Chapter 6
Correction for Instrument Response

Abstract The recorded signal from a seismic sensor will give a series of numbers which, in a given frequency range, will be proportional to velocity or acceleration. However the user wants to get the true ground motion in acceleration, velocity or displacement in the widest frequency band possible. This is also called correction for instrument response.

For a given instrument, the amplitude frequency response function (gain of the instrument at different frequencies) for e.g. displacement can be determined such that for given harmonic ground displacement $X(\omega)$, the output $Y(\omega)$ can be calculated as

$$Y(\omega) = X(\omega)\,A(\omega)$$

where ω is the frequency, $Y(\omega)$ is the recorded amplitude and $A(\omega)$ is the displacement amplitude response. In order to recover the displacement, $X(\omega)$ can simply be calculated as

$$X(\omega) = Y(\omega)/A(\omega)$$

This response function can only be used for the amplitudes of a single sine wave at a given frequency. In order to make the complete instrument correction of the seismogram, the phase response must also be used. It turns out that, in general, the complete amplitude an phase response best can be described by a complex response function $T(\omega)$. In order then to calculate the corrected complex signal spectrum, $X(\omega)$, a complex Fourier transform is calculated of $Y(\omega)$ and the complex corrected spectrum is then

$$X(\omega) = Y(\omega)/T(\omega)$$

of which the real part is the amplitude spectrum. In order to get the corrected complex signal in time domain, $X(\omega)$ is then converted back to time domain with an inverse Fourier transform and the corrected signal is then the real part of the converted signal.

The response function $T(\omega)$ can be specified in different ways of which the most common are: discrete numbers of amplitude and phase, instrumental parameters like seismometer free period, damping, generator constant and digitizer gain or as a function described by poles and zeroes. The specification of anti-alias filters is also included in the response function.

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Recorded signals, whether analog or digital, rarely give us the real ground motion, even using a constant factor. The velocity sensors will give output proportional to velocity for frequencies above the instrument natural frequency, however below the natural frequency, there is no such simple relationship (see Chap. 2). Furthermore, seismologists generally want to measure the ground displacement and one very important task is therefore to recover the ground displacement from a given recorded signal. This is also called correction for instrument response. We have seen in the sensor section that, for a given instrument, the amplitude frequency response function can be determined such that for given harmonic ground displacement $U(\omega)$, the output $Z(\omega)$ can be calculated as

$$Z(\omega) = U(\omega)\, A_d(\omega) \tag{6.1}$$

where $Z(\omega)$ can be the amplitude on a mechanical seismograph, voltage out of a seismometer or amplifier or counts from a digital system, and $A_d(\omega)$ is the displacement amplitude response. In order to recover the displacement, $U(\omega)$ can simply be calculated as

$$U(\omega) = Z(\omega)/A_d(\omega) \tag{6.2}$$

$A_d(\omega)$ has traditionally been called the magnification since for an analog recorder, (6.2) gives how many times the signal is magnified. If e.g. the ground displacement is 0.1 mm and the magnification 100, the amplitude on the paper seismogram will be $100 \cdot 0.1$ mm = 10 mm. Similarly, if we want to determine the ground velocity, we would get

$$\dot{U}(\omega) = Z(\omega)/A_v(\omega) \tag{6.3}$$

where A_v is the velocity amplitude response. This simple single frequency instrument correction has been widely used for determining maximum ground displacements from analog seismograms in order to determine magnitude. The measure assumes that the signal is nearly monochromatic, which often was the case for the older type narrow-band instruments. Figure 6.1 shows typical magnification curves for the standard WWSSN (World Wide Standard Seismic Network) short and long period (LP) seismographs. For the short period, the maximum gain or magnification near 1 s is about 70 000 times, while for the long period it is about 3000 times near the 20 s period. Typical gains used in practice are 50000 and 2000 respectively.

Figure 6.2 shows a copy of an LP record of surface waves from a distant earthquake. The maximum amplitude is about 16 mm at a period of 25 s. Using the magnification curve in Fig. 6.1, we find that the gain is 2900 and the ground displacement is thus 16 mm/2900 = 0.0055 mm. In this case, the signal is rather monochromatic, so we obtain nearly the correct ground displacement. For more complex signals, particularly if recorded with broadband sensors, the correction

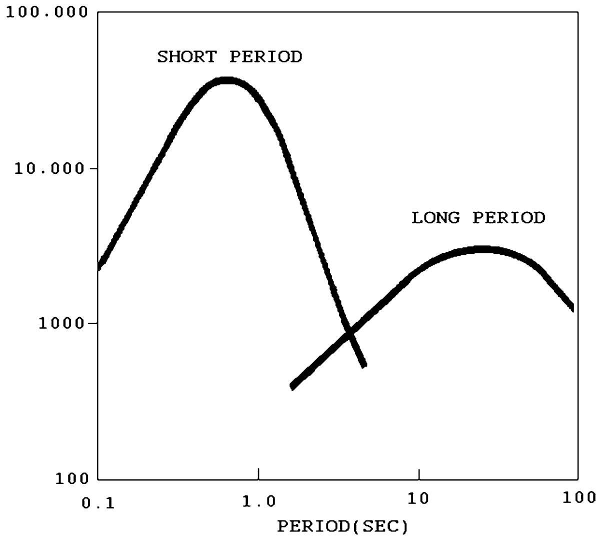

Fig. 6.1 Typical magnification curves for the WWSSN seismographs for long period (*LP*) and short period (*SP*) instruments

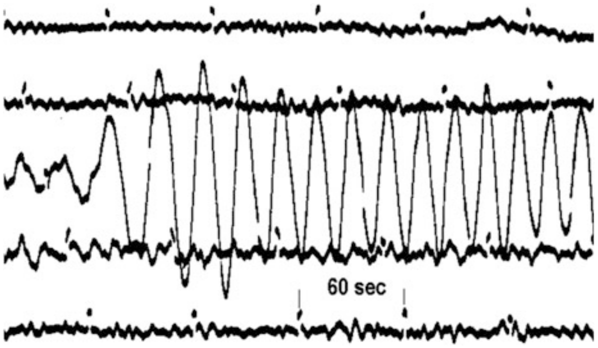

Fig. 6.2 Surface waves recorded on a WWSSN long period seismograph. The scale is 15 mm/min and there is 10 mm between the traces so a 24 h seismogram is 90 by 30 cm

cannot be done so easily and we have to deal with the whole frequency range. It is therefore necessary to briefly describe some of the general concepts of linear systems and spectral analysis.

6.1 Linear Systems

In seismology we assume that our instrument behaves as a linear system (6.4). Instruments can here be sensors, amplifiers or complete recording systems. The linearity means that there is a linear relationship between input signal and output

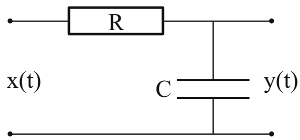

Fig. 6.3 RC low pass filter. *R* is the resistor and *C* the capacitor. Input is *x(t)* and output *y(t)*

signal. If the input signal is *x(t)* and the output *y(t),* then multiplying *x(t)* with a constant will result in an output signal multiplied with the same constant. Like if the ground velocity is doubled, then the output from the seismometer is also doubled. If two signals of different frequency and amplitude are input, then also two signals with the same frequencies (with different amplitude and phase) are output.

$$\begin{array}{ll} \text{Input} : x_1(t) & \text{Output} : y_1(t), \\ \text{Input} : x_2(t) & \text{Output} : y_2(t) \\ \text{Input} : \mathrm{ax}_1(t) + \mathrm{bx}_2(t) & \text{Output} : \mathrm{ay}_1(t) + \mathrm{by}_2(t) \end{array} \tag{6.4}$$

A linear system is also said to be time invariant if its properties (like filter constants) are time invariant. In practice, in a physical system, the parameters will change with time, but these changes are negligible in the short interval of a record.

The purpose of this section is to determine how the behavior of the linear system can be described generally so that the complete input signal can be determined from the complete output signal or vice versa, which includes the special monochromatic case illustrated above. For a more complete description, e.g. see Scherbaum (2007).

Let us first look at a very simple linear system, the RC filter. An RC low-pass filter is seen in Fig. 6.3.

It is well known that this circuit lets low frequencies pass while high frequencies are attenuated due to the frequency dependent impedance of the capacitor. The reactance R_c of a capacitor seen by the input (impedance) for a sine wave signal of frequency f is

$$R_c = \frac{1}{2\pi f C} = \frac{1}{\omega C} \tag{6.5}$$

where C is the capacitance (F). Considering that the RC filter is a frequency dependent voltage divider, using a monochromatic signal of angular frequency ω and amplitude $X(\omega)$, $x(\omega,t) = X(\omega)\cos(\omega t)$, the output signal amplitude $Y(\omega)$ can be written as (see Appendix I)

$$Y(\omega) = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} X(\omega) \quad = \quad \frac{1}{\sqrt{1 + \omega^2/\omega_0^2}} X(\omega) \tag{6.6}$$

where $\omega_0 = 1/RC$. ω_0 is also called the corner frequency of the filter and for $\omega = \omega_0$ the amplitude has been reduced to $1/\sqrt{2} = 0.707$. If the input signal is a

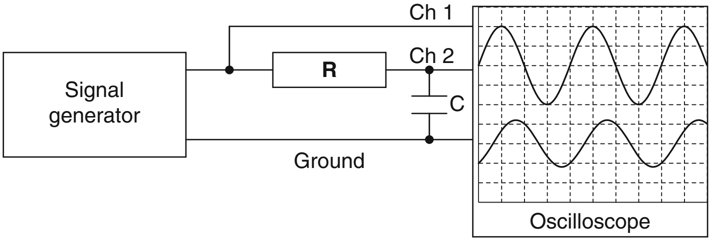

Fig. 6.4 Measuring the amplitude response function of an RC filter. The signal from a signal generator goes directly to channel 1 (Ch1, *top trace*) on the oscilloscope and to channel 2 (Ch2, *bottom trace*) through the filter so both input and output is measured.

steady state sine wave, the relation between the output and input signal amplitudes, $A(\omega)$, can be written as

$$A(\omega) = \frac{1}{\sqrt{1 + \omega^2/\omega_0^2}} = \frac{Y(\omega)}{X(\omega)} \tag{6.7}$$

$A(\omega)$ is called the amplitude frequency response function of the filter since only amplitudes are considered. If $A(\omega)$ is completely known, then the amplitude of the harmonic input signal $X(\omega)$ can be calculated from the measured signal as

$$X(\omega) = Y(\omega)/A(\omega) \tag{6.8}$$

The amplitude response can be measured very simply as shown in Fig. 6.4. By varying the frequency, both input and output amplitudes can be measured at different frequencies to produce the amplitude response function.

