Problem Set 1

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Problem 1-1.

(a) $f_1 = \log n^n = n \log n \in O(f_2)$, $f_3 = \log n^{6006} = 6006 \log n \in \Theta(\log n)$. So that $f_3 \in O(f_1)$. Meanwhile $f_4 \in O(f_2)$, and $f_5 \in O(f_3)$, the answer is $(f_5, f_3, f_1, f_4, f_2)$.

(b) Since $n \in O(2^n)$, $f_1 \in O(f_3)$. Also $n \in O(n^2), n^2 \in O(2^n)$, $f_1 = 2^n \in \Theta(6006^n) = \Theta(f_2)$ and $f_3 = 2^{6006^n} \in \Theta(6006^{2^n}) = \Theta(f_4)$, the answer is $(\{f_1, f_2\}, f_5, \{f_3, f_4\})$.

(c) Firstly,

$$\begin{aligned} f_2 = \binom{n}{n-6} &= \frac{n!}{6!(n-6)!} \\ &\in \Theta\left(n(n-1)\ldots(n-)\right) \\ &= \Theta(n^6), \end{aligned} \tag{1}$$

so $f_5 = n^6 \in \Theta(f_2)$. For f_4, by the Stirling's approximation,

$$\begin{aligned} \log f_4 = \log \binom{n}{n/6} &= \log \frac{n!}{(n/6)!(5n/6)!} \\ &\sim \log \frac{\sqrt{2\pi n}(n/e)^n}{\sqrt{2\pi n}(n/6e)^{n/6}\sqrt{2\pi n}(5n/6e)^{5n/6}} \\ &= \log \frac{(6/5^{5/6})^n}{\sqrt{2\pi n}} \\ &\in \Theta\left(n \log(6/5^{5/6}) - \frac{1}{2}(\log n + \log 2\pi)\right) = \Theta(n). \end{aligned} \tag{2}$$

since $6/5^{5/6} > 1$. Hence $f_2(\in \Theta(f_5)) \in O(f_4)$, $f_4 \in O(f_1)$ since $f_4 \in \{2^p | p \in \Theta(n)\} \subset \Omega(n^6)$ and $\log f_1 \in \Theta(n \log n)$. Also by the Stirling's approximation,

$$\begin{aligned} f_3 = (6n)! &\sim \sqrt{12\pi n}(6n/e)^{6n} \\ &\in \Theta((6n)^{6n}) \subset \Omega(n^n) = \Omega(f_1). \end{aligned} \tag{3}$$

Thus the answer is $(\{f_2, f_5\}, f_4, f_1, f_3)$.

(d)

Problem 1-2.

(a)

(b)

Problem 1-3.

Problem 1-4.

(a)

(b)

(c)

(d) Submit your implementation to `alg.mit.edu`.