Properties of the Hodgkin-Huxley equations

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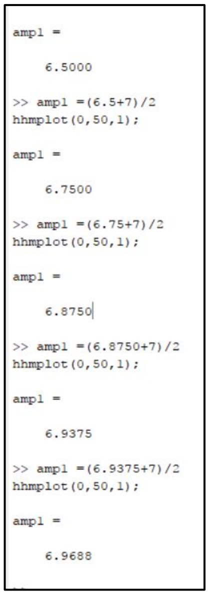
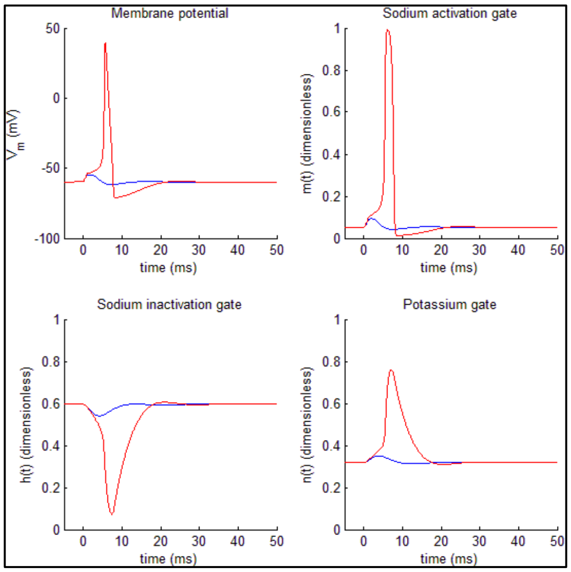
Question 1

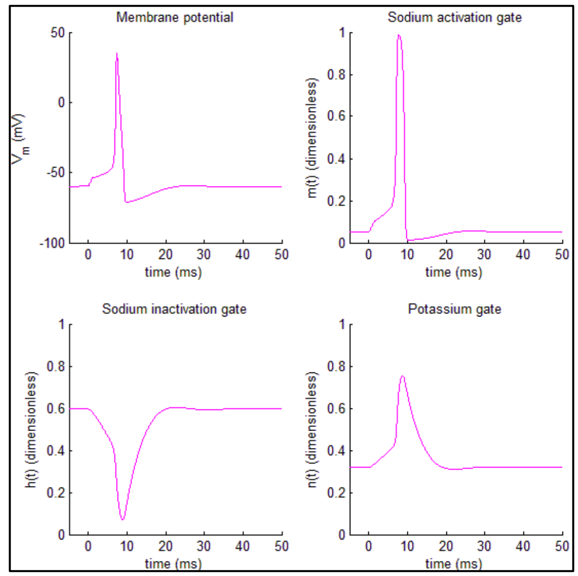
When we put values 6 $\mu A\ cm^{-2}$ and 7$\mu A\ cm^{-2}$we can observe the action potential is not generated for 6 $\mu A\ cm^{-2}$ but it is observed for 7$\mu A\ cm^{-2}$. Therefore to obtain threshold values we can apply bisection continuously until we observe an action potential. Action potential is observed at 6.9688$\mu A\ cm^{-2}$. By rounding off to lower 6.96$\mu A\ cm^{-2}$action potential is observed but for 6.95$\mu A\ cm^{-2}$ no action potential is observed.

Answer: 6.96$\mu A\ cm^{-2}$





6 $\mu A\ cm^{-2}$ and 7$\mu A\ cm^{-2}$



6.96 $\mu A\ cm$

Question 2

Consider the following net current densities with different amplitudes applied for different widths. By comparing net current observed and sum of currents $(qna + qk + ql\ )$ through each gates,

