Phys273 - Homework 3

Dillon Walton

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1 Part a

$$\vec{F} = m\ddot{x}$$
$$m\ddot{x}_1 = -k(x_1 - x_2) \tag{1}$$
$$m\ddot{x}_2 = -k(x_2 - x_1) - k(x_2 - x_3) \tag{2}$$
$$m\ddot{x}_3 = -k(x_3 - x_2) \tag{3}$$

2 Part b

Assume solution of:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} e^{i\omega t}$$

2.1 Equations for x_1

$$x_1 = A_1 e^{i\omega t}$$
$$\acute{x}_1 = i\omega A_1 e^{i\omega t}$$
$$\ddot{x}_1 = -\omega^2 A_1 e^{i\omega t}$$

$$-m\omega^2 A_1 = -k(A_1 - A_2)$$
$$-m\omega^2 A_1 + k(A_1 - A_2) = 0$$
$$-m\omega^2 A_1 + kA_1 - kA_2 = 0$$
$$(-m\omega^2 + k)A_1 - kA_2 = 0$$

2.2 Equations for x_2

$$x_2 = A_2 e^{i\omega t}$$
$$\acute{x}_2 = i\omega A_2 e^{i\omega t}$$
$$\ddot{x}_2 = -\omega^2 A_2 e^{i\omega t}$$

$$-m\omega^2 A_2 = -k(A_2 - A_1) - k(A_2 - A_3)$$
$$-m\omega^2 A_2 + k(A_2 - A_1) + k(A_2 - A_3) = 0$$
$$(-m\omega^2 + 2k)A_2 - kA_1 - kA_3 = 0$$

2.3 Equations for x_3

$$x_3 = A_3 e^{i\omega t}$$
$$\acute{x}_3 = i\omega A_3 e^{i\omega t}$$
$$\ddot{x}_3 = -\omega^2 A_3 e^{i\omega t}$$

$$-m\omega^2 A_3 = -k(A_3 - A_2)$$
$$(-m\omega^2 + k)A_3 - kA_2 = 0$$

2.4 Final Equations

$$(-m\omega^2 + k)A_1 - kA_2 = 0 \tag{4}$$
$$-kA_1 + (-m\omega^2 + 2k)A_2 - kA_3 = 0 \tag{5}$$
$$-kA_2 + (-m\omega^2 + k)A_3 = 0 \tag{6}$$

2.5 Matrix

$$\begin{bmatrix} (-\omega^2 + {\omega_0}^2) & -{\omega_0}^2 & 0 \\ -{\omega_0}^2 & (-\omega^2 + 2{\omega_0}^2) & -{\omega_0}^2 \\ 0 & -{\omega_0}^2 & (-\omega^2 + {\omega_0}^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

3 Part c

$$\begin{vmatrix} (-\omega^2 + {\omega_0}^2) & -{\omega_0}^2 & 0 \\ -{\omega_0}^2 & (-\omega^2 + 2{\omega_0}^2) & -{\omega_0}^2 \\ 0 & -{\omega_0}^2 & (-\omega^2 + {\omega_0}^2) \end{vmatrix} = 0 \tag{8}$$

3.1 Determinant

$$\Bigg((-\omega^2 + {\omega_0}^2)\Bigg[\Big((-\omega^2 + 2{\omega_0}^2)(-\omega^2 + {\omega_0}^2)\Big) - \Big((-{\omega_0}^2)(-{\omega_0}^2)\Big)\Bigg]\Bigg) - \Big((-{\omega_0}^2)(-\omega^2 + {\omega_0}^2)\Big) + 0 = 0 \tag{9}$$

$$\Big[(-\omega^2 + {\omega_0}^2)(\omega^4 - 3\omega^2{\omega_0}^2 + {\omega_0}^4)\Big] - ([{\omega_0}^4\omega^2] - {\omega_0}^6) = 0 \tag{10}$$

$$-\omega^6 - 3\omega^2{\omega_0}^4 + 4\omega^4{\omega_0}^2 = 0 \tag{11}$$

$$(\omega^2 - 3{\omega_0}^2)(\omega^2 - {\omega_0}^2) = 0 \tag{12}$$

3.2 Normal mode frequencies

$$\omega = \pm\sqrt{3}\omega_0 \tag{13}$$
$$\omega = \pm\omega_0 \tag{14}$$
$$\omega = 0 \tag{15}$$

4 Part d

4.1 $\omega = 0$

$$\begin{bmatrix} (\omega_0{}^2) & -\omega_0{}^2 & 0 \\ -\omega_0{}^2 & (2\omega_0{}^2) & -\omega_0{}^2 \\ 0 & -\omega_0{}^2 & (\omega_0{}^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{17}$$

4.2 $\omega = \pm\omega_0$

$$\begin{bmatrix} (-\omega_0{}^2 + \omega_0{}^2) & -\omega_0{}^2 & 0 \\ -\omega_0{}^2 & (-\omega_0{}^2 + 2\omega_0{}^2) & -\omega_0{}^2 \\ 0 & -\omega_0{}^2 & (-\omega_0{}^2 + \omega_0{}^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \tag{19}$$

4.3 $\omega = \pm\sqrt{3}\omega_0$

$$\begin{bmatrix} (-3\omega_0{}^2 + \omega_0{}^2) & -\omega_0{}^2 & 0 \\ -\omega_0{}^2 & (-3\omega_0{}^2 + 2\omega_0{}^2) & -\omega_0{}^2 \\ 0 & -\omega_0{}^2 & (-3\omega_0{}^2 + \omega_0{}^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \tag{21}$$

4.4 General Solution

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cos(\phi_1) + A_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \cos(\omega_0 t + \phi_2) + A_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \cos(\sqrt{3}\omega_0 t + \phi_3) \tag{22}$$