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Fingers to Proofs: A History of Early Mathematics

"Philosophy is written in this grand book--I mean the universe--which stands continually open to our glaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these one is wandering in a dark labyrinth." Galileo Galilei wrote this in his book *The Assayer,* and it aptly describes the viewpoint and the driving force behind mathematicians throughout history. From the beginning of history, mathematics has been studied as a way to attempt to understand the world around us, with each new generation and civilization building upon those that came before them.

One, two, and three, these numbers are quantities that the human brain understands effortlessly. Both language and etymology show that these first three numbers are the beginning of our relationship with the world of mathematics. In languages with case and gender inflections, such as German, these numbers are usually the only ones with inflection. They also have particular and distinct form, for example in English as ordinal numbers they do not end in "th" where as the rest of the numbers such as "fourth" and "fifth" do. "2" and "second" both generally imply another, for example the adjective secondary or the verb to second. The Indo-European root of three is synonymous with "a lot" and "beyond all others," some examples are the French “très” for very or the Italian “troppo” for too much or excessive. The Indo-Europeans may have only known “one,” “one and another,” and “a lot,” or one, two, and three or more. [1]

Naturally humanity was eventually able to count higher than three, and this next step was done using body parts. A one-to-one correspondence could be established between a set of items and one's fingers, toes, and so forth. The specific parts used came to symbolize a specific quantity, so raising three fingers symbolized the number three. Body based counting is found the world over, and some isolated communities use forms of it in the present day. For more advanced systems, naming a body part became enough to convey the quantity, for example in many communities the etymology of five involves the word hand. There was one more step between body based systems and true abstract numbers, and this was creating larger numbers from combinations of smaller ones. Body based counting suffers from the fact that the parts of the body form a small set that is around thirty. For example, instead of assigning six a random part such as “wrist”, it would be considered “one on the other hand” and since hand means five, six is expressed as five and one. This would continue to ten, which would be expressed as two hands, or two fives. In this we see the concept of any modern number system, with a defined base number (five) and larger numbers are expressed as a total of products and sums of smaller numbers. Modern day numerals were originally distinct compounds, for example eleven was one and ten, which were eventually contracted.[2]

One of the two oldest mathematical systems is that of Ancient Egypt. Their system used repetitions of symbols that stood for one, ten, one hundred, one thousand, ten thousand, one hundred thousand, and one million. This is an early example of a base ten decimal system. The

1 Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics* (New York: Oxford University Press, 1997), 92.

2 *Ibid.,* 93-95.

symbol for one was just a vertical stroke, so the numbers one through nine were just a series of the appropriate amount of strokes. The rest of the symbols were more picturesque, with no obvious connection to the number, for example the symbol for one hundred thousand was a fish. The Egyptian system had unique symbols for each number, and no place value system, so a number could be written in any order and it would mean the same thing. Nonetheless, there were strict formal rules for style, as well as informal customs such as grouping similar numbers together to assist the reader in quick counting. Because of this relative positioning, there was no need for the concept of zero. There are no empty slots, such as in 101, and if there is nothing to count then a scribe would just not write anything. [3]

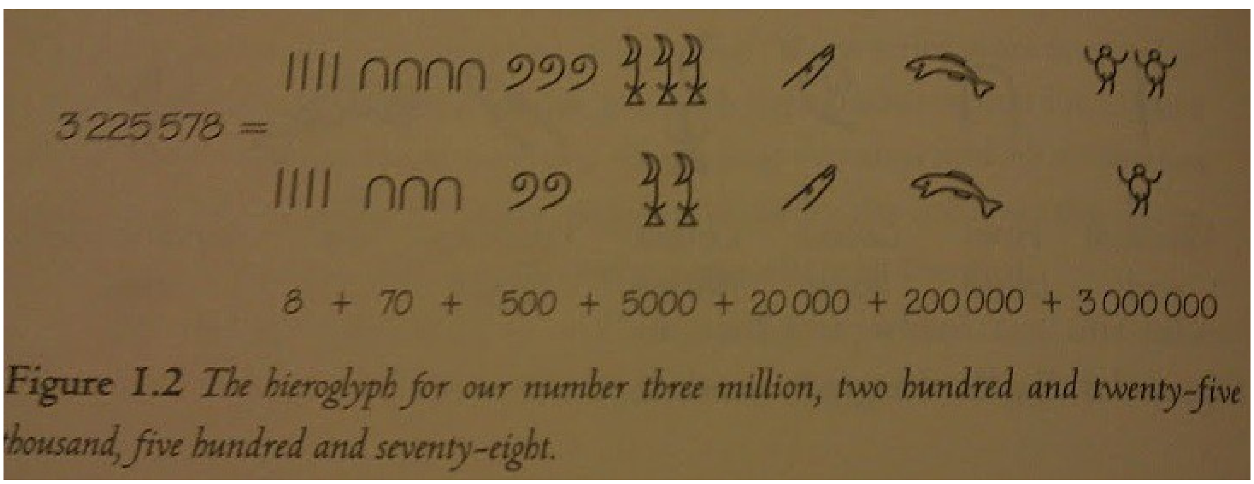
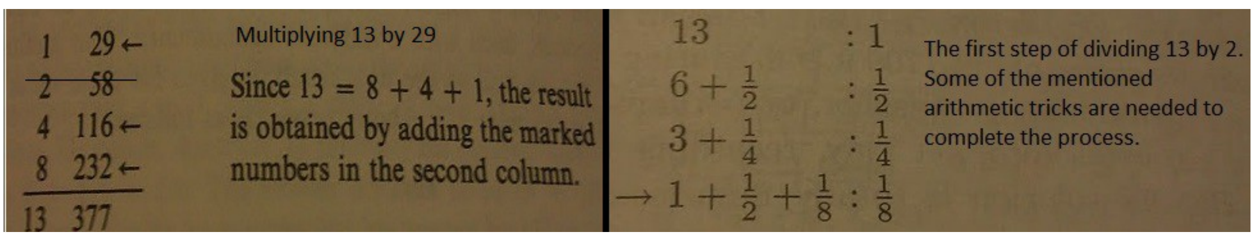


