Physics 412 - Homework 1

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1 Problem 1

$$A = -3\hat{x} + 2\hat{y} + \hat{z}$$
$$B = 2\hat{x} - \hat{y} + 4\hat{z}$$

1.1

$$\begin{aligned} A - 3B &= (-3-6)\hat{x} + (2+3)\hat{y} + (1-12)\hat{z} \\ &= -9\hat{x} + 5\hat{y} + -11\hat{z} \end{aligned} \tag{1}$$

1.2

$$A \cdot B = -4 \tag{2}$$

1.3

$$\begin{aligned} \text{Proj}_A B &= -\frac{2}{7}(-3\hat{x} + 2\hat{y} + \hat{z}) \\ &= \frac{6}{7}\hat{x} - \frac{4}{7}\hat{y} - \frac{2}{7}\hat{z} \end{aligned} \tag{3}$$

1.4

$$\begin{aligned} \theta &= \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|A||B|} \\ &= \frac{\vec{A}\cdot\vec{B}}{|A||B|} \\ &= 1.0278(radians) \end{aligned} \tag{4}$$

1.5

$$\begin{aligned} A \times B &= [(2*4) - (1*-1)]\hat{x} + [(1*2) - (-3*4)]\hat{y} + [(-3*-1) - (2*2)]\hat{z} \\ &= [8+1]\hat{x} + [2+12]\hat{y} + [3-4]\hat{z} \\ &= 9\hat{x} + 14\hat{y} - \hat{z} \end{aligned} \tag{5}$$

1.6

$$\begin{aligned} Area &= base * height = |A| * |B| \\ Area &= \sqrt{14} * \sqrt{21} \\ &= \sqrt{294} \end{aligned} \tag{6}$$

2 Problem 2

$$f(x) \approx f(x_0) + (x - x_0)\frac{df}{dx}|_{x=x_0} + \frac{(x-x_0)^2}{2!}\frac{d^2f}{dx^2}|_{x=x_0}$$

2.1

$$\begin{aligned} f(x) = \frac{1}{1-x} &\approx \frac{1}{1-x_0} + (x - x_0)\frac{df}{dx}|_{x=x_0} + \frac{(x-x_0)^2}{2!}\frac{d^2f}{dx^2}|_{x=x_0} \\ &\approx 1 + (x)(1) + \frac{x^2}{2}\frac{2}{(1-x_0)^3} \\ &\approx 1 + x + x^2 \end{aligned} \tag{7}$$

2.2

$$\begin{aligned} f(x) = \log(1-x) &\approx \log(1-x_0) + x\frac{1}{x_0 - 1} + \frac{x^2}{2}\frac{1}{(x_0-1)^2} \\ &\approx -x - \frac{x^2}{2} \end{aligned} \tag{8}$$

2.3

$$\begin{aligned} f(x) = e^x &\approx e^{x_0} + xe^{x_0} + \frac{x^2}{2}e^{x_0} \\ &\approx 1 + x + \frac{x^2}{2} \end{aligned} \tag{9}$$

2.4

$$\begin{aligned} f(x) = \sin x &\approx \sin x_0 + x\cos x_0 - \frac{x^2}{2}\sin x_0 - \frac{x^3}{3!}\cos x_0 \\ &\approx x - \frac{x^3}{3!} \end{aligned} \tag{10}$$

2.5

$$\begin{aligned} f(x) = \cos x &\approx \cos x_0 - x\sin x_0 - \frac{x^2}{2}\cos x_0 + \frac{x^3}{3!}\sin x_0 \\ &\approx 1 - \frac{x^2}{2} \end{aligned} \tag{11}$$