Physics 375 - Lab Notebook 1

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Part A

Uncertainty

Possible Uncertainties

- Uncertainty in the angular vernier, we can calculate this uncertainty since we know the precision of the vernier, and it should be very small.
- This measurement is taken by eye, and therefore is subjective by nature and there is significant uncertainty in the point I decide total internal reflection is occurring. This should account for a large amount of uncertainty.
- The optical block may have imperfections on the surface and internally which impact the trajectory of the beam, this should not account for a large amount of uncertainty.
- Any measurement and experiment entirely set up by hand will be subject to imperfections in arrangements, this should account for a large amount of uncertainty.

T he two greatest sources of uncertainty are **the subjectivity of the measurement itself**, since it is taken by eye, and the **imperfections in the experimental setup**. In order to measure these two sources of uncertainties, repeated measurements will be necessary rotating clockwise and counter-clockwise. Since we know what results we are expecting, that θ_c should be the same rotating in either direction, we can evaluate our results vs that expected result and determine if either of these sources of uncertainty are having dramatic effects in our measurements. Since we have a relatively simple set up, we will reconstruct the experiment by placing the block and angular vernier at different distances from the beam to ensure we are getting consistent results.

S ince we are taking multiple measurements in order to minimize our error, we will avoid using the $\frac{1}{3}$ rule as an estimation of the error on each measurement and instead use our standard deviation as an estimate of our error. The standard error in this case is not too relevant, as we would rather over estimate our error than underestimate it, and given the context, std error does not make for too much of a useful metric in this case.

Data

LabOneDataPartA.xlsm

Clockwise Data		
Rotation	Degrees	σ
Clockwise	42.25	0.005
Clockwise	42.92	0.005
Clockwise	43.00	0.005

$$\bar{x} = 42.72°$$
$$\sigma_x = 0.41°$$
$$\sigma_{\bar{x}} = 0.24°$$

Index of refraction

$$n_1 = 1.5$$
$$\sigma_{n_1} = 0.38$$

Counter-Clockwise Data		
Rotation	Degrees	σ
Clockwise	43.25	0.005
Clockwise	43.08	0.005
Clockwise	43.00	0.005

$$\bar{x} = 43.11°$$
$$\sigma_x = 0.13°$$
$$\sigma_{\bar{x}} = 0.07°$$

Index of refraction

$$n_1 = 1.5$$
$$\sigma_{n_1} = 0.16$$

Analysis

Final value for n_1

$$n_1 = 1.5$$
$$\sigma_{n_1} = 0.25$$

T he values for n are in agreement strong agreement with each other. I do however, believe the uncertainty in my measurements to contribute to this conclusion more than anything else. Quantitatively, I stand behind my value for n_1, qualitatively I am not so sure.

Do you get the same value for θ_c

T he two values are very close, but they are not exactly the same.

Comments

I think I could have gone about this differently if I had the χ^2 goodness of fit test in mind from the beginning. I did not consider how I was going to compare my values of n vs theory and therefore I did not structure my data or analysis in a way that made that work. What I would have done differently: Take more measurements of θ_c, instead of minimizing σ_{θ_c}, use the $\frac{1}{3}$ rule to estimate (which is a reasonable estimate anyway), and then calculate n for each measurement of θ_c. This would have given me (hopefully) a distribution for n which I could take the difference against theory - which would say my value for $n_{clockwise}$ should be equal to $n_{counter\text{-}clockwise}$. In the end I just used a graph to measure their overlap within error bars to determine if the two values where in agreement (and used qualitative analysis for determine the strength of agreement), and then took the average between the n values and their σ to determine my final value.

P lease see attached spreadsheet LabOneDataPartA.xlsm for the data pertaining to these calculations. To reduce uncertainty in this part of the lab, we used multiple measurements, took the average and then took our uncertainty to be the uncertainty in the average. Our formulas are listed below in equations (3), (4) and (5). In excel, we converted our data to time measurements to allow our averages to come out correctly in terms of minutes and seconds. Since our degrees was not going to be impacted by this shortcut, we did thought it would be a great way to ensure accuracy in our calculations. We hope you agree...