Figure 1.2 *The hieroglyph for our number three million, two hundred and twenty-five thousand, five hundred and seventy-eight.*

Besides basic counting, the Egyptians were also able to multiply, divide, and solve linear equations. Multiplying was simple enough. They would start at one, multiply it with the number to be multiplied (13 in 11 * 13), and double the multiplicand (11 in 11 * 13) until they had powers of two that would add up to the original multiplicand. Then they would add the results of the multiplications of those powers of two and discard the unused ones. For example, to

3 John D. Barrow, *The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe* (New York: Pantheon Books, 2000), 15-18.

calculate the example of 11 * 13, the process would be the following: 1 * 13 = 13, 2 * 13 = 26, 4 * 13 = 52, 8 * 13 = 104. Since 1 + 2 + 8 = 11, 13 + 26 + 104 = 143, 11 * 13 = 143. The division of a number was represented as a natural number (1, 2, 3...) plus a sum of unit fractions. This is a fraction that has a one in the numerator and some number greater than one in the denominator, such as 1/5. This was more difficult than the multiplication algorithm, but still within the realm of possibility for the Egyptians. They would take the divisor (x in x/y) and consider the ratio x : 1, and divide both sides by 2 successively. The ratio means that there are x parts in the whole, so if the ratio is preserved then if b can be found on the left then whatever sum is on the right is the answer. This complex process required a lot of trial and error and tricks of arithmetic which scribes learned after years of experience.[4]



Linear equations are a problem in the form of “ax=b” which is easy enough to solve with modern concepts of dividing and fractions, but these concepts require a large amount of mathematical gymnastics and proof[5] to establish and the Egyptians did not have them. To solve them, they used what is called the method of false position. The value of x was guessed to purposely be an incorrect answer, and then the result of the left side would be scaled to lead to the correct answer. These types of problems, as well as tables to express 2/(2n + 1) with n being between 2 and 50 inclusive to be used in division problems with odd fractions, were contained

4 Arthur Knoebel and others, eds., *Mathematical Masterpieces: Further Chronicles by the Explorers* (New York: Springer, 2007), 87-90.

5 In my intro to proofs class, it took about three and a half chapters to lead up to the division algorithm, and that was only for natural numbers. Also, this required the concept of proofs which wouldn't arise until the Greeks.

in two main sources: The Rhind Papyrus and the Moscow Papyrus. The Rhind Papyrus was a handbook written around 1700 B.C. with 84 worked problems involving arithmetic, linear equations, as well as simple areas and volumes. This document had the tables of unit fractions, which was used by scribes to make their computation work easier. A problem on the Rhind Papyrus leads to a value of pi close to 3.16. The Moscow Papyrus, which was written around 1900 B.C., had about 25 problems. The most exceptional one hides the correct formula for the volume of a truncated pyramid, which is not surprising given Egypt's fascination with them. [6]

