Physics 401 - Homework 1

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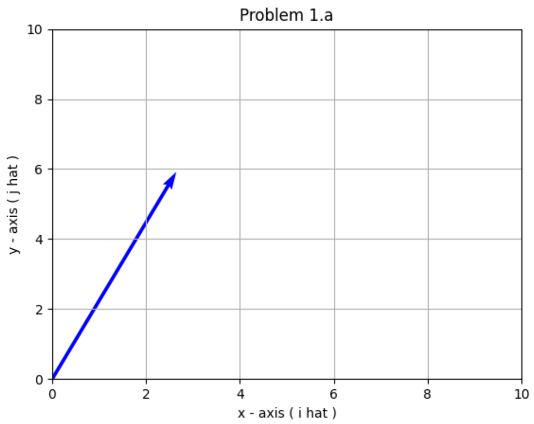
1 Problem 1

1.a



1.b

$$\vec{v} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} \tag{1}$$

1.c

$$M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

1.d

$$m = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3}$$

2 Problem 2

$$A = \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{bmatrix}$$

2.a

2.a.1 $A \cdot B$

$$\begin{aligned} A \cdot B &= \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (0 \cdot 2) + (-2i \cdot 0) & (0 \cdot 0) + (-2i \cdot 1) \\ (2i \cdot 2) + (0 \cdot 0) & (0 \cdot 2i) + (1 \cdot 0) \end{bmatrix} \\ &= \begin{bmatrix} 0 + 0 & 0 + -2i \\ 4i + 0 & 0 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -2i \\ 4i & 0 \end{bmatrix} \end{aligned} \tag{4}$$

2.a.2 $B \cdot A$

$$\begin{aligned} B \cdot A &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix} \\ &= \begin{bmatrix} (0 \cdot 2) + (2i \cdot 0) & (2 \cdot -2i) + (0 \cdot 0) \\ (0 \cdot 0) + (1 \cdot 2i) & (0 \cdot -2i) + (1 \cdot 0) \end{bmatrix} \\ &= \begin{bmatrix} 0 + 0 & -4i + 0 \\ 0 + 2i & 0 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -4i \\ 2i & 0 \end{bmatrix} \end{aligned} \tag{5}$$

2.a.3 $A \cdot B = B \cdot A$ **?**

$$\begin{bmatrix} 0 & -4i \\ 2i & 0 \end{bmatrix} ! = \begin{bmatrix} 0 & -2i \\ 4i & 0 \end{bmatrix}$$

$A \cdot B$ does not equal $B \cdot A$

2.b $A \cdot C$

$$\begin{aligned} A \cdot C &= \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{bmatrix} \\ &= \begin{bmatrix} (0 \cdot \frac{4}{5}) + (-2i \cdot \frac{-3}{5}) \\ (2i \cdot \frac{4}{5}) + (0 \cdot \frac{-3}{5}) \end{bmatrix} \\ &= \begin{bmatrix} 0 + (\frac{-6i}{5}) \\ (\frac{8i}{5}) + 0 \end{bmatrix} \\ &= \begin{bmatrix} (\frac{-6i}{5}) \\ (\frac{8i}{5}) \end{bmatrix} \end{aligned} \tag{6}$$

2.c Eigenvalues of A

$$A = \begin{bmatrix} -\lambda & -2i \\ 2i & -\lambda \end{bmatrix}$$

Take determinate

$$
\begin{aligned}
&\begin{vmatrix} -\lambda & -2i \\ 2i & -\lambda \end{vmatrix} = 0 \\
&(\lambda^2 - (-4(i^2)) = 0 \\
&(\lambda^2 - 4) = 0 \\
&\qquad \lambda = \pm 2
\end{aligned} \tag{7}
$$

2.d

You cannot because it violates the mathematical properties of matrix operations. The number of columns in the object on the left must be equal to the number of rows of the object on the right for you to be able to carry out the multiplication. Further - a vector does not operate on a matrix, that makes no sense.