Equations

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\sigma_{n_2} = \sqrt{(\cos\theta_c * \triangle\theta_c)^2 + (\sin\theta_c * \triangle n_1)^2} \tag{1}$$

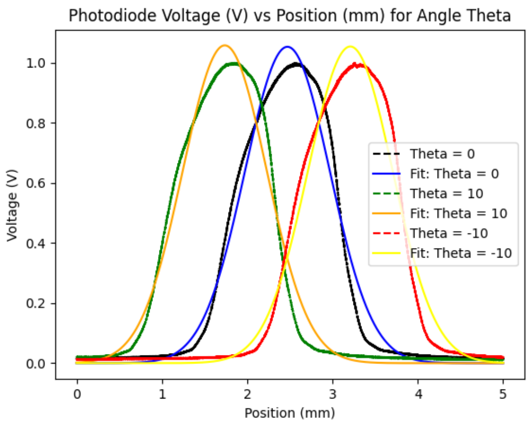
$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{2}$$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{3}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \tag{4}$$

Part B

I really never figured out exactly how to calculate the distance between the two. It made no sense to me. The reason is, I could either calculate the distance between the peaks? Or my final effort was to normalize and then fit the data, to give me a continuous distribution, which I could then loop over the voltage values and calculate the difference between the positions. At this point, I still had challenges. This part is ultimately what killed 5-7 hours of my time just trying to figure out a way to do this and I could never do it. Below is a graph of my normalized data with a fit overlayed. The only problem with this method, is I have no idea what the uncertainty would be in my d.



Graph with fit - final try to get a d.

Setup

Needed measurements:

- measurement of L (thickness of glass)
- measurement of θ_i
- measurement of n_1
- measurement of D

W hen setting up this experiment, the iris should not be incredible large. As we have seen in Lab 0 as well as listed above in our possible uncertainties, stray photons do impact our base voltage. The larger the iris, the more uncertainty there will be in our measurements. However, we do need to ensure our iris is just large enough to make a meaningful measurement. If the iris is not large enough to measure the entire diameter of the beam, we will only see a plateau of intensity as opposed to a distinguished peak as the beam moves across the photodiode.

W hen setting up this experiment, the photodiode gain should be set to a value which makes intensity of the beam easy to pick up and read. Obviously, the gain should not compromise your measurements by being to large and therefore saturating your measurements. It should only work to amplify what is already there for sake of making your data easier to work with.

Uncertainty

Systematic

- photodiode offset voltage
- imperfections in glass

Random

- motorized stepper distance
- apparatus configuration errors, human errors
- stray photons entering hitting photodiode sensor (effects of non-isolated system)

Systematic

Random

- error θ_i
- error in L
- error in measurement for D

W e are not repeating the same measurement multiple times, therefore we will use propagation of errors to find our uncertainty.

Data

Please see data final directory, it is the most organized.

Analysis

Equations

$$n_2 = \sin\theta_i \sqrt{[1 + \frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]} \tag{5}$$

$$\sigma_{n_2} = \sqrt{(\frac{\partial}{\partial\theta_i} * \sigma_{\theta_i})^2 + (\frac{\partial}{\partial d} * \sigma_d)^2 + (\frac{\partial}{\partial L} * \sigma_L)^2} \tag{6}$$

Measurements

$$\begin{aligned} L &= length \times width \times height \\ &= 3.1in \times 2.1in \times 0.5in \end{aligned} \tag{7}$$

$$\sigma_L = 0.03in \tag{8}$$

$$\sigma_{\theta_i} = 0.1minutes = 0.005^\circ \tag{9}$$

$$\frac{\partial}{\partial\theta_i} = \sqrt{2}(d - L\sin\theta_i)^3 \sqrt{\frac{-d^2 + 2dL\sin\theta_i - L^2}{-2d^2 + 4dL\sin\theta + L^2\cos 2\theta_i - L^2}} \tag{10}$$

B oth σ_L and σ_{θ_i} were calculated by using the $\frac{1}{3}$ rule. The reason is, we did not find different values for L or θ_i upon repeated measurements. This does not mean that there is not uncertainty, so we used the $\frac{1}{3}$ rule to estimate it. We used the most precise value of the measuring device and divided it by 3.