The second of the two ancient systems is that of Mesopotamia, developed by the Sumerians and Babylonians and preserved by the various cultures that conquered Mesopotamia throughout ancient history. Babylonian mathematics has been discovered as early as 2100 B.C. in Ur, was developed greatly around the time of Hammurabi in 1800 B.C. and then stagnated until around the time of the Seleucids in the 300's B.C.[7] Since the Babylonians wrote on clay tablets with wedge shaped tablets, their number symbols were less ornate than those of the Egyptians. They had two main symbols, a vertical wedge that stood for one and a corner wedge that stood for ten. Unlike the Egyptians though, the Babylonians had a base 60, or sexagesimal, system that had place value. This system should be familiar after some thought to modern readers, as it is the same system that is used for time and degree measurements. In earlier texts, there was no symbol for zero and spaces were only sometimes left where we would put a zero, meaning that the context of a number determined the difference between 65 and 605 or 65 and 650. Later texts in the Seleucid period had a symbol for zero, which was two vertical stacked wedges, but it was only used as a separator. Standard notation in history books about

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6 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 3-6.
7 *Ibid.*, 6-7.

Babylonian mathematics is to separate the value of each place by a comma. Hence, 5130, which is $1*60^2 + 25*60 + 30$, is 1,25,30. [8]

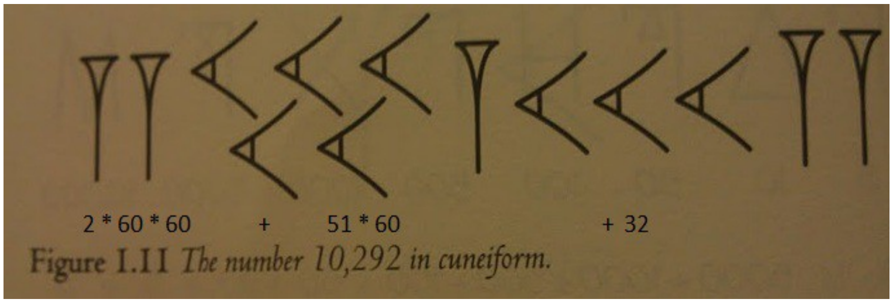

Figure I.11 *The number 10,292 in cuneiform.*

The Babylonians created reciprocal tables for all integers below sixty that contained factors of some power of sixty. The prime factorization of 60 is $2^2 * 3 * 5$, and any power of 60 is $60^n=2^{(2n)} * 3^n * 5^n$. Because every number has a unique prime factorization[9], any number that has a factor other than two, three, and five will not divide a power of sixty. Hence, numbers like seven and eleven were not on the table, but numbers like eight and twenty-seven were. These reciprocal tables show that the place system went both ways, so a number could be either a number extending into positive powers of sixty or negative powers (fractions). In modern notation, a semicolon is used where we would put a decimal in a modern number. For example, the number 1,25,30 that was mentioned above could be 5130, or $1,25;30 = 1*60 + 25 + 30/60 = 85.5$. It could also be 1;25,30. This was also generally determined by context. Two advantages to the sexagesimal system was that more fractions could be written in finite expansions than in a decimal system and that large numbers could be written very easily in comparison to a decimal system. For a fraction to be able to be written as a finite expansion in a given base b, the denominator can only have prime factors that are in b. In a decimal system, that means only numbers with the prime factors two and five. In a sexagesimal system, this

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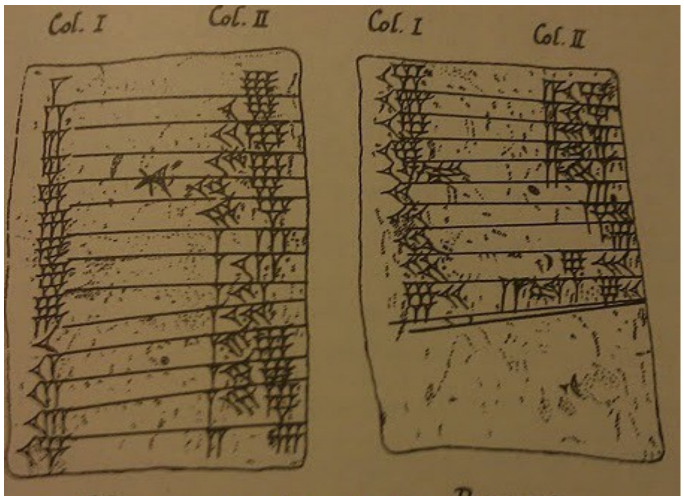
8 Asger Aaboe, *Episodes from the Early History of Mathematics* (New York: Random House, 1964), 5-10.