From Fig. 6.4, it is seen that the output signal not only has been changed in amplitude, but also has been delayed a little relative to the input signal; in other words, there has been a phase shift. Looking at the circuit diagram Fig. 6.3, it is obvious why the signal has been delayed. An instantaneous voltage over the resistor will result in a low voltage over the capacitor, since it will take some time to charge it. The charging time (see Appendix I) will depend of the size of both R and C and it can be shown that the phase shift Φ is a function of ω and RC. In this example, the phase shift is negative (see definition in (6.9)). The complete frequency response of the filter therefore consists of both the amplitude response function and the phase response function $\Phi(\omega)$, see Fig. 6.5. Considering a general input harmonic waveform $x(\omega,t) = X(\omega) \cdot \cos(\omega t)$, the output can be written as

$$y(\omega, t) = X(\omega) \cdot A(\omega) \cdot \cos(\omega t + \Phi(\omega)) \tag{6.9}$$

The phase shift is here defined as a quantity being added to the phase as seen above. Thus comparing Fig. 6.4 and (6.9), we see that the phase shift is negative. This is the most common way of defining the phase shift, but the opposite sign is sometimes seen and it may then be called phase delay or phase lag. So it is very important to know which definition has been used.

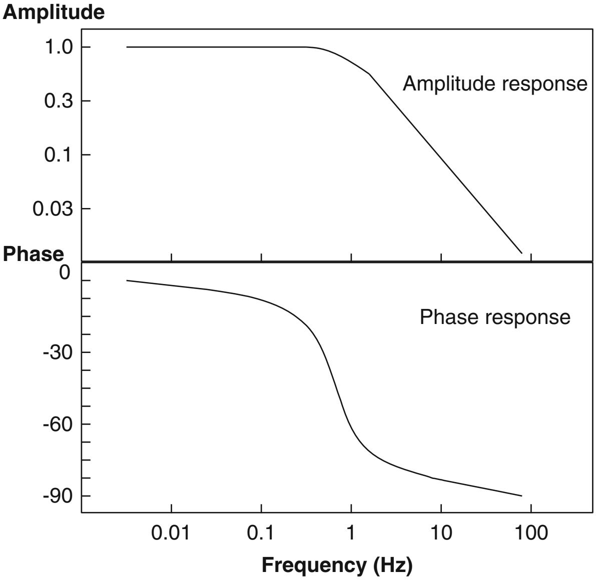

Fig. 6.5 Amplitude and phase response of an RC filter with a corner frequency of 1 Hz. The phase response (or phase shift) is given in degrees. See Appendix I

The frequency response function is a bit cumbersome to define and use. This can be considerably simplified if the complex representation of harmonic waves is used. Instead of writing *cos(ωt)*, we can use the real part of the exponential function

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t) \tag{6.10}$$

Equation (6.9) can now be written

$$y(\omega, t) = X(\omega)A(\omega)e^{i(\omega t + \Phi(\omega))} = X(\omega)A(\omega)e^{i\omega t}e^{i\Phi(\omega)} \tag{6.11}$$

y(ω,t) is now a complex number of which the real part is the actual output. This can be further simplified considering that any complex number Z can be written as

$$Z = a + ib = \sqrt{a^2 + b^2}e^{i\Phi} = |Z|e^{i\Phi} \tag{6.12}$$

where $\Phi = \tan^{-1}(b/a)$ which also follows from (6.10). We can now define the complex frequency response *T(ω)* as

$$T(\omega) = A(\omega)e^{i\Phi(\omega)} = |T(\omega)|e^{i\Phi(\omega)} \tag{6.13}$$

and (6.11) can be written

$$y(\omega, t) = X(\omega)T(\omega)e^{i(\omega t)} \tag{6.14}$$

Note that $X(\omega)$ is real. We now have only one complex function that includes the phase shift and therefore completely describes the instrument frequency response. This was also the kind of response obtained when dealing with seismometer theory (see Chap. 2). So far we have only dealt with monochromatic signals and the corrections can easily be made with both the real and complex representations (6.9 and 6.13) so it might be hard see why we have to go to complex representation of the response function. Later in this chapter, we are going to see how we correct an observed seismogram, not only for one amplitude at a time, but dealing with the amplitude and frequency content of the whole signal, in other words, we have to make spectral analysis. It will hopefully then be clear why a complex representation is needed. Before getting into how to do this we have to review basic concepts of spectral analysis.

6.2 Spectral Analysis and the Fourier Transform

When looking at seismic signals (or other signals), they often appear to consist of a superposition of harmonic signals like Fig. 6.2. Let us consider a signal of a finite duration T and defined in the time interval $-T/2$ to $T/2$, or 0 to T. It can be shown that most physical signals can be decomposed into an infinite sum of monochromatic components, the so called Fourier series, which is a sum of sine and cosine functions:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\omega_n t) + \sum_{n=1}^{\infty} b_n \sin(\omega_n t) \tag{6.15}$$

where $\omega_n = 2n\pi/T$. Strictly, the Fourier series expansion is limited to periodic functions. Therefore, this representation implicitly assumes that the signal is of infinite duration and periodic also outside this interval 0-T, but the key point is that (6.14), for a continuous signal, is an exact representation of the signal within the interval of interest. If it actually has a finite duration, we are not concerned with the Fourier series result outside this interval. The Fourier coefficients a_n and b_n can be calculated as (e.g. Stein and Wysession 2003)

$$\begin{aligned} a_n &= \frac{2}{T}\int_0^T \cos(\omega_n t)x(t)dt \\ b_n &= \frac{2}{T}\int_0^T \sin(\omega_n t)x(t)dt \end{aligned} \tag{6.16}$$

with n going from 1 to infinity. The term a_0 is simply the average signal level or the DC level.

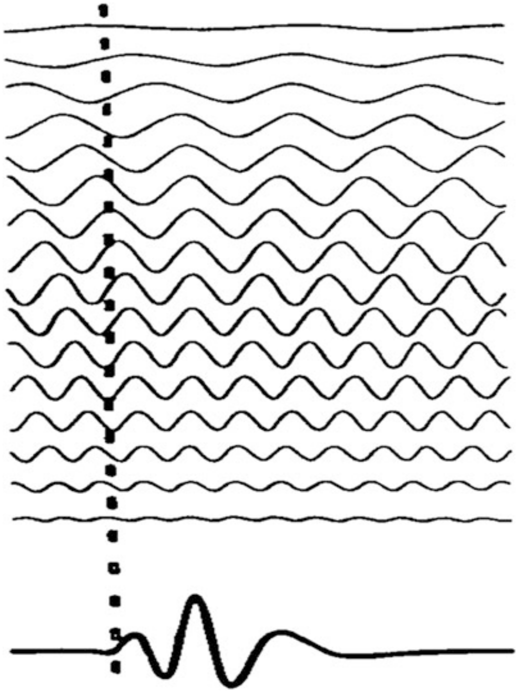

Fig. 6.6 Sum of harmonic functions (*top*) that become the wavelet seen at the *bottom* (Reprinted from *Modern Global Seismology* by Lay and Wallace (1995), p. 177; copyright (1995), with permission of Elsevier)

What does this mean physically? Imagine a simple signal of just a sine wave with amplitude unity and frequency ω_n. (6.15) would then give

$$b_n = \frac{2}{T}\int_0^T \sin(\omega_n t)\sin(\omega_n t)dt = \frac{2}{T}\cdot\frac{T}{2} = 1 \tag{6.17}$$

and the signal would be characterized by one spectral component with amplitude equal to one. Since the signal can have both cosine and sine waves, the amplitude spectrum is defined as

$$A_n = \sqrt{a_n^2 + b_n^2} \tag{6.18}$$

Note that the spectral amplitudes are normalized such that changing the length of the time window will not change the amplitudes. If the unit of the ground displacement is meter, then the unit of the amplitude spectrum is also meter.

It is conceptually easy to understand that a signal like in Fig. 6.2 can be made up of harmonic signals, but what about a short wavelet? Figure 6.6 shows an example of a sum of harmonics that gives a short wavelet by constructive and destructive interference.

Equation (6.14) has both sine and cosine terms, which is a way of being able to sum harmonics with different phase, since the sum of any sine and cosine functions with the same frequency, can be reduced to a single cosine function with a particular amplitude and phase. Considering the trigonometric identity

$$\cos(\omega_n t + \Phi_n) = \cos(\Phi_n)\cos(\omega_n t) - \sin(\Phi_n)\sin(\omega_n t) \tag{6.19}$$

and setting $a_n = A_n cos(\Phi_n)$ *and* $b_n = -A_n sin(\Phi_n)$, we can rewrite (6.14) as a sum of cosine functions multiplied with the spectral amplitude $A_n = \sqrt{a_n^2 + b_n^2}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(\omega_n t + \Phi_n) \tag{6.20}$$

where $\frac{-b_n}{a_n} = \frac{\sin(\Phi_n)}{\cos(\Phi_n)}$ and therefore $\Phi_n = \tan^{-1}\frac{-b_n}{a_n}$ and $\Phi_n(\omega)$ is called the phase spectrum. Thus the signal is composed of a sum of cosine terms with different amplitudes and phases. Equation (6.19) can be simplified by writing it in exponential form.

$$x(t) = a_0 + \sum_{n=1}^{\infty} X_n e^{i\omega_n t} \tag{6.21}$$

where $X_n = A_n e^{i\Phi_n} = A_n \cos(\Phi_n) + iA_n \sin(\Phi_n)$. X_n is now the complex spectrum whose magnitude is the true amplitude spectrum and Φ_n is the phase spectrum. *x(t)* is now formally a complex function of which the real part is the true signal. However, we will continue to call it *x(t)* remembering that only the real part is to be used.

