Problem Set 0

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Problem 0-1.

(a) $A = \{1, 6, 12, 13, 9\}$, $B = \{3, 6, 12, 15\}$, $A \cap B = \{6, 12\}$.

(b) $|A \cup B| = |\{1, 3, 6, 12, 13, 15, 9\}| = 7$

(c) $|A - B| = |\{1, 13, 9\}| = 3$

Problem 0-2.

(a) $E[X] = 3/2$

(b) $E[Y] = (7/2)^2 = 49/4$

(c) $E[X + Y] = E[X] + E[Y] = 55/4$

Problem 0-3.

(a) true

(b) false

(c) false

Problem 0-4.

For $n = 1$, $\sum_{i=1}^{1} i^3 = 1 = \left[\frac{1 \cdot 2}{2}\right]^2$.

Assume $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$, then

$$\begin{aligned}\sum_{i=1}^{n+1} i^3 &= \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3 \\ &= (n+1)^2 \left[\left(\frac{n}{2}\right)^2 + (n+1)\right] \\ &= \left[\frac{(n+1)(n+2)}{2}\right]^2.\end{aligned} \tag{1}$$

Problem 0-5.

For $|V| = 1$, given graph G is acyclic since there is no edge. Let G_n be set of all connected undirected graph $G = (V, E)$ for which $|V| = |E| + 1 = n$. Assume $(\forall G \in G_n)$ G is acyclic. Then $(\forall G \in G_{n+1}, \exists G' \subset G)\ G' \in G_n$ since G is connected. Moreover, $G - G'$ has only one edge and one vertex that is connected to some vertex of G' since G' consume $n-1$ edges. Thus G is acyclic.

Problem 0-6.

```python
def count_long_subarray(A):
    '''
    Input:  A     | Python Tuple of positive integers
    Output: count | number of longest increasing subarrays of A
    '''
    count = 0
    length = 0
    lengths = []
    prev = 0
    for a in A:
        if a < prev:
            lengths.append(length)
            length = 1
        else:
            length += 1
        prev = a
    lengths.append(length)

    return lengths.count(max(lengths))
```