Problem Set 1

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Problem 1-1.

(a) $f_1 = \log n^n = n \log n \in O(f_2)$, $f_3 = \log n^{6006} = 6006 \log n \in \Theta(\log n)$. So that $f_3 \in O(f_1)$. Meanwhile $f_4 \in O(f_2)$, and $f_5 \in O(f_3)$, the answer is $(f_5, f_3, f_1, f_4, f_2)$.

(b) $f_2 \in O(f_5)$ and $f_5 \in O(f_4)$. Also $f_1 \in O(f_2)$, $f_5 \in O(f_3)$. For f_3 and f_4,

$$\begin{aligned}\log \frac{f_3}{f_4} &= \log \frac{2^{6006^n}}{6006^{2^n}} \\ &= 6006^n \log 2 - 2^n \log 6006 \\ &= 2^n(3003^n \log 2 - \log 6006) \to \infty\end{aligned} \tag{1}$$

$f_4 \in O(f_3)$ and the answer is $(f_1, f_2, f_5, f_4, f_3)$.

(c) Firstly,

$$\begin{aligned}f_2 = \binom{n}{n-6} &= \frac{n!}{6!(n-6)!} \\ &\in \Theta\left(n(n-1)\dots(n-)\right) \\ &= \Theta(n^6),\end{aligned} \tag{2}$$

so $f_5 = n^6 \in \Theta(f_2)$. For f_4, by the Stirling's approximation,

$$\begin{aligned}\log f_4 = \log \binom{n}{n/6} &= \log \frac{n!}{(n/6)!(5n/6)!} \\ &\sim \log \frac{\sqrt{2\pi n}(n/e)^n}{\sqrt{2\pi n}(n/6e)^{n/6}\sqrt{2\pi n}(5n/6e)^{5n/6}} \\ &= \log \frac{(6/5^{5/6})^n}{\sqrt{2\pi n}} \\ &\in \Theta\left(n \log(6/5^{5/6}) - \frac{1}{2}(\log n + \log 2\pi)\right) = \Theta(n).\end{aligned} \tag{3}$$

since $6/5^{5/6} > 1$. Hence $f_2(\in \Theta(f_5)) \in O(f_4)$, $f_4 \in O(f_1)$ since $f_4 \in \{2^p | p \in \Theta(n)\} \subset \Omega(n^6)$ and $\log f_1 \in \Theta(n \log n)$. Also by the Stirling's approximation,

$$\begin{aligned}f_3 = (6n)! &\sim \sqrt{12\pi n}(6n/e)^{6n} \\ &\in \Theta((6n)^{6n}) \subset \Omega(n^n) = \Omega(f_1).\end{aligned} \tag{4}$$

Thus the answer is $(\{f_2, f_5\}, f_4, f_1, f_3)$.

(d)

Problem 1-2.

(a)

(b)

Problem 1-3.

Problem 1-4.

(a)

(b)

(c)

(d) Submit your implementation to `alg.mit.edu`.