J $(\mu A cm^{-2})$	$width$ (ms)	$\int J_{ei}$	$qna + qk + ql$			$\int \sum J_k$	Reasonable approximation
			qna	qk	ql		
6	1	6	-71.2768	236.8179	-159.5414	5.9997	6
6.5	1	6.5	-76.9595	242.2239	-158.7648	6.4996	6.5
7	1	7	1.3627x10^3	1.5003x10^3	-130.5537	7.0014	7
10	1	10	-1.4341ex10^3	1.5780x10^3	-133.9062	9.9984	10
6	2	12	-1.4316x10^3	1.5774x10^3	-133.7965	11.9998	12
6.5	2	13	-1.4342x10^3	1.5811x10^3	-133.9148	12.9996	13
7	2	14	-1.4360x10^3	1.5840x10^3	-134.0070	14.0000	14
10	2	20	-1.4421x10^3	1.5964x10^3	-134.2957	19.9994	20

We can observe the $Net\ inward\ current\ =\ algebraic\ sum\ of\ the\ currents\ through\ ionic\ gates$

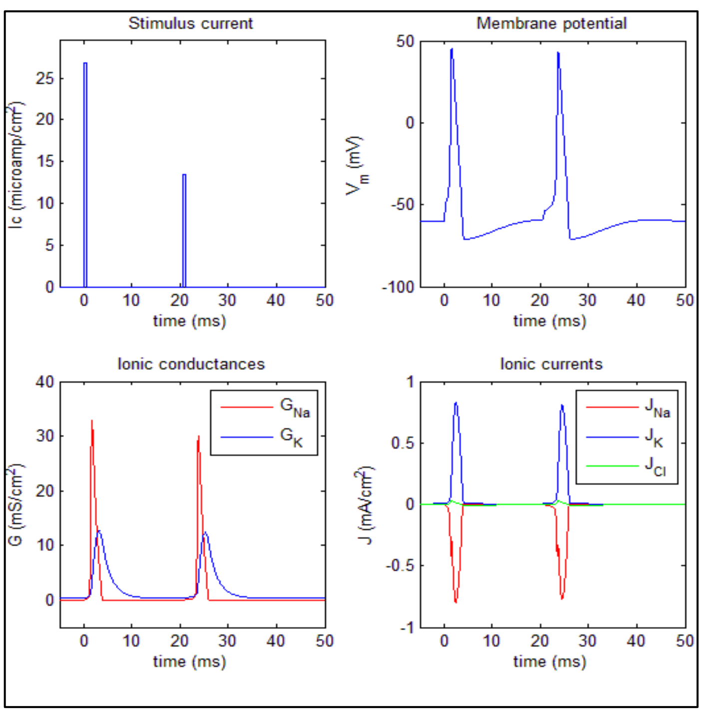
From the above table it can be observed that the value of the stimulating current is approximately equal to the net inward current of the neuron.