9 For a proof, see: Bernd S. W. Schroeder, *Fundamentals of Mathematics: An Introduction to Proofs, Logic, Sets, and Numbers* (New Jersey: John Wiley & Sons, Inc., 2010), 134-135.

means two, three, and five. Even constraining possible denominators to numbers between two and twenty, the decimal system has seven finite fractions and the sexagesimal system has thirteen. These advantages helped the Babylonians in their great achievements in astronomy, and even the Greeks continued to use a sexagesimal number system for fractions. [10]



A reciprocal table

In addition to basic reciprocal tables, the Babylonians had other tricks to help them with computations. Just like in the decimal system, there are rules and shortcuts for various operations such as testing divisibility by three, and these rules exist in the sexagesimal system as well. They had extended reciprocal tables that gave answers to several places for numbers like seven and eleven that did not have finite reciprocals. There were tables for computing compound interest at rates like 20%. Tables existed for finding squares, square roots, and cubes. They solved geometric problems mostly using algebraic solutions, and had no general formula for something like the quadratic formula. While they did not have a formula though, they did have detailed sets of instructions on how to work these types of problems to the point where it is obvious they realized there was a general pattern. They were acquainted with the fact that

10 Asger Aaboe, *Episodes from the Early History of Mathematics* (New York: Random House, 1964), 10-20.

the length of the diagonal of a square is the length of a side times the square root of two. This is a special case of the Pythagorean Theorem discovered 1200 years before the estimated time when Pythagoras lived, and they used the concept for more than just diagonals of squares. They even had techniques to approximate an accurate value of the square root of two. In addition, they also had figured out how to find the area of basic shapes such as trapezoids and triangles. Various approximations for pi existed, with one being about 3 and recently discovered ones being closer. Finally, they had discovered the basics behind the theorem of Pythagorean Triples. This is a name for a series of three numbers that are whole number solutions to the Pythagorean Theorem, such as 3, 4, and 5. The theorem that gave the equations for finding all values for Pythagorean Triples was published in 1945. There exists a table that has listed Pythagorean Triples that are so large that it is extremely unlikely they were guesses. Such an example is x = 3,31,49 = 12709, y = 3,45,0 = 13500, z = 5,9,1 = 18541. The method of calculation and reasoning behind the table are unknown, as the left side of the tablet which likely provided important clues is lost to history.[11]

"The salient and original feature of the Greek's pursuit of mathematics is that it established the foundations of logical reasoning and that it rigorously followed these rules of reasoning. . . they adhered to the rules that govern logical discourse and they established the idea that results should be derived from axioms that are accepted as true."[12] There are two main periods of history in Greek mathematics, the Classical period from 600 to 300 B.C. and the Hellenistic period from 300 B.C. to 600 A.D. The Greeks of the Classical period used a number system similar to that of Egypt. This was eventually superseded by what is called the Ionic

11 Asger Aaboe, *Episodes from the Early History of Mathematics* (New York: Random House, 1964), 22-32.

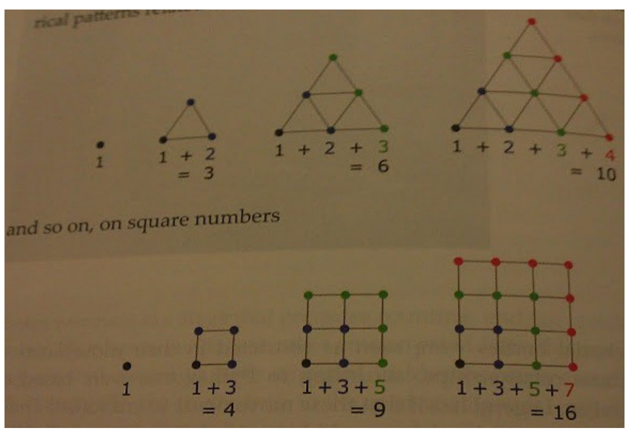
12 Bernd S. W. Schroeder, *Fundamentals of Mathematics: An Introduction to Proofs, Logic, Sets, and Numbers* (New Jersey: John Wiley & Sons, Inc., 2010), 318.

system, which used the twenty-four letters of the Greek alphabet and three obsolete letters to represent 1-9; 10, 20..90; and 100,200..900. Intermediate numbers were made by combinations. Extra markings were used for numbers over 1000. This worked well for natural numbers, but the Babylonian system was kept for fractions. Unfortunately, unlike the works of the Egyptians and Babylonians, many of the works of the Greeks did not survive, and much of what we know is from sources that were copies of copies and based on editors writing about (by the time of their writing) centuries old works. Greek mathematics consisted of three major elements. The first was the focus on deductive reasoning. This is the process of starting from a particular point and proceeding in logical steps until you arrive at a conclusion, which by the design process must be accepted. The second element was abstraction. A dominant theme of Greek philosophy was the search of abstract perfection, exemplified in Plato's concept of the ideal world of forms from which all things emulate. The third element was the focus on geometry and using geometric solutions to solve problems. The Greeks believed the world was designed and operated according to mathematical laws, with Plato saying in Timaeus that the Creator balanced the composition of the universe in accordance with the Golden Ratio, and discovering these laws was the way to comprehend the universe.[13]

