

Physics 375 - Homework 1

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September 2022

1 Problem 1

1.1

The procedure I used to measure the intensity profile of the laser beam using the lab's photodiode was quite simple. My apparatus consisted of a laser aimed at a photodiode sensor. The photodiode sensor was attached to a stepper motor whose motion I could control and generate using matlab scripts. I could therefore move the photodiode linearly along a path perpendicular to the beam of the laser. By moving the photodiode across the beam of the laser, I could map the profile of the beam as the photodiode sensor converted incident optical intensity into a proportional voltage I could measure. Collecting this data and plotting it displayed a gaussian distribution of voltage vs time as the photodiode moved into and out of the range directly perpendicular to the laser beam.

The size of the photodiode sensor played a crucial role in the data collection. The beam from my laser was not a point beam, but rather a line. It took me many trials to get an accurate distribution that showed results I should expect to see. When the photodiode was too large, I found the measured voltage distribution to plateau at a maximum before dropping again, since the line beam could stay directly pointed at the photodiode sensor for a large distance.

Additionally, the size of the photodiode sensor and opening of the iris was proportional to the base voltage that the sensor picked up. Since the system was not perfectly isolated, stray photons from additional light sources were picked up by the sensor. This base voltage was proportional to the size of the opening of the iris.

By measuring this "extra" voltage picked up by the sensor, I realized that decreasing the size of the aperture was ideal to record measurements with the highest accuracy. However, if the aperture was too small, the full intensity of the beam was never completely distributed across the sensor, which resulted in data that plateaued. Therefore, in order to get the best reading, it was necessary for me to find a size that was small enough to minimize stray photons, yet large enough to get a full distribution of the intensity profile of the beam.

1.2

Angular Vernier: $127^{\circ} 15$ minutes

Linear vernier: 3.24 mm

1.3

From a little bit of research, I believe the cause of the non-zero fluctuations in voltages stem from the photodiodes offset voltage. Reading about photodiodes, many of them are designed with a low input offset voltage. There is also input bias current, however this effect becomes more pronounced with increased gain, so I believe the input offset voltage is the source of these fluctuations. The fluctuations between multiple discrete values was confusing to me, but I believe since the offset voltage is extremely small and designed to offset any transient illumination, it is possible it is picking up on disturbances to the circuit from outside sources. Since the environment is not isolated, fluctuations in the environment are likely the cause of these measurement readings.

I would consider this a systematic error, since this error can be systematically minimized with increased precision in the measurement device. Also, this error was not revealed through repeating measurements, it is consistent to the system we are using.

2 Problem 2

2.1

Group 10 Mean: 7.05
Group 10 Std Dev: 0.034
Group 10 Uncert: 0.01

Group 20 Mean: 7.14
Group 20 Std Dev: 0.0921
Group 20 Uncert: 0.0206

2.2

Group 10 has the more precise measurement as their data recorded less fluctuation and uncertainty. All things equal, though group 20 recorded more measurements, their data fluctuated further from the mean and had a greater uncertainty, therefore we are less confident in the data group 20 presents.

3 Problem 3

3.1

Please see excel workbook "question3.xlsx" for the computed values of g.

3.2 Uncertainty using propagation of errors

$$uncert_g = \sqrt{\left(\frac{\partial}{\partial L}\sigma_L\right)^2 + \left(\frac{\partial}{\partial T}\sigma_T\right)^2}$$

$$\frac{\partial}{\partial L} = \frac{4\pi^2}{T^2}$$

$$\frac{\partial}{\partial T} = \frac{-8\pi^2 L}{T^3}$$

$$uncert_g = 0.058 \frac{m}{s^2}$$

3.3 Standard deviation

$$uncert_g = 0.026 \frac{m}{s^2} \tag{1}$$

The standard deviation is roughly half of the calculated uncertainty using propagation of errors. Though the two values are within the same order of magnitude, they differ enough to be distinct.

3.4 Final answer for g

$$g = 9.81 \pm 0.012 \frac{m}{s^2} \tag{2}$$

3.5

Her answer for g is within reasonable agreement with the accepted value of her lab. Her value for g is within $2\sigma_g$ of the expected value at the lab which is reasonable and indicates agreement between theory and data. Additionally, in calculating a χ^2 distribution as a further measure of quantitative analysis, I found the P value to be 0.23, well within the confidence interval and indicating agreement between theory and data.

4 Problem 4

4.1

$$\begin{aligned} \bar{x} &= \int_0^1 x dx \\ &= \frac{x^2}{2} \Big|_0^1 \\ &= \frac{1}{2} \end{aligned} \tag{3}$$

$$\begin{aligned}
\sigma_x^2 &= \int_0^1 (x - \bar{x})^2 dx \\
&= \int_0^1 (x - \frac{1}{2})^2 dx \\
\text{Let } u &= x - \frac{1}{2}, du = 1dx \\
&= \int_{-\frac{1}{2}}^{\frac{1}{2}} (u)^2 du \\
&= \frac{u^3}{3} \Big|_{-\frac{1}{2}}^{\frac{1}{2}} \\
\sigma_x^2 &= \frac{1}{12} \\
&= \frac{1}{2\sqrt{3}}
\end{aligned} \tag{4}$$

4.2

The rand(N,1) function generates uniformly distributed pseudorandom numbers on the open interval (0,1).

4.3

To calculate the number N , we will solve this equation:

$$\begin{aligned}
\sigma_{\bar{x}} &= \frac{\sigma_x}{\sqrt{N}} \\
0.01\sqrt{N} &= \frac{1}{2\sqrt{3}} \\
\sqrt{N} &= \frac{1}{0.03464} \\
N &= 833.3\bar{3} \\
&\approx 834 \text{ random calls}
\end{aligned}$$

4.4

Please see Matlab file "question4.m" for code. Values are listed below as output.

$$\begin{aligned}
N = 10, \bar{x} &= .4612, \sigma_{\bar{x}} = 0.0813 \\
N = 100, \bar{x} &= .4732, \sigma_{\bar{x}} = 0.0295 \\
N = 1000, \bar{x} &= .5084, \sigma_{\bar{x}} = 0.0092 \\
N = 1000000, \bar{x} &= .5003, \sigma_{\bar{x}} = 2.8856E-4
\end{aligned}$$

Yes, these results agree with my expectation. I estimated around 834 calls would yield me an uncertainty that was below 1%, this seemed to be the case as $N = 1000$ had $\sigma_{\bar{x}}$ far below 1%.

Using this line of reasoning, we'd expect around the $N = 850$ to reach a $\sigma_{\bar{x}}$ of 1%.