Problem Set 1

Name: Kim Heesuk

Problem 1-1.

(a) $f_1 = \log n^n = n \log n \in O(f_2)$, $f_3 = \log n^{6006} = 6006 \log n \in \Theta(\log n)$. So that $f_3 \in O(f_1)$. Meanwhile $f_4 \in O(f_2)$, and $f_5 \in O(f_3)$, the answer is $(f_5, f_3, f_1, f_4, f_2)$.

(b) Since $n \in O(2^n)$, $f_1 \in O(f_3)$. Also $n \in O(n^2), n^2 \in O(2^n)$, $f_1 = 2^n \in \Theta(6006^n) = \Theta(f_2)$ and $f_3 = 2^{6006^n} \in \Theta(6006^{2^n}) = \Theta(f_4)$, the answer is $(\{f_1, f_2\}, f_5, \{f_3, f_4\})$.

(c) Firstly, it is known that $\log n! \in \Theta(n \log n)$. By this,

$$\begin{aligned} \log f_3 = \log((6n)!) \in \Theta(6n \log 6n) &= \Theta(n(\log 6 + \log n)) \\ &= \Theta(n \log n). \end{aligned} \tag{1}$$

For f_2,

$$\begin{aligned} \log f_2 = \log \binom{n}{n-6} &= \log \frac{n!}{6!(n-6)!} \\ &\in \Theta\left(\log \frac{n!}{(n-6)!}\right) \\ &= \Theta(\log n + \log(n-1) + \cdots + \log(n-5)) \\ &= \Theta(\log n). \end{aligned} \tag{2}$$

And for f_4,

$$\begin{aligned} \log f_4 = \log \binom{n}{n/6} &= \log \frac{n!}{(n/6)!(5n/6)!} \\ &\sim \log \frac{\sqrt{2\pi n}(n/e)^n}{\sqrt{2\pi n}(n/6e)^{n/6}\sqrt{2\pi n}(5n/6e)^{5n/6}} \\ &= \log \frac{(6/5^{5/6})^n}{\sqrt{2\pi n}} \\ &\in \Theta\left(n \log(6/5^{5/6}) - \frac{1}{2}(\log n + \log 2\pi)\right) = \Theta(n) \end{aligned} \tag{3}$$

since $6/5^{5/6} > 1$. Lastly, $\log f_1 = \log n^n \in \Theta(n \log n)$, and $\log f_5 = \log n^6 \in \Theta(\log n)$. Now we can conclude that the answer is $(\{f_2, f_5\}, f_4, \{f_1, f_3\})$.

(d)

Problem 1-2.

(a)

(b)

Problem 1-3.

Problem 1-4.

(a)

(b)

(c)

(d) Submit your implementation to `alg.mit.edu`.