3 Problem 3

$$
e^{i\theta} = \cos\theta + i\sin\theta
$$
$$
z = Ae^{i\theta} = \cos\theta + i\sin\theta
$$

3.a

$$
\begin{aligned}
Re(z) &= \cos\theta \\
e^{i\theta} &= \cos\theta + i\sin\theta \\
e^{-i\theta} &= \cos\theta - i\sin\theta
\end{aligned}
$$

You have a system of equations. Since $Re(z) = \cos\theta$, we are looking to solve for $\cos\theta$. Therefore we will add the equations and cancel the imaginary sin terms and use algebra to solve for cos.

$$
\begin{aligned}
e^{i\theta} &= \cos\theta + i\sin\theta \\
+(e^{-i\theta} &= \cos\theta - i\sin\theta) \\
e^{i\theta} + e^{-i\theta} &= 2\cos\theta \\
\frac{e^{i\theta} + e^{-i\theta}}{2} &= \cos\theta \\
Re(z) = \cos\theta &= \frac{e^{i\theta} + e^{-i\theta}}{2}
\end{aligned} \tag{8}
$$

3.b

$$
\begin{aligned}
Im(z) &= i\sin\theta \\
e^{i\theta} &= \cos\theta + i\sin\theta \\
e^{-i\theta} &= \cos\theta - i\sin\theta
\end{aligned}
$$

We will use the same method here with our system of equations except this time we will be solving for $i\sin\theta$.

$$
\begin{aligned}
e^{i\theta} &= \cos\theta + i\sin\theta \\
-(e^{-i\theta} &= \cos\theta - i\sin\theta) \\
e^{i\theta} - e^{-i\theta} &= 2i\sin\theta \\
\frac{e^{i\theta} - e^{-i\theta}}{2} &= i\sin\theta \\
Im(z) = i\sin\theta &= \frac{e^{i\theta} - e^{-i\theta}}{2} \qquad (9)
\end{aligned}
$$

3.c

$$
\begin{gathered}
z_1 = x_1 + iy_1 \\
z_2 = x_2 + iy_2
\end{gathered}
$$

$$
\begin{aligned}
z_1z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\
&= x_1x_2 + x_1iy_2 + iy_1x_2 + i^2y_1y_2
\end{aligned} \qquad (10)
$$

For part c and part d I will use the equation above (equation 7) to deduce the correct answer.

$$Re(z_1z_2) = x_1x_2 - y_1y_2 \qquad (11)$$

3.d

$$Im(z_1z_2) = x_1y_2 + y_1x_2 \qquad (12)$$

3.e

$$
\begin{gathered}
z = x + iy \\
z^* = x - iy
\end{gathered}
$$

$$
\begin{aligned}
z^2 &= (x + iy)(x + iy) \\
&= x^2 + 2ixy - y^2
\end{aligned} \qquad (13)
$$

$$
\begin{aligned}
|z|^2 &= (x + iy)(x - iy) \\
&= x^2 - y^2
\end{aligned} \qquad (14)
$$

$$
\begin{aligned}
Re(z^2) &= x^2 + 2ixy - y^2 \\
&= x^2 - y^2 \\
Re(|z|^2) &= x^2 - y^2 = Re(z^2) \qquad (15)
\end{aligned}
$$

Equations 10 and 11 prove that $z^2 \neq |z|^2$, equation 12 proves that $Re(|z|^2) = Re(z^2)$.

3.f

$$
\begin{aligned}
z = Ae^{i\theta} &= A(\cos\theta + i\sin\theta) \\
Re(z) &= A\cos\theta \qquad (16) \\
Im(z) &= A\sin\theta \qquad (17) \\
z^* &= A(\cos\theta - i\sin\theta) \qquad (18) \\
|z| &= A(\sqrt{\cos^2\theta + \sin^2\theta}) = A \qquad (19)
\end{aligned}
$$

3.g

$$\begin{aligned} e^{i(\alpha+\beta)} &= \cos(\alpha+\beta) + i\sin(\alpha+\beta) \\ e^{-i(\alpha+\beta)} &= \cos(\alpha+\beta) - i\sin(\alpha+\beta) \end{aligned}$$

$$\begin{aligned} e^{i\alpha}e^{i\beta} &= (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) \\ &= (\cos\alpha\cos\beta) + (\cos\alpha i\sin\alpha) + (i\sin\alpha\cos\beta) + (i\sin\alpha i\sin\beta) \\ &= (\cos\alpha\cos\beta) + (\cos\alpha i\sin\alpha) + (i\sin\alpha\cos\beta) - (\sin\alpha\sin\beta) \end{aligned} \tag{20}$$

$$\begin{aligned} e^{-i\alpha}e^{-i\beta} &= (\cos\alpha - i\sin\alpha)(\cos\beta - i\sin\beta) \\ &= (\cos\alpha\cos\beta) - (\cos\alpha i\sin\alpha) - (i\sin\alpha\cos\beta) + (i\sin\alpha i\sin\beta) \\ &= (\cos\alpha\cos\beta) - (\cos\alpha i\sin\alpha) - (i\sin\alpha\cos\beta) - (\sin\alpha\sin\beta) \end{aligned} \tag{21}$$