Now consider how a_n and b_n can be calculated with a complex exponential function. Replacing the sine and cosine functions in (6.15) by the complex equivalents

$$\begin{aligned} \sin(x) &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos(x) &= \frac{e^{ix} + e^{-ix}}{2} \end{aligned} \tag{6.22}$$

it can be shown that (6.14) can be written

$$x(t) = a_0 + \sum_{n=1}^{\infty} F_n e^{i\omega_n t} + \sum_{n=1}^{\infty} F_{-n} e^{-i\omega_n t} \tag{6.23}$$

where

$$F_n = \frac{a_n - ib_n}{2} \quad \text{and} \quad F_{-n} = \frac{a_n + ib_n}{2} \tag{6.24}$$

Defining ω_{-n} as $-\omega_n = -2n\pi/T = \omega_{-n}$ and F_{-n} as the complex conjugate of F_n, the negative exponentials can be written

$$\sum_{n=1}^{\infty} F_{-n} e^{-i\omega_n t} = \sum_{n=-1}^{-\infty} F_n e^{i\omega_n t} \tag{6.25}$$

Thus by artificially using negative frequencies and setting $F_0 = a_0$, (6.22) can be written simply as

$$x(t) = \sum_{n=-\infty}^{\infty} F_n e^{i\omega_n t} \tag{6.26}$$

such that the spectral coefficients F_n are determined as

$$F_n = \frac{1}{T}\int_0^T x(t) e^{-i\omega_n t} dt \tag{6.27}$$

The amplitude spectrum can now be determined from F_n. Considering (6.23), we have

$$|F_n| = \frac{\sqrt{a_n^2 + b_n^2}}{2} = \frac{A_n}{2} \text{ or } A_n = 2|F_n| \tag{6.28}$$

An explanation for the factor 2 is that the energy artificially has been spread out over both positive and negative frequencies, which is just a way of simplifying calculations. Unfortunately, this factor is often forgotten. It has to be included if (6.19) is used for $x(t)$. Nevertheless, it should be pointed out that this factor 2 is sometimes taken as part of the normalization constant and different definitions are used. The various definitions of direct and inverse transforms are matched and the right pair has to be used for the original signal to be recovered from its spectrum. On the other hand, the fact that spectral amplitudes for negative frequencies are the complex conjugate of the positive ones is only true for real signals. So a more general formulation of Fourier series has to use both positive and negative frequencies.

For discrete data we cannot use the integration (6.26), which must be replaced by a summation. We assume an even number of N data points in the time window T and get:

$$F_n \cong \frac{1}{T}\sum_{k=0}^{N-1} x(k\Delta t) e^{-i\omega_n k\Delta t} \Delta t = \frac{\Delta t}{T}\sum_{k=0}^{N-1} x(k\Delta t) e^{-i\omega_n k\Delta t} \tag{6.29}$$

Since the data is sampled, and only available in the time window T, we know that aliasing exists (see 4.8) and in reality information is only available up to the Nyquist

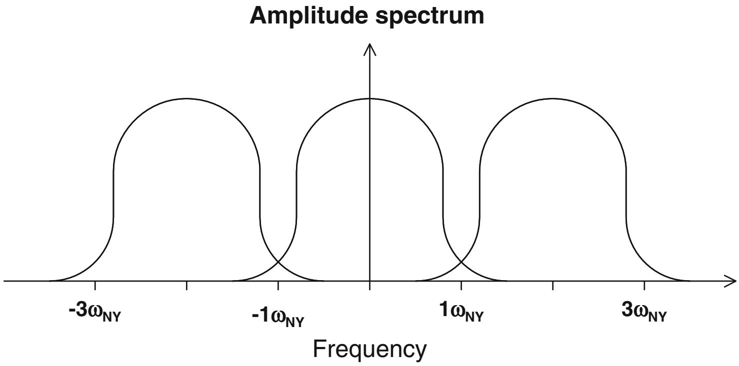

Fig. 6.7 The aliasing effect in the spectral domain. The Nyquist frequency is ω_{NY} and it is seen that the spectrum repeats itself for each $2\omega_{NY}$ and that there is overlap when the signal frequency is larger than ω_{NY}

frequency $\omega_{N/2} = \omega_{NY} = \pi N/T$ or if measured in Hz, half the sample rate $1/2\Delta t$. It can also be shown that the spectral content repeats itself, see Fig. 6.7.

This means that the spectral values for $n > N/2$ corresponds to the spectral content with the negative frequencies and the first half of the spectrum for negative frequencies is stored in the F_n – values for $n > N/2$, and the N positive frequency spectral values are:

$$\begin{array}{cccccc} \omega_0 & \omega_1 & \omega_2 & \omega_{N/2} & \omega_{-(N/2-1)} \omega_{-(N/2-2)} \\ F_0 & F_1 & F_2 & F_{N/2} & F_{-(N/2-1)} F_{-(N/2-2)} \end{array} \tag{6.30}$$

So, when writing the inverse transform, only the summation over the positive frequencies has to be done, since the values for $n > N/2$ in reality are the negative frequencies and it is assumed that there is no energy above the Nyquist frequency:

$$x(k\Delta t) = \sum_{n=0}^{N-1} F_n e^{i\omega k \Delta t} \tag{6.31}$$

Amplitude spectra are not always the most objective way of representing the spectral content, as we shall see in an example. Figure 6.8 shows the spectra of a transient signal recorded in a long and a short time interval respectively. The spectrum from the long time window has a much lower spectral level than the spectrum for the short time window. This is caused by two factors: (1) The normalization ($1/T$) over the long window causes any one spectral amplitude estimate to be smaller since a longer window does not mean more energy, (2) The longer window has a more dense spectrum since the frequency spacing $\Delta f = 1/T$ is smaller for the long window, so the energy is smeared out over more discrete amplitude estimates.

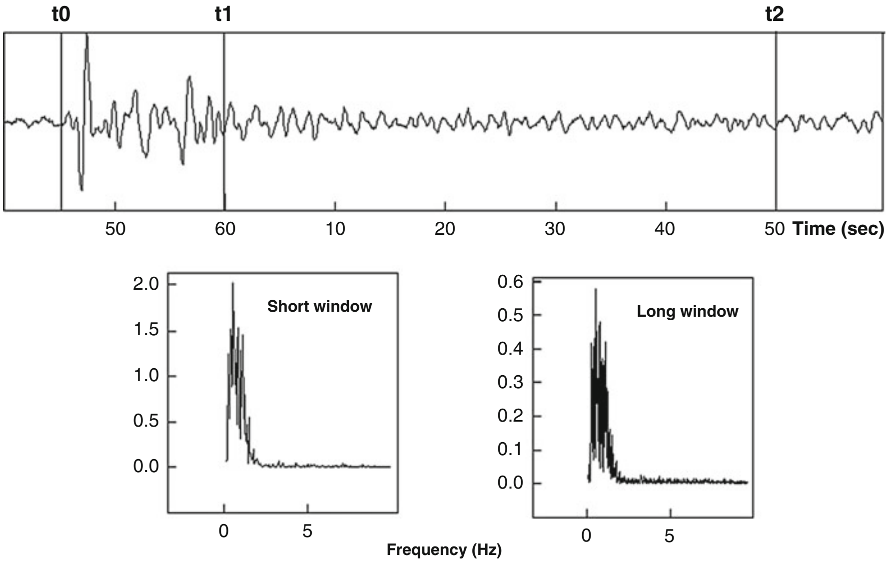

Fig. 6.8 The signal (*top*) is a teleseismic P-phase recorded using a short period seismometer. Below is shown the amplitude spectra of the short window (*t0−t1*) and the long window (*t0−t2*). The scales on the axes are linear (Note that the short window has a higher level than the long window. Also note that the frequency estimates are much closer together for the long window)

Obviously, it is desirable that the spectral amplitude is constant for a given signal irrespective of Δf, so that brings us to the definition of the amplitude density spectrum. If instead of specifying the individual amplitudes, we specify the spectral amplitude multiplied by the number of estimates per Hz, n, then we get an estimate independent of Δf since the smaller amplitudes correspond to a larger n. The number n of amplitudes per Hz can be calculated as $n = 1/\Delta f = T$. So, multiplying (6.28) by T, we get the amplitude spectral density, $F_n^d = TF_n$. The unit of the spectrum has now changed from amplitude to amplitude/Hz. This is the most common way of calculating the spectrum:

$$F_n^d = \Delta t \sum_{k=0}^{N-1} x(k\Delta t) e^{-i\omega_n k\Delta t} = \Delta t \sum_{k=0}^{N-1} x_k e^{-i2\pi nk/N} = \Delta t F_n^{DFT} \tag{6.32}$$

where

$$x_k = x(k\Delta t) \quad \text{and} \quad \omega_n k\Delta t = \frac{2\pi n}{T} k \frac{T}{N} = \frac{2\pi nk}{N} \tag{6.33}$$

and

$$F_n^{DFT} = \sum_{k=0}^{N-1} x_k e^{-i2\pi nk/N} \tag{6.34}$$

F_n^{DFT} is called the Discrete Fourier Transform (DFT) which is the way most Fourier transforms are implemented in computer programs.

