Physics 401 - Homework 3

Dillon Walton

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1 Problem 1

The first important aspect to note in this explanation is that we know nothing about the spin in the y-orientation before measurement. The number of and orientation of the analyzers that come before our first y analyzer is completely irrelevant to the probabilities of our measurements in y. We have a two state system, spin-up or spin-down, and regardless of the number of and orientation of the analyzers before our first y-analyzer, we will be starting from a state of either 100% spin-up or 100% spin-down in the orientation of the previous analyzer. From experiment, we measure the spin in the y-orientation and find we have a distribution of 50% spin-up and 50% spin-down. From our experimental results, we can deduce our coefficients of the $\left|+\right\rangle_y$ and $\left|-\right\rangle_y$.

$$
\begin{aligned}
P_{1,+y} &= |_y \langle +|+\rangle |^2 = \frac{1}{2} \\
P_{1,-y} &= |_y \langle -|+\rangle |^2 = \frac{1}{2} \\
P_{2,+y} &= |_y \langle +|-\rangle |^2 = \frac{1}{2} \\
P_{2,-y} &= |_y \langle -|-\rangle |^2 = \frac{1}{2}
\end{aligned}
\tag{1}
$$

We know the above relations must be true from experiment. Prior to measurement, there is nothing we could say about the y-orientation. From here we can work to find the coefficients for our state vectors in the y-basis which we know will be similar but not the same to equation 1.35:

$$
\begin{aligned}
\left|+\right\rangle_y &= \frac{1}{\sqrt{2}}[\left|+\right\rangle + e^{i\alpha}\left|-\right\rangle] \\
\left|-\right\rangle_y &= \frac{1}{\sqrt{2}}[\left|+\right\rangle - e^{i\beta}\left|-\right\rangle]
\end{aligned}
\tag{2}
$$

It is important to note here that everything above would be true as well if the previous analyzer was in the x-orientation as well. It is also important to note that each coefficient is complex, and therefore we can describe it with an amplitude and phase (using Euler's). Piggy backing more off McIntyre's logic (which I happen to be in full agreement with), the phase of the state vector has no meaning on the measurement and therefore we can use convention by choosing the $\left|+\right\rangle$ to be positive and real without impacting our outcome.

We cannot make the same assumption about choosing our phase to be 0 as we did in the x case. Why? Because we already chose that phase, and if we chose the same phase for x and y we would have no way to distinguish them when we translated from z.

We can make the assumption about $|_y \langle +|+\rangle_x|^2$ and $|_y \langle -|+\rangle_x|^2$ from experiments. Using these assumptions we can figure out what the phase is for our $|-\rangle$ coefficient.

$$
\begin{aligned}
{}_y\langle +|+\rangle_x &= \frac{1}{\sqrt{2}}(1 e^{-i\alpha})\frac{1}{\sqrt{2}}(11) \\
&= \frac{1}{2}(1+e^{-i\alpha})
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
|_y\langle +|+\rangle_x|^2 &= \frac{1}{\sqrt{2}}(1 e^{-i\alpha})\frac{1}{\sqrt{2}}(11) \\
&= \frac{1}{4}(1+e^{-i\alpha})\frac{1}{2}(1+e^{i\alpha}) \\
&= \frac{1}{2}(1+\cos\alpha) = \frac{1}{2}
\end{aligned} \tag{4}
$$

From this result we know α must be $\pm\frac{\pi}{2}$ and we can deduce our final equations below.

$$
\begin{aligned}
|+\rangle_y &= \frac{1}{\sqrt{2}}|+\rangle + \frac{i}{\sqrt{2}}|-\rangle \\
|-\rangle_y &= \frac{1}{\sqrt{2}}|+\rangle - \frac{i}{\sqrt{2}}|-\rangle
\end{aligned} \tag{5}
$$

2 Problem 2

2.1

Since this is a spin-1/2 system, the possible measurements are $S_z = \pm\frac{\hbar}{2}$.

