac{1}{\sqrt{46}}\hat{i} + \frac{1}{\sqrt{46}}\hat{j} - \frac{1}{\sqrt{46}}\hat{k}
\end{aligned}
$$

A lot cancels out in this since you get a unit vector when you divide a vector by its modulus, and then you are dividing that unit vector

$$
\begin{aligned}
&= \frac{3\hat{i} + \hat{j} - 6\hat{k}}{\sqrt{3^2 + 1 + 6^2}} * \frac{1}{(r_1 - r_2)} \\
&= (r_1 \hat{-} r_2) * \frac{1}{(r_1 - r_2)}
\end{aligned} \qquad (5)
$$

3 Problem 3

3.1

$$
\begin{aligned}
\int_{-\inf}^{\inf} dx(x^3 - 2)\delta(x - 1) = -1 \\
y = 1,\ f(y) = -1,\ f(x) = (x^3 - 2) \\
f(y = 1) = (1^3 - 2) = -1
\end{aligned} \qquad (6)
$$

3.2

$$
\begin{aligned}
\delta(cx) &= \frac{1}{|c|}\delta(x) \\
\int \delta(c(x - y))dx &= f(cy) \\
\int \delta(cx - cy)dx &= cf(y) \\
\int \delta(cx)dx &= cf(0)
\end{aligned} \qquad (7)
$$

$$
\begin{aligned}
\int \frac{1}{|c|}\delta(x)dx &= f(0) \\
\frac{1}{|c|}\int \delta(x)dx &= f(0) \\
\int \delta(x)dx &= |c|f(0)
\end{aligned} \qquad (8)
$$

Since $y = 0$ in both cases, these are equivalent

3.3

$$\begin{aligned} \frac{d\theta(x)}{dx} &= \delta(x) \\ \frac{d}{dx}1 &= 0 \\ \frac{d}{dx}0 &= 0 \\ \delta(x) &= 0 \end{aligned} \tag{9}$$

3.4

$$\begin{aligned} F(k) &= \int_{-\inf}^{\inf} \frac{dx}{\sqrt{2\pi}} e^{-ikx}\delta(x) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\inf}^{\inf} dx e^{-ikx}\delta(x) \\ \text{Define } f(x) = e^{-ikx} \\ f(y = 0) &= e^0 = 1 \\ F(k) &= \frac{1}{\sqrt{2\pi}} \end{aligned} \tag{10}$$

3.5

$$\begin{aligned} \int_{-\inf}^{\inf} dx f(x)\delta'(x) &= -f'(0) \\ \text{Integration by parts} \\ \int_{-\inf}^{\inf} dx f(x)\delta'(x) &= f(x)\delta(x) - \int_{-\inf}^{\inf} f'(x)\delta(x)dx \\ &= -\int_{-\inf}^{\inf} f'(x)\delta(x)dx \\ \text{Using definition of Dirac function...} \\ &= -f'(0) \end{aligned} \tag{11}$$

3.6

$$\begin{aligned} \nabla^2 \frac{1}{\sqrt{x^2 + y^2 + z^2}} \\ \nabla^2 \frac{1}{r} \\ &= (\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z})\frac{1}{r} \\ &= (\frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z})\frac{1}{r} \end{aligned} \tag{12}$$

4 Problem 4

$$Divergence = \frac{2\pi r^3}{3} \tag{13}$$

I had a little trouble doing this, but this arises from my knowledge that the divergence of a circle would simple by the volume of that circle, in this case you would divide that volume by 2.