$$\int_{t_0}^{t_f} J_{ei} \approx \int_{t_0}^{t_f} \sum J_k$$

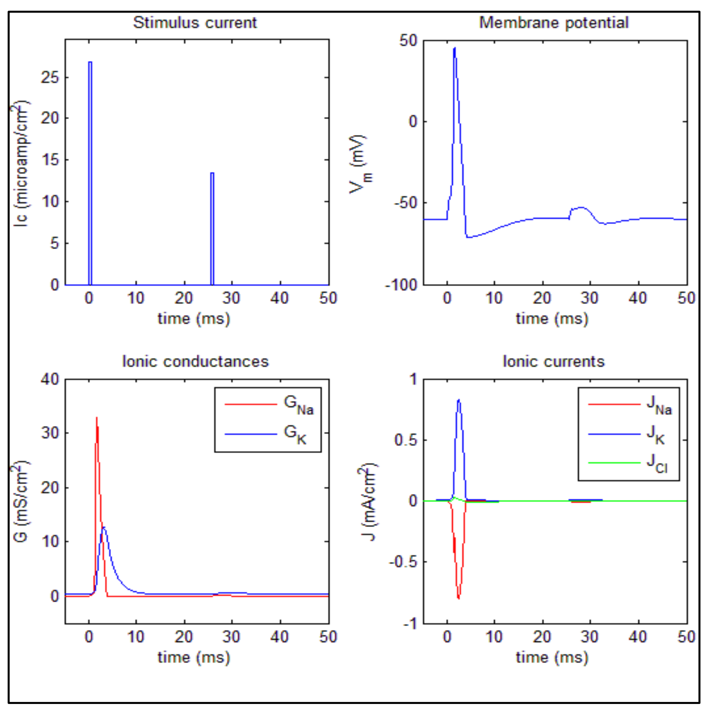
Question 3

The threshold current to for a single pulse is $I_{1th} = 13.4\ \mu Acm^{-2}$.with amplitude of thee the 1st pulse being set to twice as this threshold value, that is $26.8\ \mu Acm^{-2}$. Then, the minimum value of the second amplitude which gives a second action potential is found for different delays between 2 amplitudes for an accuracy of $0.1\ \mu Acm^{-2}$. Throughout the process width of the second amplitude is kept at $0.5ms$

$\boldsymbol{Delay\ (ms)}$	25	24	22	20	18	16	14	12	10	8	6	4
$\boldsymbol{I_{2th}(\mu Acm^{-2})}$	13.7	13.4	12.6	11.6	11.3	12.7	17.0	25.5	40.8	70.1	145.2	

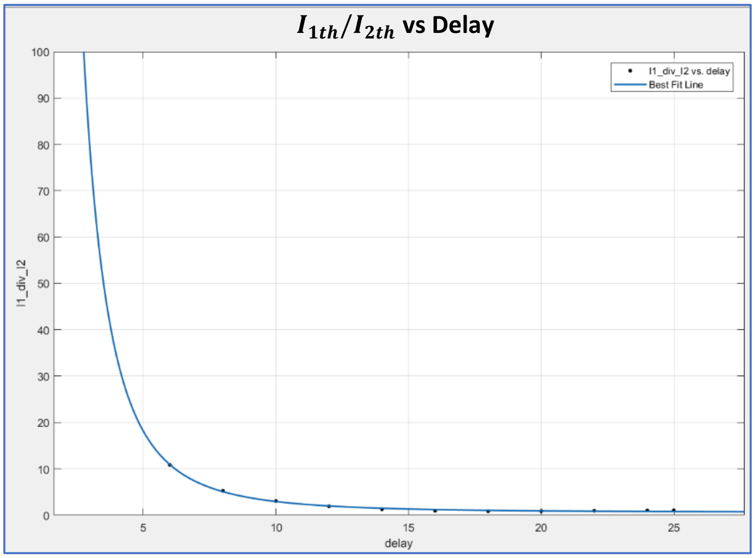


Amplitude 2 generating an action potential



Magnitude of Amplitude 2 being insufficient to generate an action potential

Question 4




```
%Plotting I1st/I2nd vs delay
%MatLab 2017a version
I1 = 13.4
I2 = [13.7 13.4 12.6 11.6    11.3    12.7    17.0 ...
    25.5    40.8    70.1    145.3];
delay  = [25 24 22 20 18    16  14  12  10  8    6];
I1_div_I2 = I2./I1;
cftool(delay, I1_div_I2);
```

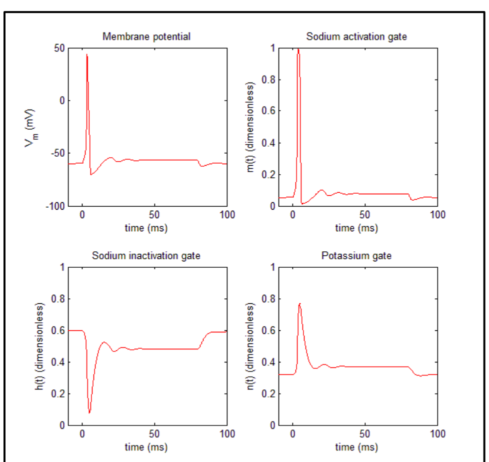
Best fit line is plotted using ctftool box in MatLAB 2017a

The period from the initiation of the action potential to immediately after the peak is referred to as the **absolute refractory period (ARP).** This is the time during which another stimulus given to the neuron (no matter how strong) will not lead to a second action potential. Since Na+ channels are inactivated during this time, additional depolarizing stimuli do not lead to new action potentials.