The Inverse Discrete Fourier Transform (IDFT) should regenerate the original signal, and since the normalization constant was removed from F_n, it now has to be put back into the inverse transform which then is

$$x(k\Delta t) = \frac{1}{T}\sum_{n=0}^{N-1} F_n^d e^{i\omega k\Delta t} = \frac{1}{N\Delta t}\sum_{n=0}^{N-1} \Delta t F_n^{DFT} e^{i\omega k\Delta t} \tag{6.35}$$

or

$$x_k = \frac{1}{N}\sum_{n=0}^{N-1} F_n^{DFT} e^{i2\pi nk/N} \tag{6.36}$$

It is up to the user to put in the correct normalization constants when using DFT, and as it can be seen from (6.33 and 6.35), no information of time window or sample rate is used when calculating DFT or IDFT. The same computer algorithms can be used by just changing the sign of the exponential.

The expression of the discrete Fourier transform is a functional equivalent to the Fourier series, although conceptually different. Series development is applicable to periodic signals. Therefore, DFT application to a non-periodic signal (e.g. a time window of a seismic record) implicitly assumes that the signal repeats outside the window. If the value at the window start, $t = 0$, is different from the end value, $t = T$, we introduce a spurious step (that really does not exist), since the end is implicitly tied to the beginning of the following period. This may produce effects as loss of definition in the spectrum and the so-called "side-lobe contamination". For a discussion of these problems and their treatments, the reader is referred to, e.g., Press et al. (1995).

If the time window goes to infinity, the Fourier density spectrum and inverse transform can be extended into a Fourier transform, which represents the function as an integral over a continuous range of frequencies. It can be shown that (6.35 and 6.31) become (e.g. Stein and Wysession 2003):

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega \tag{6.37}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \tag{6.38}$$

It is now clear that it is not possible to talk about an amplitude spectrum since the energy has been distributed over an infinite number of frequencies so, as defined above, we can only talk about an amplitude density spectrum.

6.3 Noise Power Spectrum

One of the important tasks of the spectral analysis in instrumental seismology is to calculate the seismic background noise power spectrum, which is the most standard way of quantifying the noise at a given site. The standard amplitude spectrum, as defined in (6.27), gives an average estimate and we likewise want an average power spectrum independent of window length and in its discrete form also independent of sample rate, particularly since it is assumed that the background noise is stationary. The power spectrum P_n for a periodic function is defined (e.g., Kanasewich 1973) as

$$P_n = |F_n|^2 = \frac{a_n^2 + b_n^2}{4} = \frac{A_n^2}{4}, \ -\infty < n < \infty \tag{6.39}$$

Parseval's theorem states that the average power in a Fourier series is the same as in the time series

$$\sum_{n=-\infty}^{\infty} |F_n|^2 = \frac{1}{T}\int_0^T (x(t))^2 dt \tag{6.40}$$

If we have a sine wave with amplitude A_n and frequency ω_n (as defined before) in the time window T, the average power is

$$\frac{1}{T}\int_0^T {A_n}^2 \sin^2(\omega_n t)dt = \frac{{A_n}^2 T}{2T} = \frac{A_n^2}{2} \tag{6.41}$$

Using (6.39), we have to sum the two terms F_n and F_{-n} since according to Parseval's theorem, we have to sum over all frequencies, so

$$\text{Total power} = |F_n|^2 = F_{-n}^2 + F_n^2 = 2\frac{A_n^2}{4} = \frac{A_n^2}{2} \tag{6.42}$$

which is the same as (6.40). This result is not surprising. If we have a voltage of amplitude V over a resistor $R = 1\ \Omega$, then we know that the average power is $V^2/2$, so what is calculated with (6.41) is average power. Since spectra most often are calculated as F_n^{DFT}, using (6.31), the power spectrum is calculated as

$$P_n = \left|F_n^{DFT}\right|^2 \frac{\Delta t^2}{T^2} \tag{6.43}$$

and if only positive frequencies (so called one-side spectrum) are considered it will be

$$P_n = 2\left|F_n^{DFT}\right|^2 \frac{\Delta t^2}{T^2} \tag{6.44}$$

Seismic noise is supposed to be stationary, so making the time window longer should give the same result. But if we double the window, the number of power estimates P_n will also double, however since the average energy is the same, the average energy in each P_n will be half so the spectral level will be half. To get a constant value, we must use the power density spectrum (PSD, same argument as for the amplitude density spectrum) and (6.43) must be multiplied by T. The power density spectrum is then defined as

$$\frac{1}{T}X_n^2 = \frac{1}{T}\left|\Delta t F_n^{DFT}\right|^2 \tag{6.45}$$

Considering again only positive frequencies, the seismic power density spectrum P_n^d must be calculated as

$$P_n^d = \left|F_n^{DFT}\right|^2 \frac{\Delta t^2}{T} 2 \tag{6.46}$$

Note the unit. If the amplitudes are in m, the unit is $\mathrm{m^2 s = m^2/Hz}$. Usually the amplitude is in acceleration so the unit is $(\mathrm{ms^{-2}})^2/\mathrm{Hz}$.

6.4 General Instrument Correction in Frequency and Time Domain

We have now defined a complex frequency response function $T(\omega)$ and we can define the complex spectra of the input and output signals $x(t)$ and $y(t)$ as $X(\omega)$ and $Y(\omega)$, respectively. We use the definition for the Fourier spectrum for simplicity. Knowing the complete complex output spectrum we can determine the complete complex input spectrum as

$$X(\omega) = Y(\omega)/T(\omega) \tag{6.47}$$

If $T(\omega)$ is a seismic instrument response, we can say that we have obtained the instrument corrected ground motion spectrum and since $X(\omega)$ is complex, this also includes the correction for phase. The separate amplitude and phase response can then be obtained as

$$A(\omega) = \sqrt{\mathrm{Re}(T(\omega))^2 + \mathrm{Im}(T(\omega))^2} \tag{6.48}$$

$$\Phi(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}(T(\omega))}{\mathrm{Re}(T(\omega))}\right) \tag{6.49}$$

Thus in practice, the ground displacement spectrum would be calculated by taking the complex Fourier spectrum of the output signal and dividing it with the complex displacement response function and finally taking the absolute part of the complex ground displacement spectrum.

All of this could of course easily have been done with the corresponding non-complex equations since, so far, we have not really used the information in the phase directly and usually phase spectra for earthquake signals are not used. However, the next step is to also obtain the instrument corrected ground displacement or, said in another way, get the time domain signal instead of the frequency domain signal. With the knowledge about Fourier transforms, this is now easy, since, when we have all the frequency domain coefficients or the spectrum, we can generate the corresponding signal by the inverse Fourier transform. Since our corrected signal consists of a sum of cosine signals with different phase, each of them delayed differently due to the instrument response, the shape of the output signal will depend on the phase correction. So now life is easy, since the corrected complex displacement spectrum is already corrected for phase and the ground motion can be obtained as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)/T(\omega)\cdot e^{i\omega t}d\omega \tag{6.50}$$

or with real discrete data using (6.33 and 6.35). While it is possible to use only half of the positive frequencies for making the amplitude spectrum, both positive and negative frequencies must be used for the inverse transformation. Normalization constants can be defined in different ways, but if the same routine is used for both forward and inverse transformation (like 6.31 and 6.33), the normalization constants will cancel out.

Thus in theory, we can recover the ground displacement at any frequency knowing the instrument response. In practice, one has to be careful to only do this in the frequency band where the instrument record real ground motion and not just electronic noise, since the instrument correction then becomes unstable and the output has nothing to do with the real seismic signal. Figure 6.9 shows an example.

The figure shows the influence of filtering, when estimating the ground displacement signal. In the frequency band 1–10 Hz, the signal looks very much like the original signal although a bit more low frequency, since it is converted to displacement and can nearly be considered an integration of the original signal. In the 0.1–10 Hz range, the earthquake signal almost disappears in the microseismic background noise. Why do we think it is seismic noise and not instrumental generated noise? First, the earthquake signal has about the same amplitude as

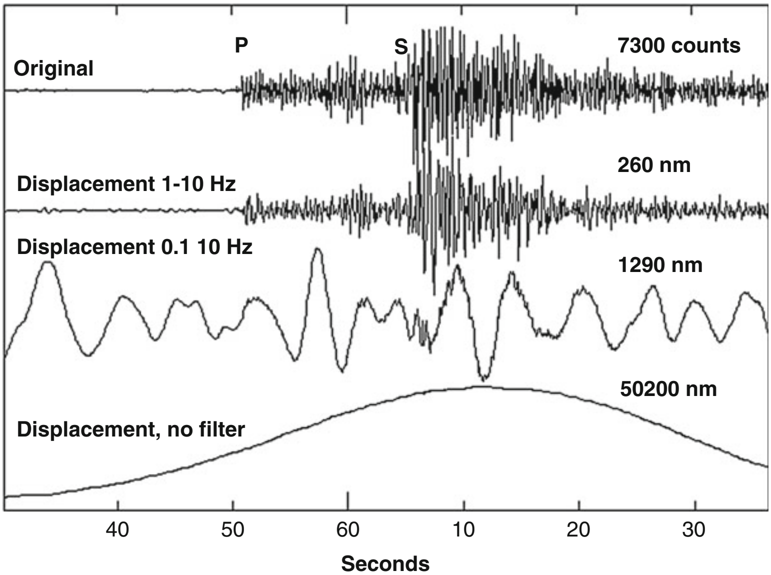

Fig. 6.9 Instrument correction in different filter bands. The *top trace* is the original recording of a small earthquake with a 1 Hz seismometer. The three *bottom traces* have been converted to displacement with different filters. The amplitudes to the right are maximum amplitudes

above, second, it 'looks' like seismic background noise and third, the amplitude at 1290 nm is at a period of 5 s (peak amplitude of microseismic noise) which looks reasonable compared to worldwide observations (see Fig. 3.3 in noise section). Note that this is how the earthquake signal would have looked being recorded on a broadband sensor, hardly noticeable. The last trace shows the calculation of the displacement without filtering so the lowest frequency used is *1/T*, where T is the length of the window, here 80 s (only 65 s shown) so $f = 0.0125$ Hz. The amplitude is now more than 50,000 nm and the signal looks 'funny'. The large amplitude obviously cannot be right since the microseismic noise has its largest amplitude around 5–10 s and it was 1290 nm. So we have a clear case of trying to make a displacement signal at frequencies lower than where seismic signals exist in the data. The ratio of the displacement gain for a 1 Hz seismometer at 1 Hz and 0.0125 Hz is $1/0.0125^3 = 5\ 10^5$. In other words, if the gain at 1 Hz is 1.0 we have to multiply by 1.0 to get the displacement, while at 0.0125 Hz we have multiply by $5\ 10^5$. So any tiny amount of instrumental noise present at low frequencies will blow up in the instrument correction. In the above example, it seems that the displacement signal can be recovered down to 0.1 Hz with a 1.0 Hz sensor.

