Contents

Applications of Graph Theory and Combinatorics in Computer Science

Dylan Galea

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1 The Travelling Salesman Problem

1.1 Important Definitions and Problem Background

To define the Travelling Salesman Problem , first the concept of a Hamiltonian Cycle must be defined. This concept is defined in definition 1.1.1 below.

Definition 1.1.1 (Hamiltonian Cycle)**.** *Given a graph G(V,E), a Hamiltonian Cycle in G is a cycle in G such that* $\forall$ $v \in V$*, v is in the cycle and is visited only once . A graph that contains a Hamiltonian cycle is called a Hamiltonian Graph [1].*

The Travelling Salesman problem can now be defined as shown in definition 1.1.2 below.

Definition 1.1.2 (Travelling Salesman Problem)**.** *Given a simple graph G(V,E) such that* $\forall$ $v,w \in V$ *,*$v \neq w$ $\{v, w\} \in E$*, the Travelling Salesman Problem is the task of finding a minimum weight Hamiltonian Cycle in G [2].*

Definition 1.1.2 suggests that in the Travelling Salesman Problem it is already known that the graph to be evaluated is Hamiltonian, otherwise a minimum weight Hamiltonian Cycle can never be found.This uncertainty can be tackled by proving that any complete graph on more than 3 vertices is Hamiltonian. This fact is proved in lemma 1.1.1 below.

Lemma 1.1.1. *For n >= 3, The complete graph on n vertices is Hamiltonian*

Proof. Let Kn be the complete graph on n >= 3 vertices labelled v1,v2,...,vn. Order all the vertices in the order v1,v2,...,vn with no repetitions of vertices.Then C=(v1 v2 ... vn v1) must be a cycle in Kn because because $\forall$ vi,vj $\in$ C, vi $\neq$ vj then $\{vi, vj\} \in$ E(Kn). Also since $\forall$ v $\in$ V(Kn) ,v is a vertex in the cycle with only one occurence in C(except for v1 which has 2 occurences) then C must be a Hamiltonian Cycle in Kn . Thus Kn must be Hamiltonian. □

ToDo define NP HARD problems .. link with hamiltonian cycle and do example

References

[1] E. Weisstein, "Hamiltonian cycle," Oct 2018. [Online]. Available: http://mathworld.wolfram.com/HamiltonianCycle.html

[2] "Travelling salesman problem — set 1 (naive and dynamic programming)," Sep 2018. [Online]. Available: https://www.geeksforgeeks.org/travelling-salesman-problem-set-1/