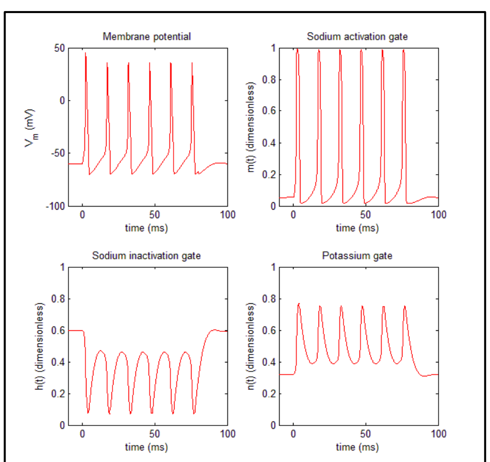
According to the graph we can observe that the ratio increases rapidly as delay is limited towards $5ms$. That is I_{2th}become very large compared to I_{2th}, therefore we can assume that delay below $5ms$. Do nott generate action potentials. That is $\boldsymbol{Absolute\ refractory\ period\ (ARP)\ =\ 5ms}$

During the **relative refractory period(RRF)**, the neuron can be excited with stimuli stronger than that needed to bring a resting neuron to threshold. That is relative refractory period is the period which I_{2th} is greater than I_{1th}. That is ratio is greater than 1. According to the graph we can see that ratio reaches 1 as delay is increased to $20ms$. Therefore, relative refractory period lies between $5ms$ and $20ms$.(length of relative refractory period is $15ms$)