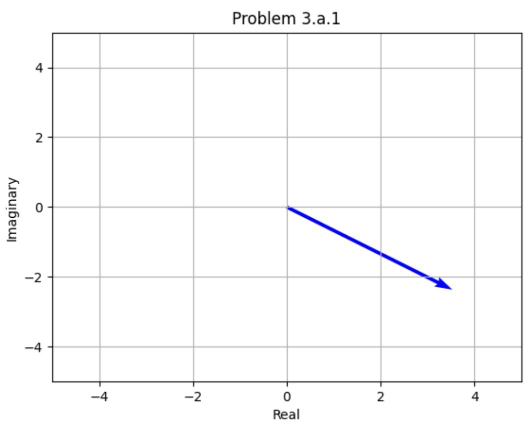
I stared at the equations above for about an hour, and looking at the proofs online they seemed to mainly prove geometrically, which I was hoping for something more elegant. I know I am missing a way of looking at this so it would be great if you could leave a note about how to prove 3.g. I have listed the answers below.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \tag{22}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \tag{23}$$

3.h

3.h.1



3.h.2

$$\begin{aligned} A = |z| &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16+4} \\ &= 2\sqrt{5} \end{aligned} \tag{24}$$

$$
\begin{aligned}
A = |z| &= \sqrt{4^2 + 2^2} \\
&= \sqrt{16 + 4} \\
&= 2\sqrt{5}
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
\theta &= \tan^{-1} \frac{-2}{4} \\
&= -26.565
\end{aligned} \tag{26}
$$

$$
z = 2\sqrt{5}e^{i(-26.565)} \tag{27}
$$

3.i

$$
z = ie^{-i\frac{3\pi}{4}}
$$

3.i.1

$$
\begin{aligned}
z &= i \cos \frac{3\pi}{4} - i^2 \sin \frac{3\pi}{4} \\
&= i \cos \frac{3\pi}{4} + \sin \frac{3\pi}{4} \\
&= \sin \frac{3\pi}{4} + i \cos \frac{3\pi}{4}
\end{aligned} \tag{28}
$$

cos and sin are orthogonal:

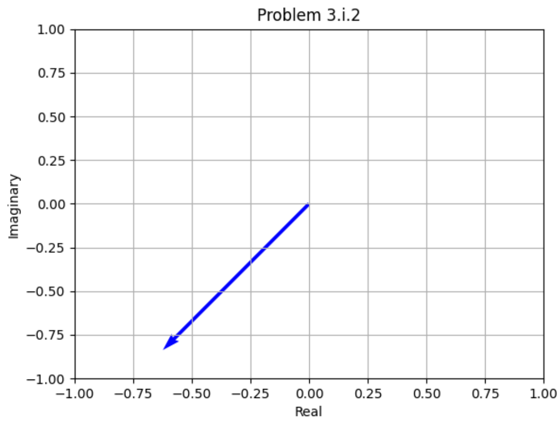
$$
\begin{bmatrix} \cos \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \sin \end{bmatrix} = (\cos \cdot 0) + (0 \cdot \sin) = 0 \tag{29}
$$

Therefore we know the phase angle between cosine and sin is $\frac{\pi}{2}$, and we can shift any cosine or sin term by $\frac{\pi}{2}$ to get it in terms of the other. Continuing from equation 22...

$$
\begin{aligned}
z &= \sin (\frac{3\pi}{4}) + (\frac{\pi}{2}) + i \cos (\frac{3\pi}{4}) + (\frac{\pi}{2}) \\
&= \cos (\frac{3\pi}{4}) + (\frac{2\pi}{4}) + i \sin (\frac{3\pi}{4}) + (\frac{2\pi}{4}) \\
&= \cos (\frac{5\pi}{4}) + i \sin (\frac{5\pi}{4}) \\
z &= e^{i\frac{5\pi}{4}}
\end{aligned} \tag{30}
$$

$A = 1$ and $\theta = \frac{5\pi}{4}$

3.i.2



4 Problem 4

This should be a relatively easy calculation since each toss is independent and has the same probability of being heads or tails. There is 2^8 different outcomes, each one with a likelihood $\frac{1}{2^8}$ of occurring. Therefore there is a $\frac{1}{256}$ probability of this set of outcomes occurring.