The earliest known Greek mathematician known to history is Thales of Miletus, who lived from 634 to 548 B.C. and is credited with predicting the solar eclipse of May 28, 585 B.C. and is said to have proved various geometrical theorems such as the equality of base angles of an isosceles triangle. The next major mathematician, Pythagoras of Samos, was likely inspired by Thales rational approach to explaining the world. Pythagoras came to the conclusion that

13 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 12-14.

everything was made of number, because it is only when a concept is reduced to numbers that he felt it could be truly understood. He discovered that when he divided a single string into ratios of small natural numbers, they formed musical scales. Celestial bodies were looked upon as moving within spheres whose relationships had numerical ratios that formed chords and were considered supernatural “music of the spheres.” One had a special status among the numbers; it was considered a unit number and a symbol of the indivisible and the divine. Two was the first genuine number, and symbolized opposition and disunion, such as man and woman or left and right. While the Pythagoreans delved greatly into numerology, they also discovered geometrical patterns with numbers. Two such are that the sum of the first n numbers is 1/2*n*(n+1), and the sum of the first n odd numbers is n^2. [14]



The Greek mathematical world hit its first major stumbling block with the concept of irrational numbers. An irrational number is a number that can not be expressed as a simple fraction such as ¾. This came into play because of the Greek's focus on geometry. The best example is the ratio of the diagonal of a square to a side. The proof that this is irrational is included at the end of the paper. This proof uses the concept of proof by contradiction, where

14 Rudolf Taschner, *Numbers at Work: A Cultural Perspective* (Massachusetts: A K Peters, 2007), 1-6.

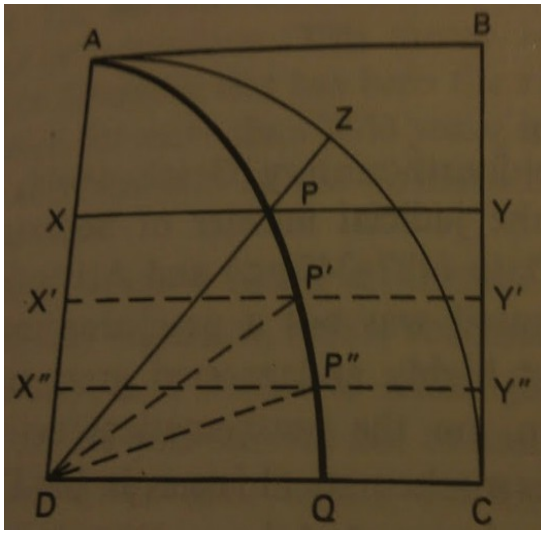
one assumes the theorem to be proved is false and shows that the theorem can't be false by a contradiction. This was a favorite method of Greek mathematicians and is still used widely by mathematics students everywhere today. The origins of this discovery are shrouded in mystery; it is guessed that it came from the diagonal to side ratio of either the square or the regular pentagon. The proof that there are irrational numbers greatly disturbed the Greeks, and there is a well known story of unknown accuracy that the Pythagoreans drowned a colleague who revealed this discovery to the public. The biggest result of this crisis is that the focus of Greek mathematics changed from the Pythagorean obsession with number to full on geometry[15]

Around the second half of the fifth century B.C., a burst of creative activity in mathematics happened, corresponding with the Golden Age of Periclean Athens. Around 430 B.C., Hippocrates (not the Father of Medicine one) left his homeland for Athens, where he started studying geometry. One of the subjects he worked on was the quadrature of lines, where a lune is the crescent shaped figure that results from the intersection of two circles and quadrature means to find the area. In one of his constructions, he uses a theorem that he previously proved to show that a semicircle circumscribed by a right angled isosceles triangle can result in a lune that has an area equal to the area of the isosceles triangle, which is itself equal to the radius of the semicircle squared. This was the first quadrature of a curvilinear figure to be proven in a rigorous format. During this period, geometric figures were constructed by the use of a straight edge and a drawing compass, which allowed the four basic arithmetic operations and the creation of square roots to be shown geometrically.[16]

There existed three great problems that influenced the development of Greek

15 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 17-22.
16 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 22-24.