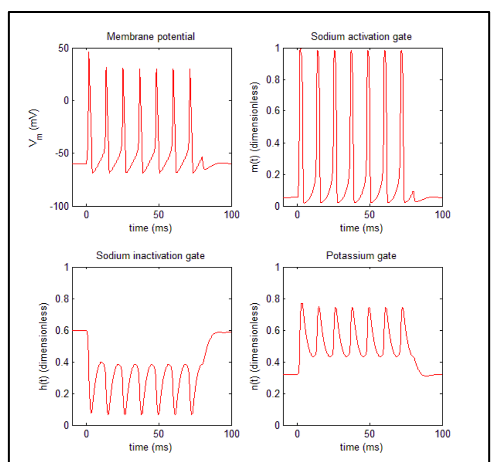
Question 5



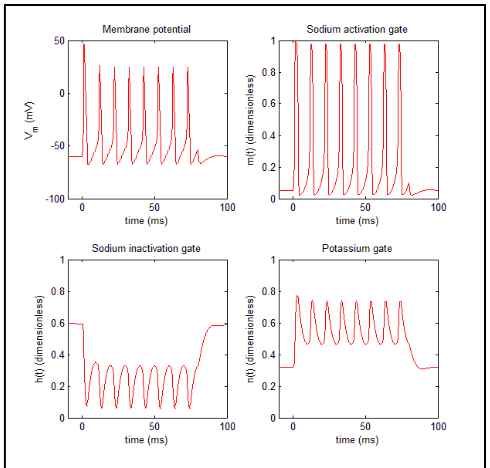
$5\mu A\ cm^{-2}$, APs = 1



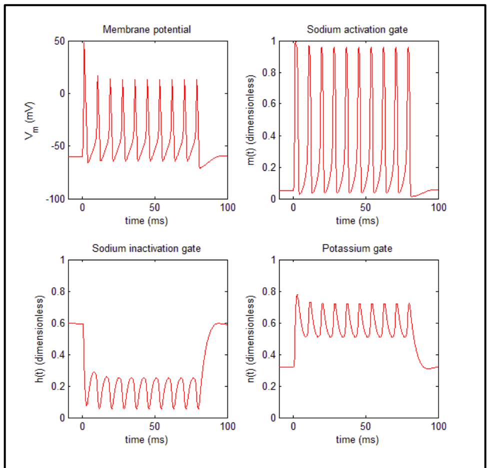
$10\mu A\ cm^{-2}$, APs = 6



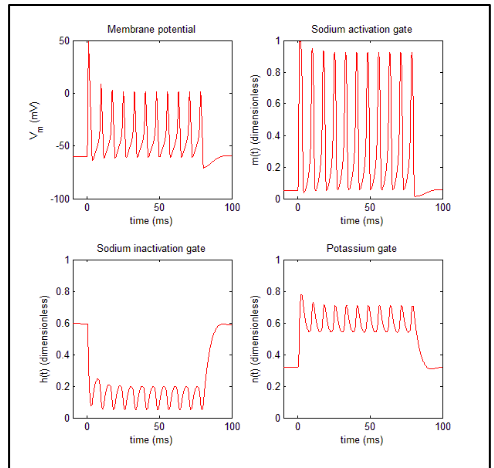
$20\mu A\ cm^{-2}$, APs = 7



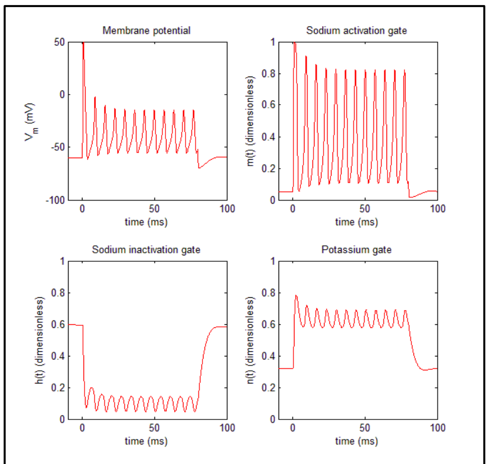
$30\mu A\ cm^{-2}$, APs = 8



$50\mu A\ cm^{-2}$, APs = 10



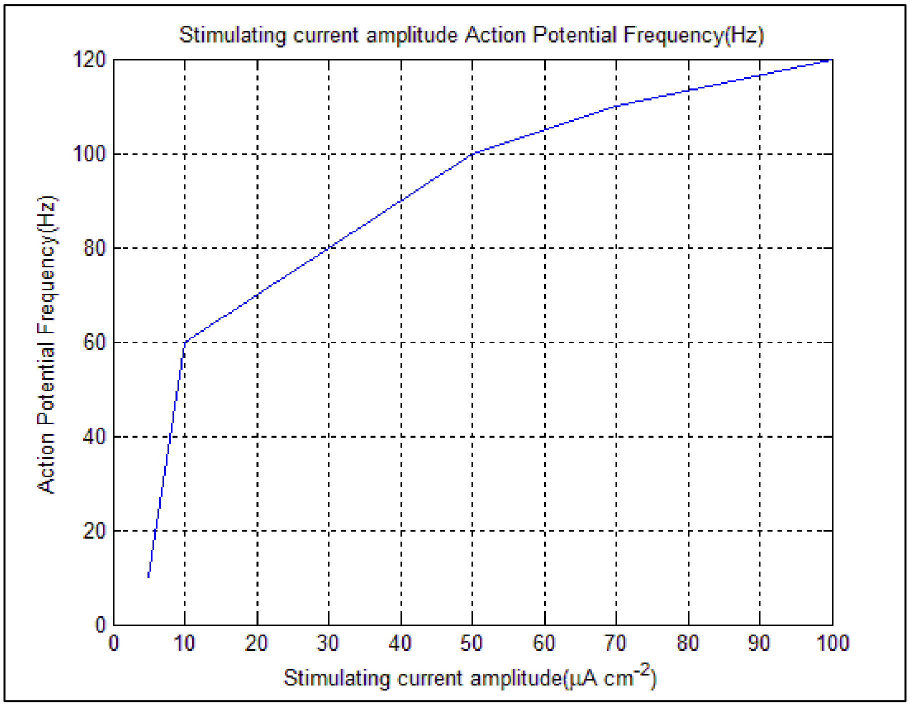
$70\mu A\ cm^{-2}$, APs = 11



$100\mu A\ cm^{-2}$, APs = 12

Stimulating current amplitude $(\mu A cm^{-2})$	5	10	20	30	50	70	100
Action Potential Frequency (Hz)	10	60	70	80	100	110	120

The plot of the action potential frequency as a function of the stimulus current.