$$P_+ = |\frac{3}{\sqrt{34}}|^2 = \frac{9}{34} \approx 26.5\% \tag{6}$$

$$P_- = |\frac{5}{\sqrt{34}}|^2 = \frac{25}{34} \approx 73.5\% \tag{7}$$

2.2

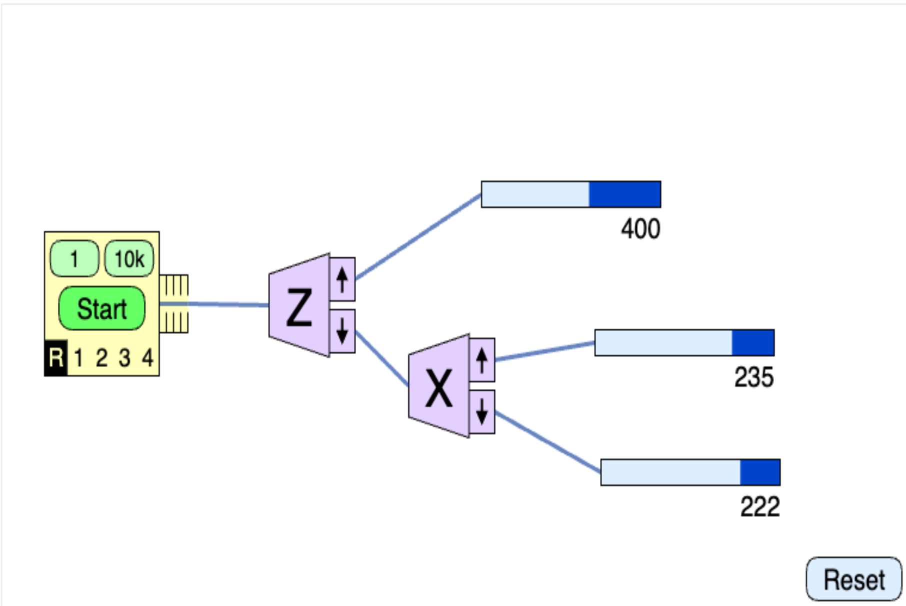
Since this is a spin-1/2 system, the possible measurements are $S_x = \pm\frac{\hbar}{2}$.

We will use equation 1.36 of McIntyre to solve this problem.

$$
\begin{aligned}
P_{+,x} &= |_x \langle +|-\rangle|^2 \\
&= |\left\langle \frac{1}{\sqrt{2}}[\langle +| + \langle -|] \middle| - \right\rangle|^2 \\
&= |(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \cdot [0, 1]|^2 \\
&= |\frac{1}{\sqrt{2}}|^2 \\
&= \frac{1}{2}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
P_{-,x} &= |_x \langle -|-\rangle|^2 \\
&= |\left\langle \frac{1}{\sqrt{2}}[\langle +| - \langle -|] \middle| - \right\rangle|^2 \\
&= |(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \cdot [0, 1]|^2 \\
&= | - \frac{1}{\sqrt{2}}|^2 \\
&= \frac{1}{2}
\end{aligned} \tag{9}
$$

2.3



3 Problem 3

3.1

Since this is a spin-1/2 system, the possible measurements are $S_x = \pm\frac{\hbar}{2}$.

$$P_{x_+} = |\frac{3}{5}|^2 = \frac{9}{25} = 36\% \tag{10}$$

$$P_{x_-} = |\frac{4}{5}|^2 = \frac{16}{25} = 64\% \tag{11}$$

3.2

Since this is a spin-1/2 system, the possible measurements are $S_z = \pm\frac{\hbar}{2}$.

Using McIntyre 1.37

$$\begin{aligned} P_+ &= |\left\langle + \middle| \psi \right\rangle_x|^2 \\ &= |\left\langle \frac{1}{\sqrt{2}}[\langle+| + \langle-|] \middle| \psi \right\rangle_x|^2 \\ &= |(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \cdot [\frac{3}{5}, -\frac{4}{5}]|^2 \\ &= |\frac{3}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}|^2 \\ &= |-\frac{1}{5\sqrt{2}}|^2 \\ &= \frac{1}{50} \end{aligned} \tag{12}$$

$$\begin{aligned} P_- &= |\left\langle - \middle| \psi \right\rangle_x|^2 \\ &= |\left\langle \frac{1}{\sqrt{2}}[\langle+| - \langle-|] \middle| \psi \right\rangle_x|^2 \\ &= |(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \cdot [\frac{3}{5}, -\frac{4}{5}]|^2 \\ &= |\frac{3}{5\sqrt{2}} + \frac{4}{5\sqrt{2}}|^2 \\ &= |\frac{7}{5\sqrt{2}}|^2 \\ &= \frac{49}{50} \end{aligned} \tag{13}$$