We now have all the elements needed to correct our recorded signals in frequency or time domain, and the main problem is to obtain and describe the frequency response function $T(\omega)$.

6.5 General Representation of the Frequency Response Function

We have already seen examples of frequency response functions like the *RC* filter (see Appendix I) and the standard inertial seismometer. A seismic station has more elements which might not fit any of these two and a general description of the frequency response function is needed (hereafter called response function for simplicity) which can cover any system used. We will start with some examples.

The response function for the *RC* low pass filter can be written

$$T_{RC}(\omega) = \frac{1}{1 + i\omega RC} \tag{6.51}$$

Similarly, the displacement response function for a mechanical seismometer (see Chap. 2) is

$$T_d(\omega) = \frac{\omega^2}{\omega_0^2 - \omega^2 + i2\omega\omega_0 h} \tag{6.52}$$

In general, the response function could be any complex function. It turns out that $T(\omega)$ for all systems made from discrete mechanical or electrical components (masses, springs, coils, capacitors, resistors, semiconductors, etc.) can be represented exactly by rational functions of $i\omega$ like

$$T(\omega) = \frac{a_0 + a_1(i\omega) + a_2(i\omega)^2 + \ldots\ldots}{b_0 + b_1(i\omega) + b_2(i\omega)^2 + \ldots..} \tag{6.53}$$

where a_i and b_i are constants. The number of terms in the polynomials will depend on the complexity of the system.

It is seen from (6.50) that the RC filter exactly looks like (6.52), while the mechanical seismometer displacement response must be slightly rewritten to

$$T_d(\omega) = \frac{-(i\omega)^2}{\omega_0^2 + 2i\omega\omega_0 h + (i\omega)^2} \tag{6.54}$$

So for a seismometer $a_0 = 0$, $a_1 = 0$, $a_2 = -1$, $b_0 = {\omega_0}^2$, $b_1 = 2\omega_0 h$ and $b_2 = 1$. This general representation is sometimes used and is one of the accepted ways of giving response in SEED format (Standard for Exchange of Earthquake Data, defined by Federation of Digital Seismograph Networks (FDSN), IRIS (2012)). However, (6.52) can be written in an alternative and somewhat simpler way. Considering that a polynomial can be factorized, (6.52) can be written

$$T(\omega) = c\frac{(i\omega - z_1)(i\omega - z_2)(i\omega - z_3)\ldots..}{(i\omega - p_1)(i\omega - p_2)(i\omega - p_3)\ldots..} \tag{6.55}$$

where c is the combined normalization constant for nominator and denominator polynomials, z are the zeros (or roots) of the nominator polynomial while the zeros of the denominator polynomial (poles) are p. Equations (6.52 and 6.54) are exactly identical so either might be used to represent $T(\omega)$. Using (6.54) to represent $T(\omega)$ is the so-called poles and zeros representation, which has become the most standard way. Representing response curves in terms of poles and zeros is often described in a very complicated way, where it is necessary to understand terms like Laplace transforms and complex s-plane. In reality, as seen above, it is quite simple.

Equation (6.50) for the low pass filter can be rewritten as

$$T_{RC}(\omega) = \frac{1/RC}{i\omega + 1/RC} \tag{6.56}$$

and it is seen that the filter only has one pole at $-1/RC$ and the normalization constant is $1/RC$. For the seismometer, the denominator polynomial must be factorized by finding the roots p_1 and p_2 of the second order in $i\omega$

$$\omega_0^2 + 2i\ \omega\ \omega_0 h + (i\ \omega)^2 = 0 \tag{6.57}$$

It turns out that

$$\begin{aligned} p_1 &= -\omega_0\left(h + \sqrt{h^2 - 1}\right) \\ p_2 &= -\omega_0\left(h - \sqrt{h^2 - 1}\right) \end{aligned} \tag{6.58}$$

so $T_d(\omega)$ can be written

$$T_d(\omega) = \frac{-(i\omega - 0)(i\omega - 0)}{(i\omega - p_1)(i\omega - p2)} \tag{6.59}$$

So in addition to the poles p_1 and $p_{2,}$ the seismometer response function has a double zero at $z = 0$ and the normalization constant is -1. Note that since h usually is smaller than 1, the poles are usually complex (see example later in this chapter). Complex poles always appear as conjugate pairs.

For the standard velocity transducer with a generator constant of 1, the equation for displacement response is

$$T_d^v(\omega) = \frac{(i\omega - 0)(i\omega - 0)(i\omega - 0)}{(i\omega - p_1)(i\omega - p_2)} \tag{6.60}$$

and there is thus one more zero and the normalization constant is now 1 instead of –1 due to the polarity caused by the velocity transducer (see Chap. 2).

For the standard accelerometer, the displacement response is simply $(i\omega)^2$, which corresponds to two zeros, for frequencies below f_0.

6.5.1 Other Filters

Butterworth Filter The RC filter is simple, but it is not very suitable for higher orders. Ideally, a filter should be very sharp (a square in the frequency domain) but this is not possible in practice. There are many different types of analog filters, but one of the best and most widely used, is the Butterworth filter. This is because the filter has a nice response function, with the minimum ripple (or maximum flatness) for the pass-band, is easy to construct for any order up to 10 and easy to describe. The amplitude response function of the normalized low pass Butterworth filter is (e.g. Kanasewich 1973)

$$|B(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^{2n}}} \tag{6.61}$$

where ω_0 is the corner angular frequency of the filter (−3 dB point or where amplitude has decreased to 0.707) and n is the order of the filter. The amplitude response function for the high pass filter is

$$|B(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^{-2n}}} \tag{6.62}$$

A nice property of this filter is that the amplitude response at the corner frequency remains constant for any order of the filter. The complex response function can be described with a series of poles and zeros. For the first 2 orders, the high pass response can be written on polynomial form as

1. order

$$B(\omega) = \frac{1}{1 - i\frac{\omega_0}{\omega}} \tag{6.63}$$

2. order

$$B(\omega) = \frac{-1}{-1 + i\sqrt{2}\frac{\omega_0}{\omega} + \left(\frac{\omega_0}{\omega}\right)^2} = \frac{-\omega^2}{\omega_0^2 - \omega^2 + i\sqrt{2}\omega\omega_0} \tag{6.64}$$

Comparing the 2-order Butterworth filter response to the seismometer velocity response for a seismometer with a velocity transducer (2.36), we see that the response is exactly the same, (except for the sign, see Chap. 2), if the damping constant $h = \sqrt{2}/2 = 0.707$. This value of damping is frequently used since it gives the most flat response function, so we can describe the seismometer response as a simple high pass 2-order Butterworth filter. Since many modern seismometers shape their output using a Butterworth filter, the equivalent output is described using the normal seismometer response function with a damping of 0.707.

Digital Filters Digital filters operate on sampled signals as described in 4.9 If x_i represents the input series of the filter and y_i the output series, the most general representation of a linear, causal, digital filter is

$$y_i = \sum_{j=0}^{N} a_j x_{i-j} + \sum_{k=1}^{M} b_k y_{i-k} \tag{6.65}$$

In this equation, the first sum is the non-recursive part (it depends only on the present and past input values) and the second sum is the recursive part (depends on the past output values). A FIR filter has only the first part, while an IIR filter has both. If the sum is extended also to future input values (j may also be negative) then the filter will be non-causal. Symmetric ($a_j = a_{-j}$) non-causal filters, often used in seismic digitizers, have the advantage that their phase response is a linear function of frequency and so they do not produce phase distortion, but just a known time delay (see e.g. Scherbaum 2007).

The shift property of Fourier transform states that a time shift of Δt in the time domain is equivalent to multiplying the transform by $z \equiv e^{i\omega\Delta t}$. By using this, the frequency response of a filter described by the differences equation (6.64) may be written in a simple way as a rational function of z:

$$\frac{Y(\omega)}{X(\omega)} = \frac{\sum_j a_j z^{-j}}{1 - \sum_k b_k z^{-k}} = \frac{\sum_j a_j e^{-i\omega\Delta t \cdot j}}{1 - \sum_k b_k e^{-i\omega\Delta t \cdot k}} \tag{6.66}$$

where $z = e^{i\omega\Delta t}$ and Δt is the sampling period of the series.

The polynomial $X(z) = \sum_i x_i z^{-i}$ is known as the z-transform of the series x_i. See also Scherbaum (2007).

6.6 Anti Alias Filters

Analog anti alias filters have been described in the digitizer section. Analog filters are usually Butterworth filters, while the more common digital anti alias filters are FIR or rarely IIR filters. These can only be represented by poles and zeros in the variable $i\omega$ for a limited number of filter coefficients, although a more suitable representation for them is in terms of the variable z, using the z-transform (see above). So, to specify these filters one has to either use the time domain filter coefficients (the most common way) or give discrete amplitude and phase values of the response function. Since FIR filters are very sharp and usually have no phase shift, there is rarely a need to correct for them since the corner frequency is very close to the Nyquist frequency (see Fig. 4.15). However, in special cases a correction is made to remove the filter effect of sharp impulsive onsets, where the onset can be masked by a precursor caused by the filter (Fig. 6.10).