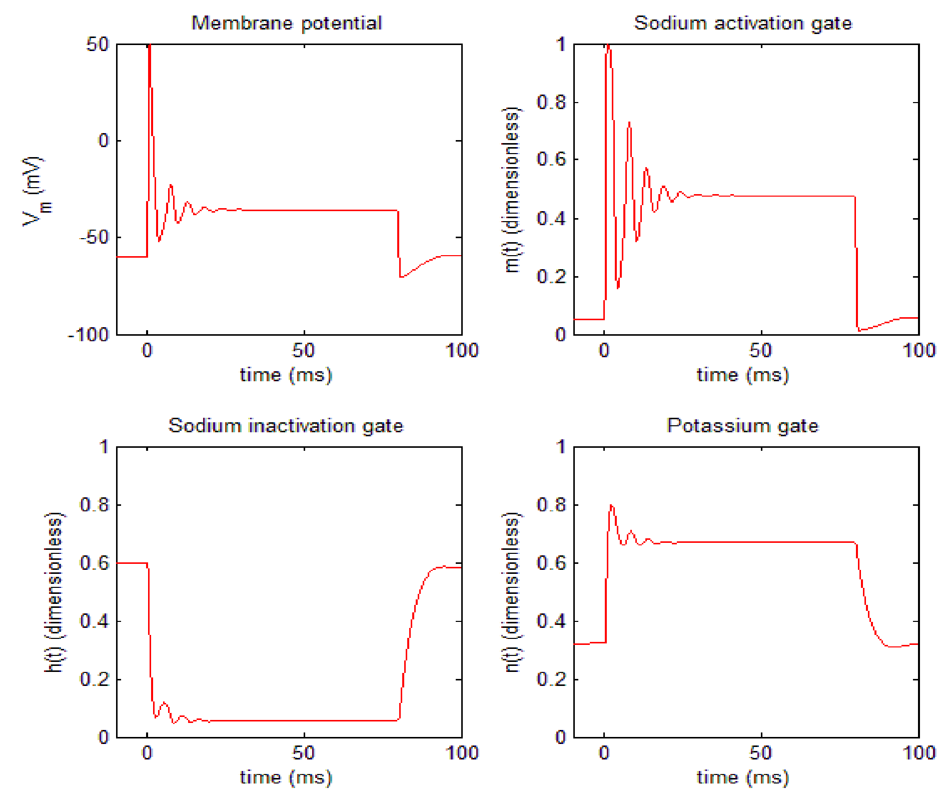

```
%Plot of action potential frequency as a function of
%stimulating current amplitude

sim_I   = [5    10  20  30  50  70  100];
ap_freq = [10   60  70  80  100 110 120];
figure();
plot(sim_I,ap_freq);
grid on;
xlabel('Stimulating current amplitude(\muA cm^{-2})');
ylabel('Action Potential Frequency(Hz)');
title('Stimulating current amplitude Action Potential Frequency(Hz)');
```

When action potentials are considered we can observe that as the stimulating current amplitudes increase amplitude of action potentials decrease but 1st action potential generated is similar in all the above plots. That is decrease in amplitude of action potential can be observed form 2nd and onwards.

From the second plot we can observe that as the simulating current amplitude increases action potential frequency increases. Further the rate of increment in action potential frequency decreases as the simulating current increases.