mathematics at this point in time. The Greeks originally tried to solve them with solutions using the ruler and compass method, but their lack of success eventually led them to the proof of the impossibility of said solutions for these problems. The first problem was known as the duplication of the cube, this was to construct a cube whose volume is twice that of a given cube. This was solved by Archytas around 400 B.C. by finding the intersection in three dimensional space of a cone, a cylinder, and a torus (donut). This was an impressive feat, considering he did not have the use of coordinates or equations and had to work with pure geometry. The second problem was to trisect an angle, which is to construct an angle one third the size of a given angle. The third problem was to square a circle. These were both solved by Hippias of Elis, a notable Sophist. He discovered a particular curve, called the quadratix, that is generated when you have a square ABCD with the side AB moving uniformly and parallel towards side DC and simultaneously having the side AD rotate uniformly around the point D until it coincides with DC. Let the line XY by the position of AB at a given point in time, and the line DZ be the position of AD at that same point, and the point P is their intersection. The quadratix is then the trace of all the points P from A to Q, which lies on DC. To trisect an angle, all you needed were three lines XY, X'Y', and X''Y'' parallel to AB such that XX' = X'X''=X''D. At the points P, P', and P'' where these three lines intersect the quadratix, the lines PD, P'D, and P''D will trisect the angle PDQ. The third problem is solved by proving that the ratio DQ:DC is equal to 2 / pi. There were two main objections to this argument. One, that the ratio of the speeds of AB and AD need to be known, which is only possible if one knows the ratio of AB to the arc AC, which is the point of the construction and therefore a circular argument. The second involves the point Q, it is intuitive that the path is approaching that point, but it is not rigorously proved that it does so. [17]



The next great mathematician was Eudoxus, who was part of Plato's Academy in Athens. He is credited with the creation of several new concepts and achievements. The first is his theory of proportion. He made a distinction between magnitudes, for example line segments or angles, which vary continuously, and numbers, which are discrete (separate). He did not assign numerical values to magnitudes or their ratios, so therefore he could compare two ratios for equality regardless of whether the magnitude was rational or irrational. His theory, which by the time of Archimedes was considered an axiom, had three conditions with only one needing to be satisfied for two ratios to be true. If you have two ratios a:b and c:d and two natural numbers m and n, then they are equal if and only if 1) ma > nb implies mc > nd, 2) ma=nb implies mc = nd, or 3) ma < nb implies mc < nd. Unequal ratios are considered by the idea that if ma > nb and mc < nd, then a:b is greater than c:d. The shining point of this theorem is that it excluded both zero and any sort of infinity. Eudoxus' second contribution was the first proto-integration method, called the method of exhaustion. This method involves inscribing a given shape with a series of polygons whose areas converge to the area of the shape with the excess area between the

17 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 24-28.

polygons and the shape decreasing by less than half at each inscription. This can be repeated to any given degree of accuracy. His third contribution was a geometrical model of the solar system involving rotating spheres. Each sphere was carried along by the rotation of the sphere outside of it. In total, Eudoxus' model required twenty-seven spheres to create an accurate model. [18]

“The commenter Proclus, writing in the 5th century A.D., remarked that 'the *Elements* bears the same relation to the rest of mathematics as do the letter of the alphabet to the language.' In his elegance, clarity, and (for the most part) logical rigor, Euclid was a true Greek of the Golden Age.” Euclid lived during the third century B.C. in Alexandria and was the first great mathematician of the Hellenistic period. He wrote about a dozen works, but only five have survived to the modern day. One of them, his magnum opus, is the *Elements*. This was written as a textbook for students, was organized into thirteen books, and contained 467 propositions. The format was presented as a sequence of propositions that were either theorems to be proved or problems involving a straight edge and compass construction. The model proved to be so successful that it continued until very recently, the works of Newton were organized on the same principles. In Book I, there are twenty-three definitions of basic things such as a point or a line, followed by five postulates and five common notions. The postulates are specific to geometry, but the common notions are really just that. They are ideas so self-evident, such as 'If equals be added to equals, the wholes are equal', that they are considered axioms today, in addition with the postulates. The proofs of his theorems consist of a sequence of deductions based upon either axioms or previously proved propositions. The proof of the Pythagorean

18 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 28-33.

Theorem, at the end of Book I, uses 24 of the other 46 propositions proved by that point. This concept of back referencing is a defining feature of Greek mathematics, and was widely used after and still is today. There is a story about Thomas Hobbes discovering a love for geometry at age 40 by reading about the Pythagorean Theorem, disagreeing with it, and looking back at the referenced propositions, and continuing this process until he was convinced of the correctness of it. The various other books concern such topics as geometrical algebra, properties of circles, expounding upon the theories of proportions from Eudoxus, a proof that there are infinite prime numbers, and a proof that there are only five Platonic solids.[19] He is also credited with the Euclidean Algorithm, a fast process used to find the greatest common divisor, or the largest natural number that will divide two given numbers without remainder, that is still used today as part of RSA encryption which keeps Internet transactions secure.[20]