4 Problem 4

4.1

4.1.1 Orthogonality

$$\begin{aligned} \langle a_1|a_2\rangle &= 0 \\ \langle a_1|a_3\rangle &= 0 \\ \langle a_2|a_3\rangle &= 0 \end{aligned} \tag{14}$$

4.1.2 Normalization

$$
\begin{aligned}
\langle a_1|a_1\rangle &= 1 \\
\langle a_2|a_2\rangle &= 1 \\
\langle a_3|a_3\rangle &= 1
\end{aligned}
\tag{15}
$$

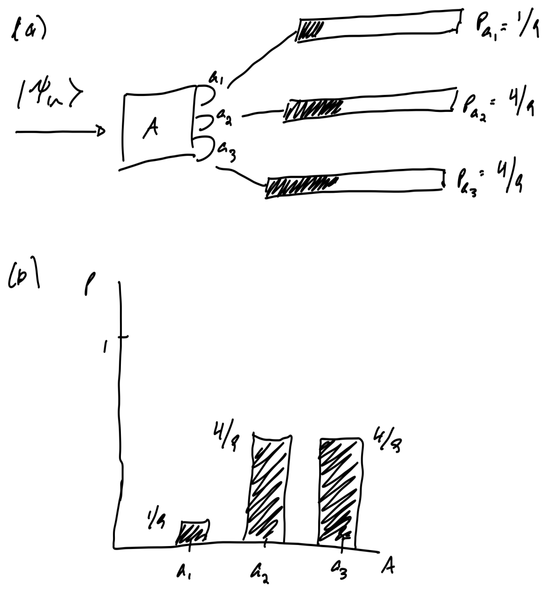
4.1.3 Completeness

$$|\psi\rangle = c_1\,|a_1\rangle + c_2\,|a_2\rangle + c_3\,|a_3\rangle \tag{16}$$

4.2

$$
\begin{aligned}
1 &= |C|^2(|1|^2 + |-2i|^2 + |2|^2) \\
1 &= |C|^2(1+4+4) \\
1 &= |C|^2 9 \\
\frac{1}{9} &= |C|^2 \\
C &= \frac{1}{3}
\end{aligned}
\tag{17}
$$

4.3



4.4

First we will see if the state is normalized. Since to be measurable, the operator that produces these measurements has to be hermitian and therefore, we know hermitian matrices have orthogonal basis. Therefore we know automatically, if we have measurable results, the basis vectors are orthogonal.

$$
\begin{aligned}
1 &= (\frac{1}{\sqrt{39}})^2(|3|^2 + |-i|^2 + |2e^{\frac{i\pi}{7}}| + |5|^2) \\
&= \frac{1}{39}(9 + 1 + (2e^{\frac{i\pi}{7}})(2e^{\frac{-i\pi}{7}}) + 25) \\
&= \frac{1}{39} * 39 \\
&= 1
\end{aligned} \tag{18}
$$

4.4.1 2 eV

$$\begin{aligned} |\langle 2eV|\psi\rangle|^2 &= (\frac{3}{\sqrt{39}}) \\ &= \frac{9}{39} \end{aligned} \tag{19}$$

4.4.2 4 eV

$$\begin{aligned} |\langle 4eV|\psi\rangle|^2 &= (\frac{-i}{\sqrt{39}}) \\ &= \frac{1}{39} \end{aligned} \tag{20}$$

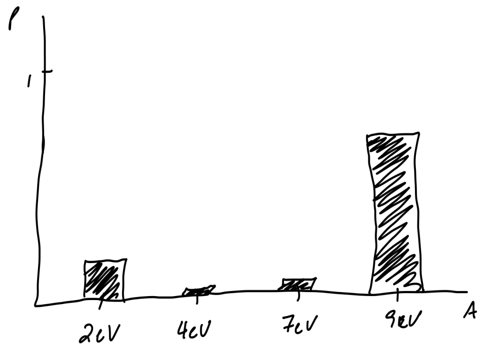
4.4.3 7 eV

$$\begin{aligned} |\langle 7eV|\psi\rangle|^2 &= (\frac{2e^{\frac{i\pi}{7}}}{\sqrt{39}}) \\ &= \frac{4}{39} \end{aligned} \tag{21}$$