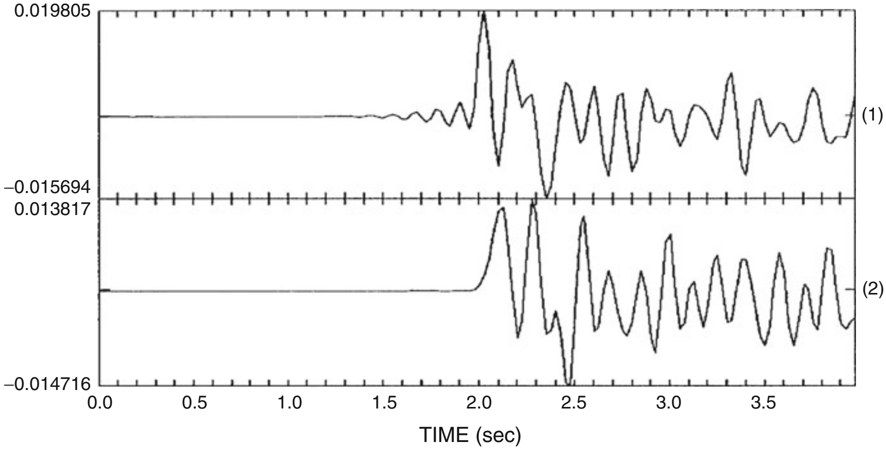

Fig. 6.10 Precursory effect of a FIR filter. The *top trace* shows the original recorded signal, and the bottom trace the corrected signal (Reprinted from Scherbaum (2001), with kind permission of Kluwer Academic Publishers)

Discussion on how to correct for digital anti alias filters is beyond the scope of this book, see Scherbaum (2007). However, the user of modern digitizers with digital anti alias filters should be aware of the pitfalls and observe precursors to a sharp onset with suspicion. See also 4.9.

6.7 Instrument Correction and Polarity

The question often comes up whether polarity (first motion is up or down) depends on the type of sensor or more generally on the response function. The answer is NO. If the sensors have been correctly installed, a first motion UP, North or East should always produce a positive signal. However, the response function, particularly digital anti alias filters, might generate precursory signals, so that it might not be easy to observe the first motion as illustrated in Fig. 6.10. Figure 6.11 shows an example of a broadband signal (velocity trace) of a teleseismic P-wave which has been converted to displacement and acceleration. Although the signals look different, the same positive polarity is clearly seen on all 3 traces.

Filters can change the onset as seen with the FIR filter example in Fig. 6.10. Figure 6.12 shows an example of a synthetic 1 Hz signal with an onset at 2 s. It has been filtered with an 8 pole zero phase shift Butterworth filter (passing the filter forwards and backwards in time domain) and a 4 pole Butterworth filter with a phase shift (passing only forwards). As it can be seen, the filter with the phase shift has preserved the polarity while the zero phase shift filter has shifted some of the energy to before the onset and it is no longer possible to observe the correct polarity and the onset time would be wrong. This is what sometimes happens with digital zero phase shift anti alias filters. An analog anti alias filter would only pass one way

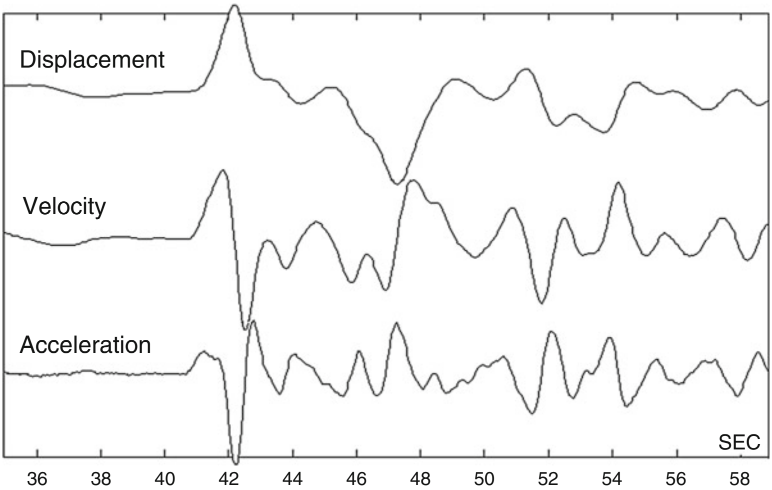

Fig. 6.11 A P-wave from a teleseismic event recorded on the velocity trace. The two other traces show displacement and acceleration (Note that the polarity of the onset is the same on all three traces)

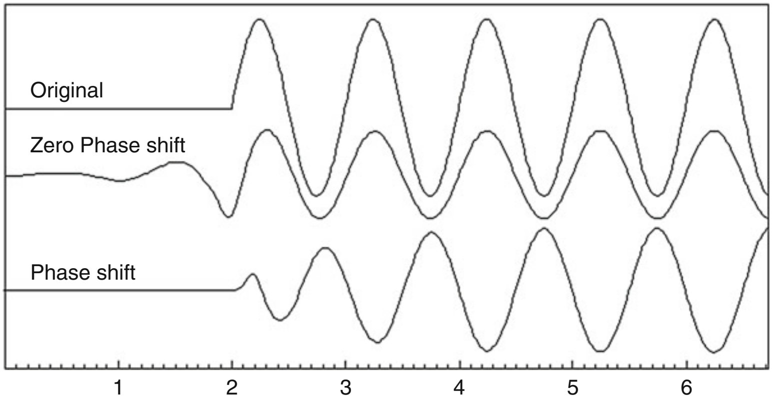

Fig. 6.12 The effect of using filters on a signal with a sharp onset. The signal is a 1 Hz sine wave with an onset at 2 s (*top trace*). The middle trace shows the signal filtered with a zero phase shift Butterworth filter while the *bottom trace* shows the signal filtered with a Butterworth filter with a phase shift. Both filters are band pass with corners at 1 and 5 Hz

and therefore not obscure the onset polarity, but the phase delay may hide the true onset in the background noise if the SNR is not high. High-order analog filters also produce a group delay that may be non-negligible. For more discussion on anti alias filters, see Chap. 4.

6.8 Combining Response Curves

We have now described the response of several individual elements, but a seismic recorder consists of several elements. How do we get the complete response? Simply by multiplying together the response functions of the individual elements. This would also be the case where the response function cannot be represented by a rational function, in which case the values of the function would just have to be evaluated at discrete frequencies. If e.g. a recorder has the following test response parameters:

Seismometer free period :	5.0 sec
Seismometer damping :	0.7
Seismometer loaded generator constant :	$200\ \mathrm{V/ms^{-1}}$
Low pass filter for anti aliasing :	25 Hz, 6 poles
High pass filter to cut DC :	0.01 Hz, 1pole
Amplifier gain :	1000 times or 60 dB
ADC converter sensitivity :	2000 counts/V

We can calculate the response of each element and combine it to the total response T_{tot} as

$$T_{tot} = T_s \cdot T_a \cdot T_{25\ Hz} \cdot T_{0.01Hz} \cdot T_{ADC} \tag{6.67}$$

where T_s is the seismometer response, T_a the amplifier response, $T_{25\ Hz}$ the 25 Hz filter response, $T_{0.01Hz}$ the 0.01 Hz filter response and T_{ADC} is the ADC response. Figure 6.13 shows the response of the elements and the combined response. A similar figure could have been made for the phase response as the sum of the phase responses of all elements.

In addition to the 3 curves shaping the response function, there is also the constant gain of the seismometer, the amplifier and the ADC. It is assumed that the filters have unity gain, but that is not always the case. The curves were made with SEISAN (Havskov and Ottemöller 1999) and the total gain at 1 Hz is $2.51 \cdot 10^9$ counts/m, which is a typical value for many sensitive SP seismic stations. This value corresponds to 2.51 counts/nm. It can often be useful to calculate such gain constants manually for checking and to get an idea of their values. We will illustrate this with the above values.

Seismometer: Since the natural frequency is at 0.2 Hz, and the damping is 0.7, we can assume that the response curve is flat for velocity at 1.0 Hz so the gain for displacement is gain for velocity multiplied by the angular frequency:

$$\begin{array}{ll}
\text{Seismometer :} & G_s = 200\ \mathrm{V/ms^{-1}} 2\pi \cdot 1.0\ \mathrm{Hz} = 1257\ \mathrm{V/m} \\
\text{Amplifier :} & G_a = 1000\ \mathrm{V/V} \\
\text{Filters :} & G_{0.01Hz} = 1\ \mathrm{V/V} \\
 & G_{25Hz} = 1\ \mathrm{V/V} \\
\text{ADC :} & G_{ADC} = 2000\ \mathrm{counts/V}
\end{array}$$

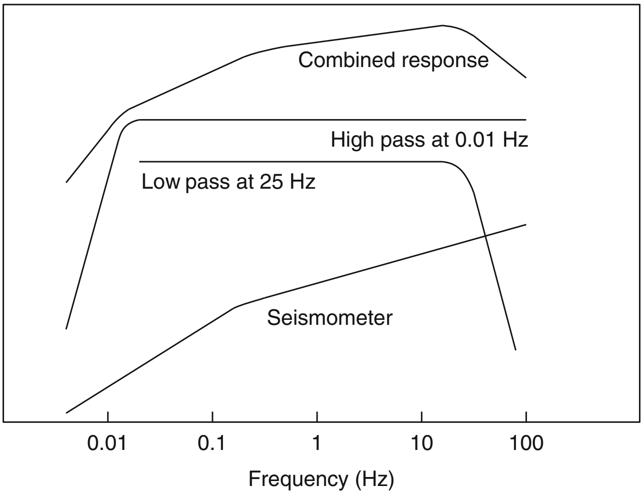

Fig. 6.13 Combining response functions. The figure shows the displacement response function of the seismometer and the amplitude response functions of the two filters. Both scales are logarithmic. The y-axis is arbitrary

Combining all, we get a total gain G_{tot}:

$$G_{\mathrm{tot}} = G_{\mathrm{s}} \cdot G_{\mathrm{a}} \cdot G_{0.01\mathrm{Hz}} \cdot G_{25\,\mathrm{Hz}} \cdot G_{\mathrm{ADC}} = \tag{6.68}$$

1257 V/m •1000 V/V •1 V/V • 1 V/V • 2000 Counts/V = 2.51 •10^9 counts/m at 1.0 Hz.

Note how all the units cancel out to give the correct unit. This is a good way of checking if all numbers have the correct units.