Apollonius, also known as the 'Great Geometer', worked in the end of the third century B.C. and was known both as a great geometer and a great astronomer. He created a model of the solar system that used circles and the epicycle instead of concentric spheres, and Apollonius' system eventually superseded it and was later enhanced. His masterpiece, *Conics*, contains of eight books and 487 propositions, organized in the Euclidean fashion and proved in the standard rigorous deductive methods. In his *Conics*, he establishes many properties about conic sections such as parabolas, hyperbolas, and ellipses that paved the way for analytical geometry.[21]

Archimedes, who lived around the same time as Apollonius, had a large range of

19 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 34-49.
20 Bernd S. W. Schroeder, *Fundamentals of Mathematics: An Introduction to Proofs, Logic, Sets, and Numbers* (New Jersey: John Wiley & Sons, Inc., 2010), 189.
21 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 57-64.

interests which included astronomy, hydraulics, and mechanics. He was one of the first great mathematicians to do practical math instead of drawing triangles with pebbles in the sand, and for this he was well renowned in his time. He invented the compound pulley, the Archimedean screw for raising water, had a firm grasp on the concept of levers, and created many war machines to help defend his native city of Syracuse. At the same time, he was in line with Greek tradition in asserting the importance of abstract thought, putting his focus on his mathematical discoveries. Unlike the works of Euclid, and to a lesser extent Apollonius, which were compilations or extensions of previous results, the works of Archimedes were original contributions. His works extended from parabolas to the surface area of a sphere, the theorem of the latter he considered one of his finest and had engraved on his tombstone. He gave a method of trisecting an angle which did not involve the quadratix. Using the method of exhaustion of inscribed polygons of up to ninety-six sides, he shows that pi (about 3.1416) is between 223/71 and 22/7, or about 3.1408 and 3.1429. He created a system for writing very large orders of numbers that was a precursor of our index notation. Finally, he seems to have understood the basics behind the concept of the logarithm.[22]

In the Hellenistic age, mathematics had a central place as a practical and pragmatic art, thanks in no small part to the influence of Archimedes and the later practical mindset of the Romans. Applications such as optics, mechanics, and hydraulics were focused on. After the death of Archimedes and Apollonius at the end of the third century B.C., the only major achievement in the realm of classical mathematics was the creation of trigonometry by Hipparchus, Menelaus, and Ptolemy. Ptolemy, who lived in the second century A.D., wrote a

22 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 64-81.

treatise called *Almagest* ('the best') which was for astronomy as *Elements* was for geometry. In this treatise contains a table of chords that used the Babylonian number system and was equivalent to a modern sine table. He also refined the concept of the epicycle from Apollonius. This decline of classical mathematics had a brief interruption from 250 – 300 A.D. Two main figures dominated this period. The first, Diophantus of Alexandria, introduced some algebraic symbolism and worked on equations of both the determinate and indeterminate type, that is equations with a finite and infinite amount of solutions, with examples being 9x^2=4 (x=2/3) and xy=24 (with possible answers being 1 and 24, 2 and 12, 4 and 6,...). The other, Pappus of Alexandria, was the last in the series of great Greek geometers. A treatise of his, *Synagoge* (Mathematical Collection), consisted of ten books, eight of which survived until the modern day. In *Synagoge*, there are records of Greek mathematics that would otherwise be considered lost, as well as alternate proofs and commentaries on these previous works and a number of new discoveries Pappus made. These results would be utilized in the seventeenth century with the advent of projective geometry. This attempt by Pappus to revive the study of geometry failed. The execution of Hypatia, a Euclidean commentator, by a Christian mob in 415 A.D. in Alexandra and Boethius, a Roman statesman who authored elementary mathematics texts, in 524 A.D. marked the end of classical mathematics in Western Europe until the late Middle Ages.[23]

While the Greeks accomplished many great things in the field of mathematics, there are two related areas where they failed, that of zero and infinity. The concept of zero as we know it today and our modern number system are credited to the Indians. Their base ten positional system developed by the 500s A.D., and was so superior that every culture that came into

23 Stuart Hollingdale, *Makers of Mathematics* (New York: Dover Publications, 2006), 83-91.

contact with the Indians eventually adopted it. By 628 A.D. the Indian astronomer Brahmagupta formally defined zero and gave rules for using it in basic arithmetic. In Indian culture, there was a wide variety of concepts already existing for the idea of nothing that were in widespread use. They did not need to make major alterations to their worldview to incorporate zero as a number. In contradiction, the Hebrew tradition viewed the void as the state from which God created the world, and hence was viewed in an unpleasant light as poverty and separation from God. In the Greek tradition, it was a serious philosophical quandary and the concept of nothing was treated as if it was something, leading to conclusions such as there is no empty space because one can only speak of what is, not about what is not. This made it extremely difficult for the idea of a zero to rise in Greece.[24]