Part B.1

Figure 1: Voltage vs Position Graph - Part B.1

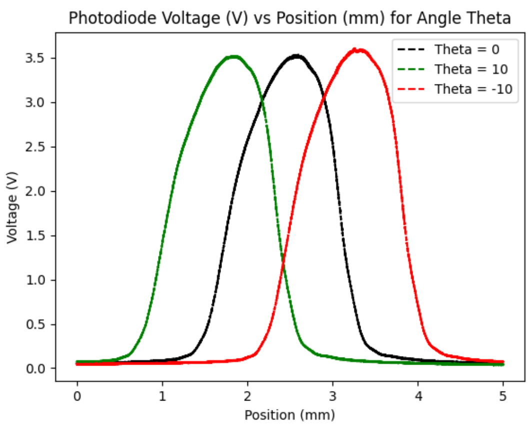
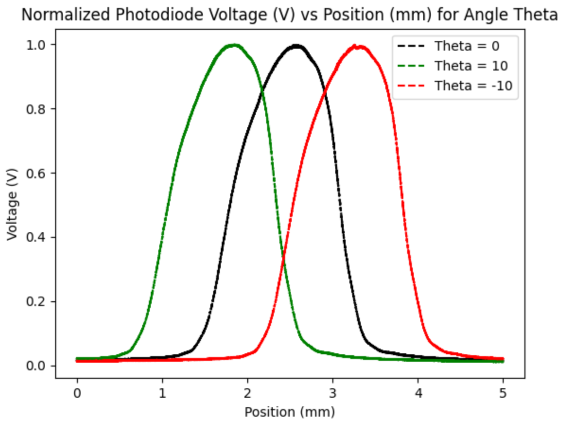


Figure 2: Normalized Voltage vs Position Graph - Part B.1



Clockwise

T o calculate d, I had a lot of trouble because each distribution was not exactly the same. Therefore I had to find specific points on each distribution and compare the position at those specific voltages. Since there was an issue with the peak voltage occurring at different heights, I normalized the distributions. When I looped through the distributions looking for similar values, I found that the only value I could compare was the peak. Therefore, for our d, I only calculated the distance between the peaks.

$$d = 0.1496mm \text{ or } 0.0001496m$$
$$L = 0.5in \text{ since our glass is vertical}$$

$$\begin{aligned} n_2 &= \sin\theta_i \sqrt{[1 + \frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]} \\ &= \sin 10 \sqrt{[1 + \frac{\cos^2 10}{(\sin 10 - \frac{0.0001496}{0.5})^2}]} \\ &= 0.1736 \sqrt{[1 + \frac{\cos^2 10}{(\sin 10 - \frac{0.0001496}{0.5})^2}]} \end{aligned} \tag{11}$$

Counter-Clockwise

$$d = 0.132mm \text{ or } 0.000132m$$
$$L = 0.5in \text{ since our glass is vertical}$$

$$
\begin{aligned}
n_2 &= \sin\theta_i \sqrt{[1 + \frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]} \\
&= \sin 10 \sqrt{[1 + \frac{\cos^2 10}{(\sin 10 - \frac{0.000132}{0.5})^2}]} \\
&= 0.1736 \sqrt{[1 + \frac{\cos^2 10}{(\sin 10 - \frac{0.000132}{0.5})^2}]}
\end{aligned}
\tag{12}
$$