Question 6



This is the grapgh obtained when the simulating current amplitude is increased to $200\mu A\ cm^{-2}$

(Other parameters are kept as same as the in Question5)

The differential equation of the Hodgkin-Huxley model

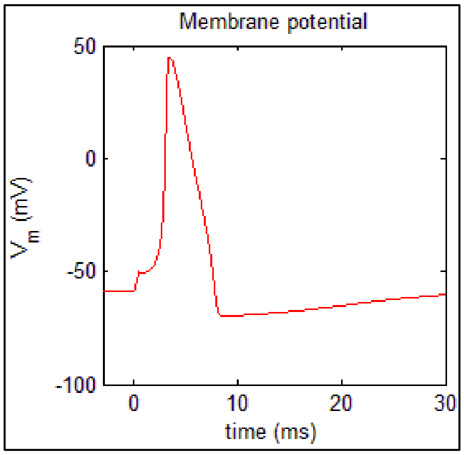
$$\frac{1}{2\pi a(r_o + r_i)v^2}\frac{d^2 Vm(z,t)}{dt^2} = C_m\frac{dV}{dt} + \overline{G_{K+}}n^4(V - V_{K+}) + \overline{G_{Na+}}m^3(V - V_{Na+})h(V - V_{Na+}) + \overline{G_L}(V - V_L)$$

"n" and "h" are voltage dependent parameters of the ordinary differential equation (ODE) of the Hodgkin-Huxley model

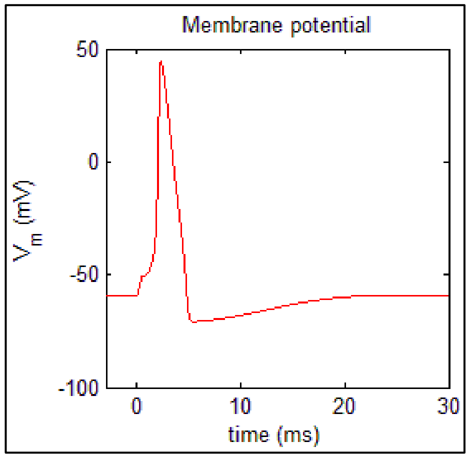
We can observe that as ODE gives an underdamped oscillatory response for simulating currents above $5\mu Acm^{-2}$. As the amplitude of the simulating current increases action potential amplitudes get affected yet, due to having low damp effect, repetitive activity of action potentials are visible. That is action potentials after 1st appearance tends to have lower amplitudes but action potentials are identifiable. As simulating current amplitude reaches $200\mu A\ cm^{-2}$, damping effect becomes higher, action potentials are not visible.

This is due to the voltage dependencies of 'n' and 'h'. According to equation degree of effect of n is 4 and that of h is 1. As higher simulating currents occur incremental effect of 'h' become lesser than damping effect of 'n'. Therefore action potentials get damped.