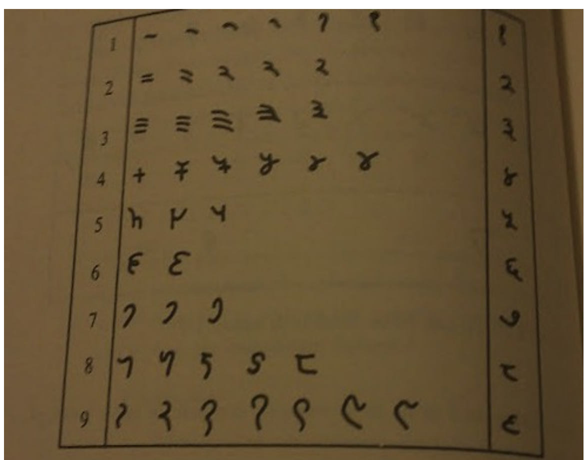


Figure 1.24 The evolution of the Nāgarī numerals. Notice how similar many of them are to the numerals we use today.

The Greeks were very close to the concept of a limit. The method of exhaustion is basically a limit argument. The Greeks weren't comfortable with the idea of infinity though, and avoided it similarly to how they avoided zero, negative numbers, and irrational numbers. They even avoided it in language; in the proof that there are infinitely many prime numbers they

24 John D. Barrow, *The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe* (New York: Pantheon Books, 2000), 32-42.

merely stated that they are not finite. This avoidance of infinity caused them to sometimes come to the wrong conclusions. The most famous error is that of Achilles and the turtle, posed by Zeno. This problem states that if Achilles races against a turtle and gave the turtle a head start, Achilles would never catch up. The argument is that every time Achilles reaches the point the turtle was, the turtle has moved forward by some amount, which repeats indefinitely. This is a false argument, based on the premise that the result of an infinite sum is infinite, which is not necessarily true. The time it takes for Achilles to catch up gets shorter and shorter each time, and their infinite sum is a finite number. Thought about another way, the number 1/3 = 0.33333..... = 0.3 + 0.03 + 0.003 + …, which the right hand side having an infinite amount of terms but resulting in a finite sum. [25]

While it has been explained how mathematics evolved past basic counting, one must wonder why it did so. At first, mathematics seems to have mainly been driven by pragmatism. The Egyptians and Babylonians needed to be able to calculate things such as tax money, crops grown, and weapons and other supplies for their armies. In addition, to figure out an accurate calendar to plan farming properly, accurate mathematics was needed. Geometry certainly helped the Egyptians build the Great Pyramids. A lot of the things the Greeks did, though, did not have any practical applications, or at least they wouldn't for several thousand years. Knowing the properties of an ellipse or that the square root of two is irrational does not feed the children or defend against Persians. A potential answer may be found in the Greek's love of abstraction and the pursuit of knowledge for knowledge's sake. They felt that by understanding mathematics, they were getting a glimpse into the nature of the universe and their Creator.

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25 Bernd S. W. Schroeder, *Fundamentals of Mathematics: An Introduction to Proofs, Logic, Sets, and Numbers* (New Jersey: John Wiley & Sons, Inc., 2010), 319-320.

Anyone who has figured out a complicated problem and had that “Eureka!” moment should be able to understand this drive.

Humanity has had at least a rudimentary grasp of mathematics since the beginning of our existence, and each group learned from and built upon previous results. Egypt took the basics of counting and created a system of arithmetic that allowed them to build the pyramids and predict floods and astronomical events, Babylon created a sexagesimal system and knew about concepts such as the Pythagorean Theorem and quadratic formula, and the Greeks took the knowledge from these two systems and created the foundations of logical reasoning that we still use today. As Newton said, “If I have been able to see further, it was only because I stood on the shoulders of giants.”

<u>Proof that the square root of two is irrational</u>

Note: Square root of two will be denoted as sqrt(2).

“Proof: Suppose for a contradiction that sqrt(2) is rational. Then, because rational numbers are fractions that can be written in lowest terms, there are natural numbers n and d such that sqrt(2) = n/d so that n and d have no common prime factors. Then n^2 / d^2 = (n/d) ^2 = 2, which means that n^2 = 2d^2. But then 2 is a prime factor of n^2 and hence 2 is a prime factor of n. Therefore, n = 2m and 2d^2 = n^2 = 4m^2, that is d^2 = 2m^2. But then 2 is a prime factor of d^2 and hence 2 is a prime factor of d, contradicting the assumption that n and d have no common prime factors. Therefore, sqrt(2) can not be rational. QED.”[26]

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26 Bernd S. W. Schroeder, *Fundamentals of Mathematics: An Introduction to Proofs, Logic, Sets, and Numbers* (New Jersey: John Wiley & Sons, Inc., 2010), 35.

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