Part B.2

Figure 3: Voltage vs Position Graph - Part B.2

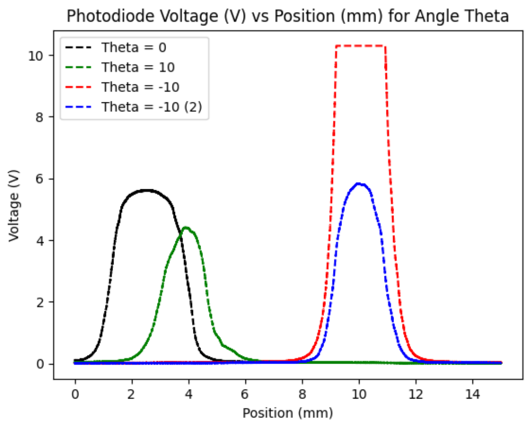
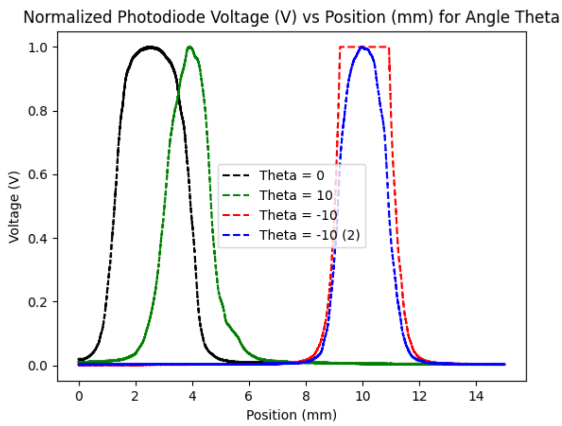


Figure 4: Normalized Voltage vs Position Graph - Part B.2



Are indices of refraction consistent? Which has the smaller uncertainty and why?

Part B.3

Figure 5: Voltage vs Position Graph - Part B.3

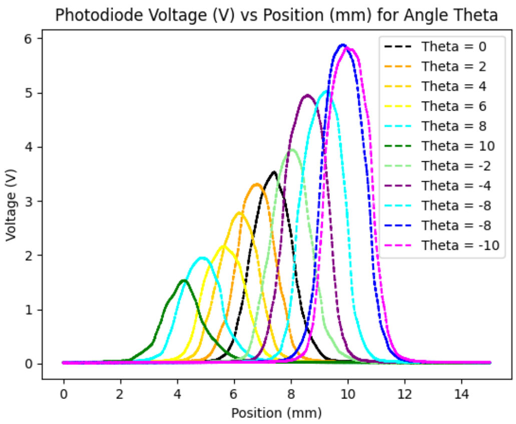
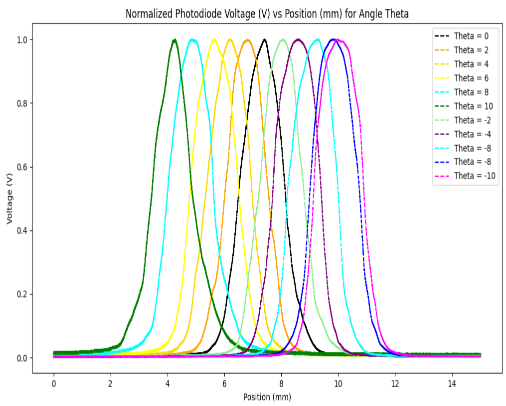


Figure 6: Normalized Voltage vs Position Graph - Part B.3



Endnotes

I will mention in this lab I had trouble with the photodiode gain. As you will see in my data, my intensity measurements were linearly increasing with time, which did not make too much sense. Therefore, as I adjusted the incident angle in part B, I found that my peak voltage readings were different. For this particular experiment, this did not jeopardize my ability to make conclusions from the data, however I was certainly disturbed by this data and I hesitated to adjust the photodiode gain for each angle as I did not want to add an additional variable (and therefore uncertainty) to my measurements.