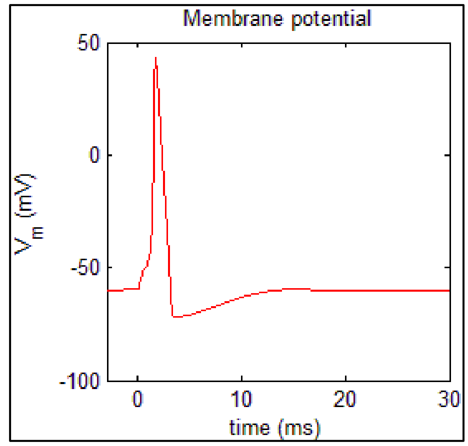
Question 7



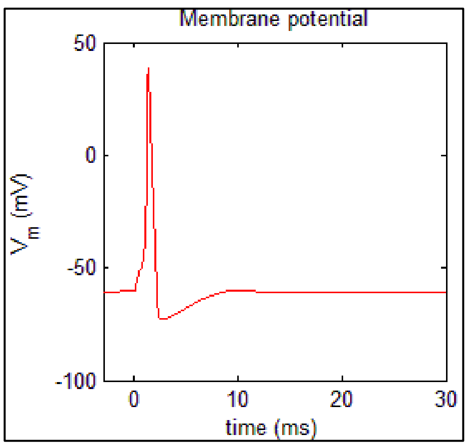
$0^{o}C$



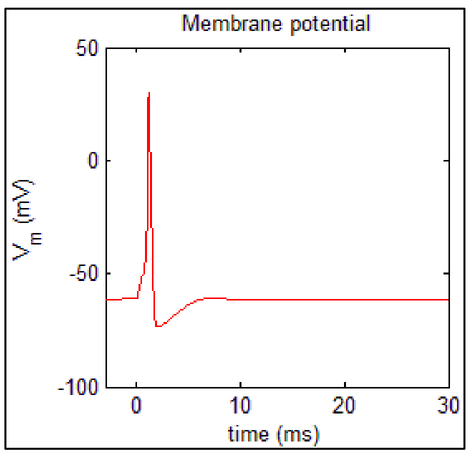
$5^{o}C$



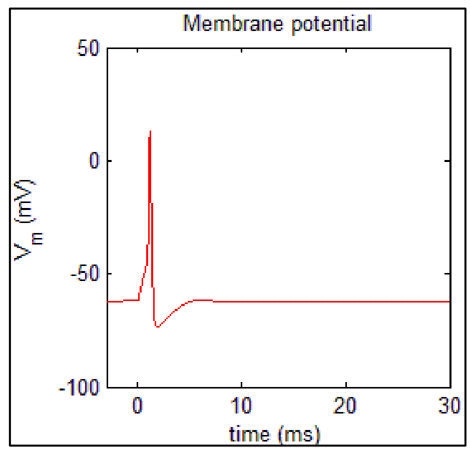
$10^{o}C$



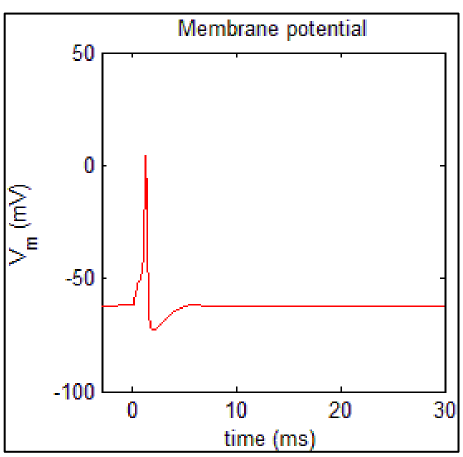
$15^{o}C$



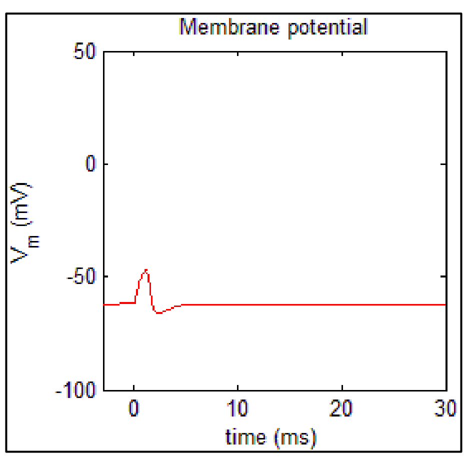
$20^{o}C$



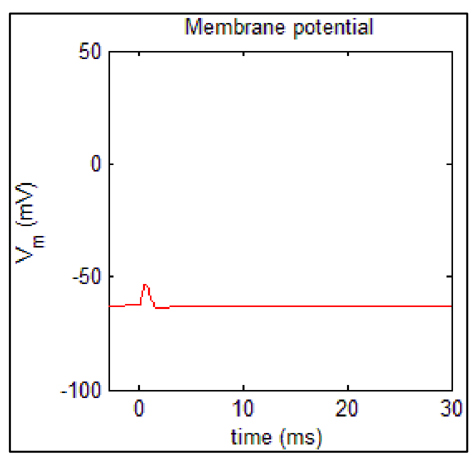
$24^{o}C$



$25^{o}C$



$26^{o}C$



$30^{o}C$

From these plots we can observe that,

- Upon increasing temperature duration of action potential decreases. Therefore absolute and relative refractory periods decrease. Therefore membranes achieve resting potential quicker at higher temperatures. Therefore action potential waveforms can be transferred at higher rate/frequency.
- Amplitude of the action potential decreases as temperature increases. At temperature above $26^{o}C$ action potential is not generated.
- When Ionic characteristic observed, current conducting durations decreases as temperature increases. After $26^{o}C$ amplitude of current generated decreases significantly.