We can now look at how the displacement response function looks like in poles and zero representation. The frequency is assumed to be in radians/s, if in Hz, a conversion must be made, see below. The seismometer has the poles (real and imaginary part):

```
-0.87964594E+00 0.89741838E+00
-0.87964594E+00 -0.89741838E+00
```

and the zeros:

```
0.00000000E+00 0.00000000E+00
0.00000000E+00 0.00000000E+00
0.00000000E+00 0.00000000E+00
```

and a normalization constant of 1.0 ms^{-1}/m not taking the generator constant into account. These values can be obtained directly from (6.57). Note the unit: The seismometer input is displacement and the output a measure of velocity. The generator constant then converts this to Volts. For the 25 Hz low pass Butterworth filter, we have the 6 poles:

```
-40.6552    151.727
-111.072    111.072
-151.727    40.6552
-151.727   -40.6552
-111.072   -111.072
-40.6552   -151.727
```

For a low-pass filter of order n and angular frequency ω_f, the normalization frequency is ω_f^n, if its flat gain is 1. So the normalization constant is $\omega_{25}^6 = 1.50 \bullet 10^{13}$ V/V. Thus to get of gain of 1.0, the response calculated with poles and zeros must be multiplied with the normalization constant.

For the 0.01 Hz high pass Butterworth filter, we have 1 pole and 1 zero:

```
Pole:
 -0.6283E-01 0.5493E-08

Zero:
  0.00000    0.00000
```

Normalization constant = 1.0 V/V. This is true for any high-pass with passband gain 1.

Total normalization constant = 200 V/ms^{-1} • 1.0 ms^{-1}/m • 1.5 10^{13} V/V • 1.0 V/V • 1000 V/V • 2000 counts/V = 6.0 • 10^{21} counts/m. NOTE: This is not the gain at any one frequency but the number to multiply with once the total pole and zero response calculation has been made.

This looks very different from the constant we calculated for 1.0 Hz for the total gain 2.51 10^9 counts/m. How can we compare one to the other? We again calculate at 1 Hz. The filters used have no effect at 1 Hz (their responses are flat at this frequency) so we can divide the normalization constant by the total filter gain (filter normalization constants). The normalized displacement response curve has a gain of 6.28 (=2π•1) at 1 Hz, so the total gain at 1 Hz is

$$6.0 \cdot 10^{21} \text{counts/m} \cdot 6.28/\left(1.5 \cdot 10^{13}\right) = 2.51 \cdot 10^9 \text{ counts/m}$$

Normalization constants used with poles and zeros representation of response curves will then often have little in common with an understandable physical number.

6.8.1 Poles and Zeros Given in Hz

Some manufactures give the poles and zeros in Hz. Since radian is the most used unit, these values must be converted by multiplying the poles and zeroes by 2π. The normalization constant in radians, C_{radian}, can be obtained from the normalization constant in Hz, C_{Hz} as $C_{radian} = C_{Hz} \bullet (2\pi)^{(\text{number of poles}-\text{number of zeros})}$

6.9 Common Ways of Giving Response Information

Working with response information supplied with some data can be quite frustrating, since it can be given in so many ways and different conventions can be used. One would think that the most natural way of supplying information should be in terms of displacement response with units of counts/m. Unfortunately, life is not so easy. In addition, different processing systems use different ways of specifying the response and also using different formats.

Apart from using different conventions, the response can be given both as velocity and displacement response and sometimes even as acceleration response. The last is of course tempting for accelerometers, since it is so simple, however, *it is strongly recommended to, at least within the same processing system, always use the same type of response curves and preferably displacement*. Doing it differently can create endless frustrations. Like when going from velocity to displacement response, it is not always easy to remember whether we have to multiply or divide by ω, and it is certainly done the wrong way many times.

In summary, the most common ways of representing response information are, in order of importance:

– Poles and zeros (PAZ)
– Individual parameters (free period, ADC gain etc.)
– Polynomials
– Combination of the above
– Discrete frequency, amplitude and phase values (FAP)
– Time domain filter coefficients

The discrete amplitude and phase values can be measured in a practical calibration (see Chap. 10) and the software used for correction for the calibration must then interpolate between the discrete values.

The two most common international waveform formats are SEED and GSE, both created with the help of many experts. However, the response information is not provided in the same way for the two formats.

The most common open source processing systems are SAC (IRIS 2014) and SEISAN. SEISAN can use GSE, SAC and SEED (ASCII RESP-files) formats (in addition to native SEISAN format) while SAC has its own format and may use GSE also. SEISAN, SEED and GSE formats can include the response within the

Table 6.1 An example of a GSE2.0 response file

```
CAL2 BERG   S Z       Sensor    0.40E+00      1.   50.00000 2000/ 1/1 0: 0
PAZ2   1 V   0.30043413E+07                   9    4 Laplace transform

 -0.87964594E+00  0.89741838E+00
 -0.87964594E+00 -0.89741838E+00
 -0.40655220E+02  0.15172728E+03
 -0.11107207E+03  0.11107207E+03
 -0.15172728E+03  0.40655182E+02
 -0.15172728E+03 -0.40655205E+02
 -0.11107205E+03 -0.11107209E+03
 -0.40655193E+02 -0.15172728E+03
 -0.62831856E-01  0.54929354E-08
  0.00000000E+00  0.00000000E+00
  0.00000000E+00  0.00000000E+00
  0.00000000E+00  0.00000000E+00
  0.00000000E+00  0.00000000E+00
DIG2   2   0.20000000E+07     50.00000 DigModel
```

All numbers are from the test response information in 6.8. First line gives station and component (BERG, S Z), sensitivity at reference period (1 s) in nm/count (0.40), sample rate and date. Second line (PAZ2) gives total normalization factor of seismometer and filter in V/nm (0.3e7) and number of poles (9) and zeros (4). The poles are listed first and then the zeros. Last line gives total gain for amplifiers and AD converter in counts/V (0.2e7) and sample rate (50). Sensitivity and sample rate are not needed. If line DIG2 is not there, it is assumed that DIG2 factors are in PAZ2 line and the normalization constant is more properly called a scaling factor. The first line lists the sensor model (Sensor) and the last line the digitizer model (DigModel)

data, while SAC cannot. Examples of response files will be given in the following as well as examples of how to switch from one format to another, which is not always a trivial matter.

6.9.1 GSE

The combined response, from the test response example above is defined in the GSE2.0 format as follows (Table 6.1)

The gain and scaling factors are:

Sensitivity: 10^9 nm/m/2.51 • 10^9 counts/m = 0.40 nm/count since at one Hz the total gain is 2.51 • 10^9 counts/m

Total normalization factor of seismometer and filter: 200 V/ms^{-1} 1.5 • 10^{13} ms^{-1}/m • 10^{-9} m/nm = 0.3 • 10^7 V/nm

Total gain for amplifiers and AD converter: 2000 V/count • 1000 V/V = 0.2 • 10^7 counts/V

The GSE file has the sum total of the individual elements' poles and zeros as seen above. However, looking at the complete response file in Table 6.1, it is difficult to see which parts originate from which section of the recorder. In

SEED, the different parts are usually separated (Table 6.2). This is also possible in GSE format. For a complete description of GSE format, see GSETT-3 (1997). Since the GSE format is an ASCII format, the response information will appear as shown above. In GSE, FAP (frequency, amplitude and phase) can also be used. The GSE format ONLY works with displacement response.

6.9.2 SEED

The SEED response is part of the SEED volumes and is well defined. It can be given in many ways and all stages in the response are described separately. Table 6.2 shows an example from a Quanterra station with a STS2 seismometer. The values are typical for the GSN network. The main parameters are:

$$\begin{array}{ll}
\text{Generator constant :} & 1500\ \ \mathrm{V/ms^{-1}} \\
\text{Period :} & 120\ \ \mathrm{s} \\
\text{Damping :} & 0.7 \\
\text{ADC sensitivity :} & 0.41 \cdot 10^6 \mathrm{counts/V}
\end{array}$$

In addition, there are antialias filters and other filters. The response information is only contained in the binary SEED headers and has been dumped with an IRIS read program rdseed into an ASCII file, the so called SEED RESP file. Only a small part of the file is shown giving the main information and some of the filter coefficients.

In SEED, displacement, velocity and acceleration response can be specified. The response has been divided into stages. Stage 1 is the seismometer which has a velocity response described with 2 zeros and 5 poles and a normalization constant A0 of $6.0 \cdot 10^7$. We recognize the 2 zeros and the 2 first poles as coming directly from the seismometer parameters (6.58). What are the last 3 poles? Since a seismometer has a normalization constant of 1.0, A0 must be related to the last 3 poles. Taking out the normalization constant and the last 3 poles and plotting the response, we can get an idea of what the last 3 poles represent (Fig. 6.14).