4.4.4 9 eV

$$\begin{aligned} |\langle 9eV|\psi\rangle|^2 &= (\frac{5}{\sqrt{39}}) \\ &= \frac{25}{39} \end{aligned} \tag{22}$$

4.4.5 Histogram



5 Problem 5

5.1

5.1.1 Estimation

For an estimate, I took the count of the first analyzer and multiplied it by two.

$$count = 29883 * 2 = 59766 \tag{23}$$

5.1.2 Calculation

For the calculation, I added up the counts on all the analyzers.

$$count = 29883 + 15070 + 7473 + 7574 = 60000 \tag{24}$$

5.2

$$|\psi\rangle_a = (1)\,|+\rangle + (0)\,|-\rangle \tag{25}$$

$$|\psi\rangle_d = (0)\,|+\rangle + (1)\,|-\rangle \tag{26}$$

For $|\psi_b\rangle$ and $|\psi_c\rangle$, we will use an interesting method since I did not feel comfortable doing direct translation...

5.2.1 ψ_b

$$\begin{aligned} P_{+,z} &= |a|^2 \\ &= |\,\langle +|+\rangle_y\,|^2 \\ &= |(1,0) * (\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}})|^2 \\ &= |\frac{1}{\sqrt{2}}|^2 \\ a &= \frac{1}{\sqrt{2}} \end{aligned} \tag{27}$$

$$\begin{aligned} P_{-,z} &= |b|^2 \\ &= |\,\langle -|+\rangle_y\,|^2 \\ &= |(0,1) * (\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}})|^2 \\ &= |\frac{i}{\sqrt{2}}|^2 \\ b &= \frac{i}{\sqrt{2}} \end{aligned} \tag{28}$$

$$|\psi_b\rangle = \frac{1}{\sqrt{2}}\,|+\rangle + \frac{i}{\sqrt{2}}\,|-\rangle \tag{29}$$

5.2.2 ψ_c

$$\begin{aligned} P_{+,z} &= |a|^2 \\ &= |\,\langle +|-\rangle_y\,|^2 \\ &= |(1,0) * (\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}})|^2 \\ &= |\frac{1}{\sqrt{2}}|^2 \\ a &= \frac{1}{\sqrt{2}} \end{aligned} \tag{30}$$

$$
\begin{aligned}
P_{-,z} &= |b|^2 \\
&= |\left\langle - | - \right\rangle_y|^2 \\
&= |(0,1) * (\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}})|^2 \\
&= |-\frac{i}{\sqrt{2}}|^2 \\
b &= -\frac{i}{\sqrt{2}}
\end{aligned} \tag{31}
$$

$$
|\psi_c\rangle = \frac{1}{\sqrt{2}}\,|+\rangle - \frac{i}{\sqrt{2}}\,|-\rangle \tag{32}
$$