5 Problem 5

$$E_n = \frac{-m_e e^4}{32\pi^2\epsilon_0^2\hbar^2 n^2} = -13.6eV(\frac{1}{n^2})$$

Homework states ϵ_0^4, the correct value is ϵ_0^2

$$c = \lambda\nu$$
$$\nu = \frac{c}{\lambda}$$
$$E = \hbar\nu$$
$$E = \hbar\frac{c}{\lambda}$$
$$\lambda = \frac{\hbar c}{E}$$

Since the Bohr model uses an infinite mass assumption given the mass of the proton vs that of the election, and the positron does not have proton but rather a positron at its center, our equation for E_n is altered to use the reduced mass:

$$\begin{aligned}\mu &= \frac{m_e m_e}{m_e + m_e} \\ &= \frac{m_e^2}{2m_e} \\ &= \frac{m_e}{2}\end{aligned} \tag{31}$$

Since the positron has the same mass as the electron, we can calculate the reduced mass by using the mass of the electron twice.

$$\begin{aligned}
E_n &= -\mu \frac{e^4}{32\pi^2\epsilon_0^2\hbar^2n^2} \\
&= -\frac{m_e}{2}\frac{e^4}{32\pi^2\epsilon_0^2\hbar^2n^2} \\
&= \frac{-1}{2}\frac{m_e e^4}{4\epsilon_0^2h^2n^2} \\
&= \frac{-1}{2n^2}\frac{m_e e^4}{4\epsilon_0^2h^2} \\
&= \frac{-1}{2n^2}\frac{m_e e^4}{4\epsilon_0^2h^2}
\end{aligned} \tag{32}$$

$$\epsilon_0 = e^2eV^(-1) \tag{33}$$

$$m_e = 1.022 * 10^3 eV$$
$$h = 4.135 * 10^{-15} eVs$$
$$\epsilon_0 = 8.85 * 10^{-12}\frac{F}{m}$$
$$F = \frac{s^4e^2}{kgm^2}$$

$$\begin{aligned}
E_n &= \frac{(1.022 * 10^3 eV)(e^4)}{4(4.135^2 * 10^{(-15*2)}eV^2s^2)(8.85^2 * 10^{-12*2}\frac{s^8e^4}{kg^2m^6})} \\
&= \frac{(1.022 * 10^3)}{4(4.135^2 * 10^{(-15*2)}eVs^2)(8.85^2 * 10^{-12*2}\frac{s^8}{kg^2m^6})} \\
&= \frac{(1.022 * 10^3)}{4(4.135^2 * 10^{(-30)}eVs^2)(8.85^2 * 10^{-24}\frac{s^8}{kg^2m^6})} \\
&= \frac{(1.022 * 10^{57})}{4(4.135^2 eV)(8.85^2\frac{s^{10}}{kg^2m^6})}
\end{aligned} \tag{34}$$

I tried to do dimensional analysis like 5 times on problem above, I know answer and will list below but just wanted to simplify that equation. Clearly there is something wrong in my calculation, if you could spot it and let me know that would be great. Thank you

$$E_n = \frac{-6.8eV}{n^2}$$

$$E_1 = -6.8eV,\ E_2 = \frac{-6.8eV}{4}$$

$$
\begin{aligned}
E_2 - E_1 &= \frac{-6.8eV}{4} - -6.8eV \\
&= \frac{-6.8eV}{4} + 6.8eV \\
&= \frac{-5.1eV}{4} \\
\lambda &= \hbar c(\frac{-5.1eV}{4})^{-1} \\
&= \hbar c(\frac{4}{-5.1kg\frac{m^2}{s^2}}) \\
&= (1.054 * 10^{-34}kg\frac{m^2}{s})(3 * 10^8\frac{m}{s})(\frac{4}{-5.1kg\frac{m^2}{s^2}}) \\
&= (1.054 * 10^{-26}\frac{m^2}{s})(\frac{m}{s})(\frac{12}{-5.1\frac{m^2}{s^2}}) \\
&= (10^{-26}\frac{m^3}{s^2})(\frac{12}{-5.1\frac{m^2}{s^2}}) \\
\lambda &= (\frac{12}{-5.1})(10^{-26}m)
\end{aligned}
\tag{35}
$$