Figure 6.14 shows the response of the three last poles, with the normalization constant. These poles are due to the feedback circuit in the seismometer and behave like a third order low-pass built into the sensor. The stage sequence number 1, second part also gives the seismometer generator constant. Stage sequence 2 gives the ADC sensitivity and stage sequence 3 is the filter coefficients for the first anti alias filter, note it has no gain. Now several filters are given (not shown in Table 6.2) and finally at the end comes sequence number 0 which is all gain coefficients multiplied together, A_{all}, except the poles and zero normalization constant:

$$1500\ \ \mathrm{V/ms^{-1}} \cdot 0.41 \cdot 10^6 \mathrm{counts/V} = 6.2 \cdot 10^8 \mathrm{counts/ms^{-1}}$$

Table 6.2 Example of response information for a SEED file

```
----------break---------------
#             +          +------------------------------------------+
#             +          |   Response (Poles & Zeros),   CART ch BHZ  |
#             +          +------------------------------------------+
#
B053F03     Transfer function type:   A [Laplace Transform (Rad/sec)]
B053F04     Stage sequence number:    1
B053F05     Response in units lookup: M/S - Velocity in Meters per
Second
B053F06     Response out units lookup: V - Volts
B053F07     A0 normalization factor:   6.0077E+07
B053F08     Normalization frequency:   0.02
B053F09     Number of zeroes:          2
B053F14     Number of poles:  5
#              Complex zeroes:
#                i  real          imag         real_error    imag_error
B053F10-13    0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
B053F10-13    1  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
#              Complex poles:
#                i  real          imag         real_error    imag_error
B053F15-18    0 -3.700400E-02 -3.701600E-02  0.000000E+00  0.000000E+00
B053F15-18    1 -3.700400E-02  3.701600E-02  0.000000E+00  0.000000E+00
B053F15-18    2 -2.513300E+02  0.000000E+00  0.000000E+00  0.000000E+00
B053F15-18    3 -1.310400E+02 -4.672900E+02  0.000000E+00  0.000000E+00
B053F15-18    4 -1.310400E+02  4.672900E+02  0.000000E+00  0.000000E+00
----------break----------------
#             +        +--------------------------------------+
#             +        |       Channel Gain,  CART ch BHZ      |
#                      +--------------------------------------+
----------break----------------
B058F03     Stage sequence number:                  1
B058F04     Gain:                                   1.500000E+03
B058F05     Frequency of gain:                      2.000000E-02 HZ
B058F06     Number of calibrations:
----------break----------------
#             +        +--------------------------------------+
#             +        |       Channel Gain,  CART ch BHZ      |
#             +        +--------------------------------------+

B058F03     Stage sequence number:                  2
B058F04     Gain:                                   4.117280E+05
B058F05     Frequency of gain:                      0.000000E+00 HZ
B058F06     Number of calibrations:                 0
----------break-----------------
#                +--------------------------------------+
#                 |   Channel Sensitivity,  CART ch BHZ   |
#                 +--------------------------------------
#
B058F03     Stage sequence number:                  0
B058F04     Sensitivity:                            6.175900E+08
B058F05     Frequency of sensitivity:               2.000000E-02 HZ
B058F06     Number of calibrations:                 0
----------break------------------------
```

Only a small part of the file is shown, break indicates that a part is missing in between

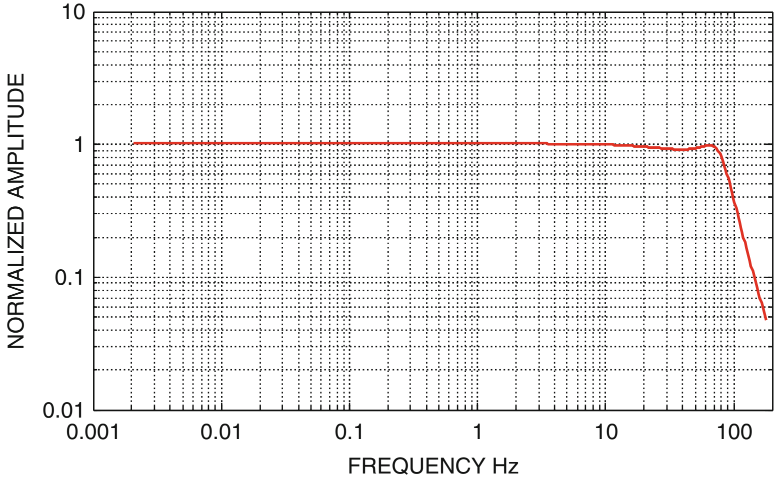

Fig. 6.14 Response curve corresponding to the last three poles and the normalization constant as given in Table 6.2 for the seismometer in stage 1

If the data from Table 6.2 are to be used in a GSE response file, one zero has to be added to the poles and zero representation to make it displacement. The other constants are calculated as:

Sensitivity in CAL2 line:

$$\begin{aligned} 10^9/\,(\mathrm{A_{all}} \bullet\ 2\pi\,\mathrm{f}) &= 10^9\ \mathrm{nm/m}/\left(6.2 \bullet 10^8 \mathrm{counts/ms^{-1}} \bullet 1 \bullet 2\pi \bullet 1\ \mathrm{Hz}\right) \\ &= 0.26\ \mathrm{nm/counts} \end{aligned}$$

Total normalization factor of seismometer and filter in PAZ2 line:

$$1500\mathrm{V/ms^{-1}} \bullet\ 6.0\ \bullet\ 10^7\mathrm{ms^{-1}/m}\ \bullet\ 10^{-9}\mathrm{m/nm}\ =\ 90\ \mathrm{V/nm}$$

Total gain for amplifiers and AD converter in DIG2 line: $0.41 \bullet 10^6$ counts/V

6.9.3 SAC

Seismic Analysis Code (SAC) is a general-purpose interactive program designed for the study of time sequential signals (Goldstein et al. 2003; Goldstein and Snoke 2005). SAC has a number of ways to correct for instrumentation, e.g. for predefined instruments, response by the program EVALRESP (by IRIS) and the response given as PAZ in a text file per channel. The units are meters to counts like SEISAN and GSE. The response files can be written in different ways like

```
ZEROS 3 POLES 4 -0.0123 0.0123 -0.0123 -0.0123 -39.1800 49.1200
-39.1800 -49.1200 CONSTANT
3.832338e+12
```

where the zeros are assumed zero and therefore not written. If the zeros are not zero they must of course be written like

```
ZEROS 5
867.0800 904.7790
867.0800 -904.7790
POLES 4
-0.1480 0.1480
-0.1480 -0.1480
-314.1590 202.3190
-314.1590 -202.3190
CONSTANT 7.028933e+07
```

So the format is flexible and sometimes a bit confusing.

6.9.4 SEISAN

SEISAN is using an ASCII format for response files. SEISAN can use GSE, SAC and SEED, but it also has a native SEISAN format which can use PAZ, FAP or parameters. SEISAN only use displacement response. Assuming a Butterworth filter with 3 poles and a corner frequency of 80 Hz, the SEISAN parameter format with the same information as in Table 6.2 (without the antialias filters) is (Table 6.3):

Table 6.3 SEISAN response file with parameters from Table 6.2

```
CART BH Z101    1  1  1  1  0  0.000

 120.000  0.7000  1500.0     0.0 .41E+06 .39E+10 80.0      3.00
```

The first line gives station, component and date from which response is valid. The parameters are from left to right: Seismometer period (s), damping, generator constant (V/ms^{-1}), amplifier gain (dB), ADC gain (counts/V), total gain at 1.0 Hz (counts/m), filter frequency and number of poles. The total gain is calculated by the response file program and is therefore not a required parameter

Table 6.4 SEISAN response file with poles and zeros from Table 6.3

```
CART BH Z101    1   1   1   1   0   0.000

    5     3 0.3685E+17-0.3700E-01-0.3702E-01-0.3700E-01 0.3702E-01
  -251.3       0.000      -131.0      -467.3       -131.0       467.3
   0.000       0.000       0.000       0.000        0.000       0.000
```

The first 3 values are the number of poles, number of zeros and total gain constant (counts/m) respectively. Following are first the poles and then the zero pairs

The SEISAN poles and zeros representation is seen in Table 6.4.

The SEISAN poles and zeros representation look almost as the SEED file except that one more zero has been added to make it a displacement response. The total gain constant has been calculated as stage 0 gain times poles and zero normalization constant A0

$$\text{Total gain} = 6.2 \cdot 10^{8}\text{ms}^{-1}/\text{m}\ \ 6.0 \cdot 10^{7}\text{counts}/\text{ms}^{-1} = 0.37 \cdot 10^{17}\text{counts}/\text{m}$$

6.9.5 How to Make and Maintain Response Files

There are thus many ways of representing the response information. SEED is the most complete and best defined but also the most complex. SEED information MUST be part of the SEED data file, while MiniSeed does not have response information. Using GSE, MiniSeed, SAC and SEISAN waveform formats, the response information can be added later and kept in separate files. The advantage with SEED response integrated in the SEED waveform file is that the information always is there, but if it is wrong, it is hard to change. The SEED RESP files can also be used with some analysis programs so this is a simple way of using SEED response information. SEISAN and GSE formats are probably the least confusing since they always are in displacement and there are not so many variations of the format.

When operating more than a few stations, it is common to have changes in response information and it is unfortunately not always updated when station parameters are changed. In that case, it is an advantage to be able to add and modify response information afterwards and keep a data base of changing response information with time. If this is too complicated to do, it will often not be done and there are many networks without proper calibration records available for processing. It is therefore a help to have a processing system that easily can generate calibration files from any kind of instrumental information. In SEED, the complete response history is kept in the response section, as a header for the SEED file or in separate RESP file. In SEISAN, each change requires an additional response file. Other systems use data bases to keep track of the response files.

A response file can be made manually following the format description for the file. However, it is much easier to use a program. IRIS/DMC (http://www.iris.edu/pub/programs) has a program PDDC to generate data-less SEED files (RESP files) and the program has built in constants for the most used instruments. So it is just a question of selecting the correct instruments.

SEISAN has a program to generate response files in SEISAN and GSE formats. The SEISAN program requires that the user know all the parameters, however this also makes it more versatile since any combination of parameters can be used like discrete measured values for one element and poles and zeros for another element.

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