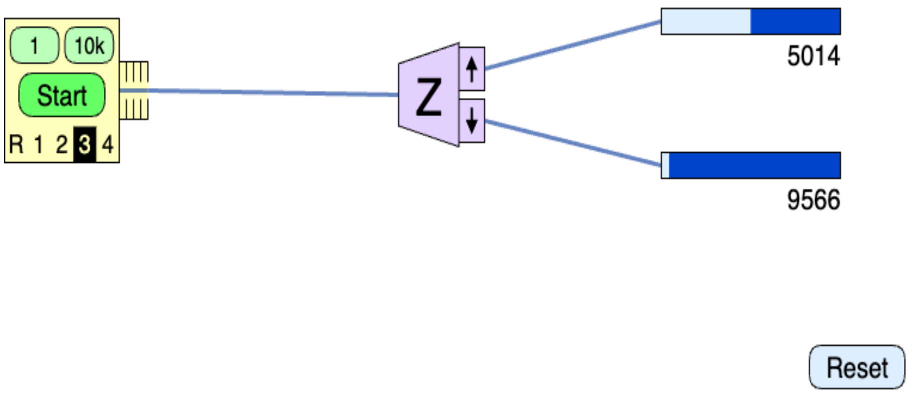
5.3

I understand that you would like a diagram on this, however I grasped and really understood this phenomenon through the math. The reason that the second analyzer "creates" a distribution in the z-orientation, despite all the particles previously being spin-up in the z-axis is simply because the measurement in the y-orientation is a change of basis. Since linear algebra has been observed to describe some quantum mechanical phenomena, we can follow the properties of the math to understand why this happens. Linear algebra operations do not commute (and more specifically, rotations do not commute), so if I have a basis in the z-orientation, and then go to y, and then go back to z, its not the same as if I went through y, then z twice. The math shows that you will find coefficients in your ψ_z, it is just a property of linear algebra, which happens to describe some quantum mechanics (up until the time I wrote this), and there is no reason why. It is just math, and we accept it only because it is the most accurate way to describe what we measure. Student C is wrong because they didn't talk about math and they are talking about physics. Student C's argument is philosophical, and has no place in a conversation about physics. To add to this, by measuring the spin, you have interacted and fundamentally changed the quantum state. This measuring is a change of basis - that is if you are changing basis. If you change from z to z you have not changed at all and your measurement will remain the same.

6 Problem 6

$$
|\psi_z\rangle = \frac{1}{\sqrt{3}}\,|+\rangle + \frac{\sqrt{2}}{\sqrt{3}}\,|-\rangle \tag{33}
$$

I made 3 measurements of $|\psi_3\rangle$ before feeling confident the distribution in equation 30 was accurate. It was clear after the third measurement that the probability was distributed 33% $|+\rangle$ and 67% $|-\rangle$. I then normalized the state vector to get ψ_3 above.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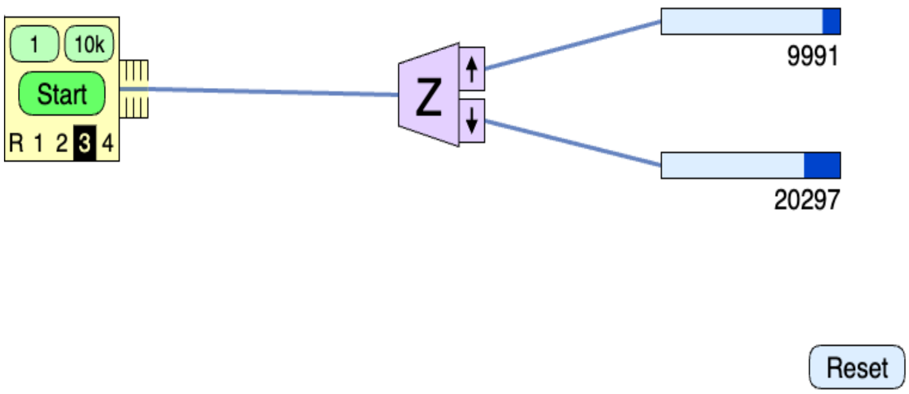
R 1 2 3 4
Z
5014
9566
Reset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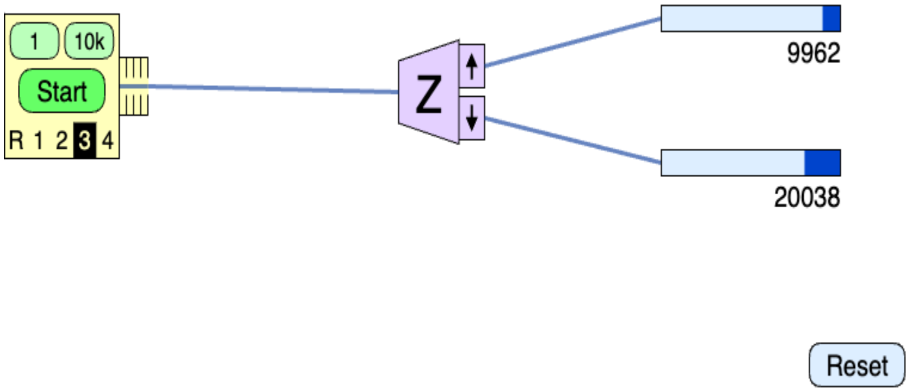
10k
Start
R 1 2 3 4
Z
9991
20297
Reset


1
10k
Start
R 1 2 3 4
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9962
20038
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