Appendix

0.1 Key equations

Reflection

$$\theta_i = \theta_r \tag{13}$$

$$\text{Rays in same plane with normal}$$

$$\vec{r}(x, y, z) \rightarrow \vec{r}(x, y, -z)$$

Refraction

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_t}{v_t} \tag{14}$$

Least time

$$\frac{\partial t}{\partial x} = 0 \tag{15}$$

Snell's Law

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{16}$$

$$n = \tfrac{c}{v}$$

Total Internal Reflection

$$n_1 \sin\theta_c = n_2 \tag{17}$$

0.2 Notes

Everything is measured with respect to the normal of the surface

0.3 Bad intensity

0.4 Uncertainty Propagation

Part A

$$\begin{aligned}
\sigma_{n_1} &= \sqrt{\left(\frac{\partial}{\partial\theta_c} * \sigma_{\theta_c}\right)^2 + \left(\frac{\partial}{\partial n_2} * \sigma_{n_2}\right)^2} \\
n_2 \textit{ is air, therefore } \sigma_{n_2} = 0 & \\
&= \sqrt{\left(\frac{\partial}{\partial\theta_c} * \sigma_{\theta_c}\right)^2} \\
&= \left|\frac{\partial}{\partial\theta_c} * \sigma_{\theta_c}\right|
\end{aligned} \tag{18}$$

Partial with respect to θ_c

$$\begin{aligned}
\frac{\partial}{\partial\theta_c}\frac{n_2}{\sin(\theta_c)} &= n_2 \frac{\partial}{\partial\theta_c} \csc\theta_c \\
\frac{\partial}{\partial\theta_c} &= -n_2 \cot(\theta_c) \csc(\theta_c) \\
n_2 \textit{ is air} & \\
\frac{\partial}{\partial\theta_c} &= -\cot(\theta_c) \csc(\theta_c)
\end{aligned} \tag{19}$$

Part B and Part C

Calculation for: $\frac{\partial}{\partial \theta_i}$

$$
\begin{aligned}
\frac{\partial}{\partial \theta_i} &= \frac{\partial}{\partial \theta_i}\left(\sin\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}\right) \\
&= \left(\cos\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}\right) + \left(\sin\theta_i \frac{\partial}{\partial \theta_i} \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}\right) \\
&= \left(\cos\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}\right) + \left(\sin\theta_i \frac{1}{2 * \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} * (-\frac{\sin(2\theta_i)}{(\sin\theta_i - \frac{d}{L})^2} + \cos^2(\theta_i) * \frac{\partial}{\partial \theta_i}(\sin\theta_i - \frac{d}{L})^{-2})\right) \\
&= \left(\cos\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}\right) + \left(\sin\theta_i \frac{1}{2 * \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} * (-\frac{\sin(2\theta_i)}{(\sin\theta_i - \frac{d}{L})^2} + \cos^2(\theta_i) * \frac{(-2)}{(\sin\theta_i - \frac{d}{L})^3}\cos\theta_i)\right)
\end{aligned} \tag{20}
$$

Calculation for: $\frac{\partial}{\partial d}$

$$
\begin{aligned}
\frac{\partial}{\partial d} &= \frac{\partial}{\partial d}[\sin\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}] \\
&= \sin\theta_i \frac{\partial}{\partial d}([1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}])^{\frac{1}{2}} \\
&= \sin\theta_i \frac{1}{2 * \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} \cos^2\theta_i * \frac{\partial}{\partial d}(\sin\theta_i - \frac{d}{L})^{-2} \\
&= \sin\theta_i \frac{1}{2 * \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} \cos^2\theta_i \frac{(-2)}{(\sin\theta_i - \frac{d}{L})^3} \frac{\partial}{\partial d}(\sin\theta_i - \frac{d}{L}) \\
&= \sin\theta_i \frac{1}{\sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} \cos^2\theta_i \frac{(-1)}{(\sin\theta_i - \frac{d}{L})^3} \frac{1}{L}
\end{aligned} \tag{21}
$$

Calculation for: $\frac{\partial}{\partial L}$

$$
\begin{aligned}
\frac{\partial}{\partial L} &= \sin\theta_i \sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]} \\
&= \sin\theta_i \frac{1}{\sqrt{[1+\frac{\cos^2\theta_i}{(\sin\theta_i - \frac{d}{L})^2}]}} \cos^2\theta_i \frac{(-1)}{(\sin\theta_i - \frac{d}{L})^3} d
\end{aligned} \tag{22}
$$