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BM2101 Analysis of physiological systems

❖ Question 1

- At Steady state

$$\frac{dV}{dX} = V$$

- The general solutions are given by

$$V_1(X) = A_1 e^{-X} + B_1 e^{X} \qquad 0 \le X \le L_1$$
$$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X} \qquad L_1 \le X \le L_{21}$$
$$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X} \qquad L_1 \le X \le L_{22}$$

- Consider the boundary conditions
  - For parent branch at $X = 0$, applied current at the beginning of parent cylinder

$$\left.\frac{dV_1}{dX}\right|_{X=0} = -(r_i\lambda_c)_1 I_{app}$$

$$\frac{dV_1(X)}{dX} = (-1)A_1 e^{-X} + (1)B_1 e^{X}$$

$$\frac{dV_1(0)}{dX} = (-A_1) + (B_1) = -(r_i\lambda_c)_1 I_{app}$$

$$\boldsymbol{(A_1) - (B_1) = (r_i\lambda_c)_1 I_{app}}$$

  - For Terminal ends of the daughter branches that are held at rest

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0$$

$$V_{21}(L_{21}) = 0$$

$$\boldsymbol{A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0}$$

  - Similarly,

$$V_{22}(L_{22}) = 0$$

$$\boldsymbol{A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0}$$

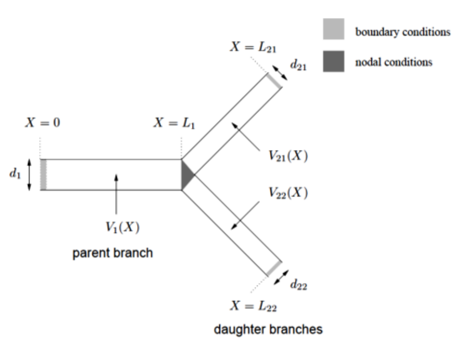


Figure 1: First order binary cable consisting of a parent branch and two daughter branches.

- Consider the Nodal Conditions
  - Continuity of membrane potential at branch nodes

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

$$V_1(L_1) = V_{21}(L_1)$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$\boldsymbol{A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0}$$

  - Similarly,

$$V_{21}(L_1) = V_{22}(L_1)$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$\boldsymbol{A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1}}$$

  - Conservation of current

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1}{dX}\bigg|_{X=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dX}\bigg|_{X=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dX}\bigg|_{X=L_1}$$

$$\frac{dV_1(X)}{dX} = (-1)A_1 e^{-X} + (1)B_1 e^{X}$$

$$\frac{1}{(r_i\lambda_c)_1}\frac{dV_1(L_1)}{dX} = \left(\frac{-1}{(r_i\lambda_c)_1}\right)A_1 e^{-L_1} + \left(\frac{1}{(r_i\lambda_c)_1}\right)B_1 e^{L_1}$$

$$\frac{dV_{21}(X)}{dX} = (-1)A_{21} e^{-X} + (1)B_{21} e^{X}$$

$$\frac{1}{(r_i\lambda_c)_{21}}\frac{dV_{21}(L_1)}{dX} = \left(\frac{-1}{(r_i\lambda_c)_{21}}\right)A_{21} e^{-L_1} + \left(\frac{1}{(r_i\lambda_c)_{21}}\right)B_{21} e^{L_1}$$

$$\frac{dV_{22}(X)}{dX} = (-1)A_{22} e^{-X} + (1)B_{22} e^{X}$$

$$\frac{1}{(r_i\lambda_c)_{22}}\frac{dV_{22}(L_1)}{dX} = \left(\frac{-1}{(r_i\lambda_c)_{22}}\right)A_{21} e^{-L_1} + \left(\frac{1}{(r_i\lambda_c)_{22}}\right)B_{22} e^{L_1}$$

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1(L_1)}{dX} = \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}(L_1)}{dX} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}(L_1)}{dX}$$

$$\frac{1}{(r_i\lambda_c)_1}\frac{dV_1(L_1)}{dX} + \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}(L_1)}{dX} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}(L_1)}{dX} = 0$$

$$\boldsymbol{-\frac{A_1 e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i\lambda_c)_{22}} = 0}$$

- **Question 2**
  - Let

$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} \qquad b = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{and}$$

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix}$$

    - $Ax = b$ gives

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix} \times \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \times A_1 & -1 \times \boldsymbol{B}_1 & 0 \times A_{21} & 0 \times B_{21} & 0 \times A_{22} & 0 \times B_{22} \\ 0 \times A_1 & 0 \times \boldsymbol{B}_1 & e^{-L_{21}} \times A_{21} & e^{L_{21}} \times B_{21} & 0 \times A_{22} & 0 \times B_{22} \\ 0 \times A_1 & 0 \times \boldsymbol{B}_1 & 0 \times A_{21} & 0 \times B_{21} & e^{-L_{22}} \times A_{22} & e^{L_{22}} \times B_{22} \\ e^{-L_1} \times A_1 & e^{L_1} \times \boldsymbol{B}_1 & -e^{-L_1} \times A_{21} & -e^{L_1} \times B_{21} & 0 \times A_{22} & 0 \times B_{22} \\ 0 \times A_1 & 0 \times & e^{-L_1} \times A_{21} & e^{L_1} \times B_{21} & -e^{-L_1} \times A_{22} & -e^{L_1} \times B_{22} \\ -e^{-L_1}/(r_i\lambda_c)_1 \times A_1 & e^{L_1}/(r_i\lambda_c)_1 \times \boldsymbol{B}_1 & e^{-L_1}/(r_i\lambda_c)_{21} \times A_{21} & -e^{L_1}/(r_i\lambda_c)_{21} \times B_{21} & e^{-L_1}/(r_i\lambda_c)_{22} \times A_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \times B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A_1 & -\boldsymbol{B}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_{21}} & B_{21}e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{22}e^{-L_{22}} & B_{22}e^{L_{22}} \times \\ A_1e^{-L_1} & \boldsymbol{B}_1e^{L_1} & -A_{21}e^{-L_1} & -B_{21}e^{L_1} & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_1} & B_{21}e^{L_1} & -A_{22}e^{-L_1} & -B_{22}e^{L_1} \\ -A_1e^{-L_1}/(r_i\lambda_c)_1 & \boldsymbol{B}_1e^{L_1}/(r_i\lambda_c)_1 & A_{21}e^{-L_1}/(r_i\lambda_c)_{21} & -B_{21}e^{L_1}/(r_i\lambda_c)_{21} & A_{22}e^{-L_1}/(r_i\lambda_c)_{22} & -B_{22}e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

    - By equalizing matrices

$$\boldsymbol{(A_1) - (B_1) = (r_i\lambda_c)_1 I_{app}}$$

$$\boldsymbol{A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0}$$

$$\boldsymbol{A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0}$$

$$\boldsymbol{A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0}$$

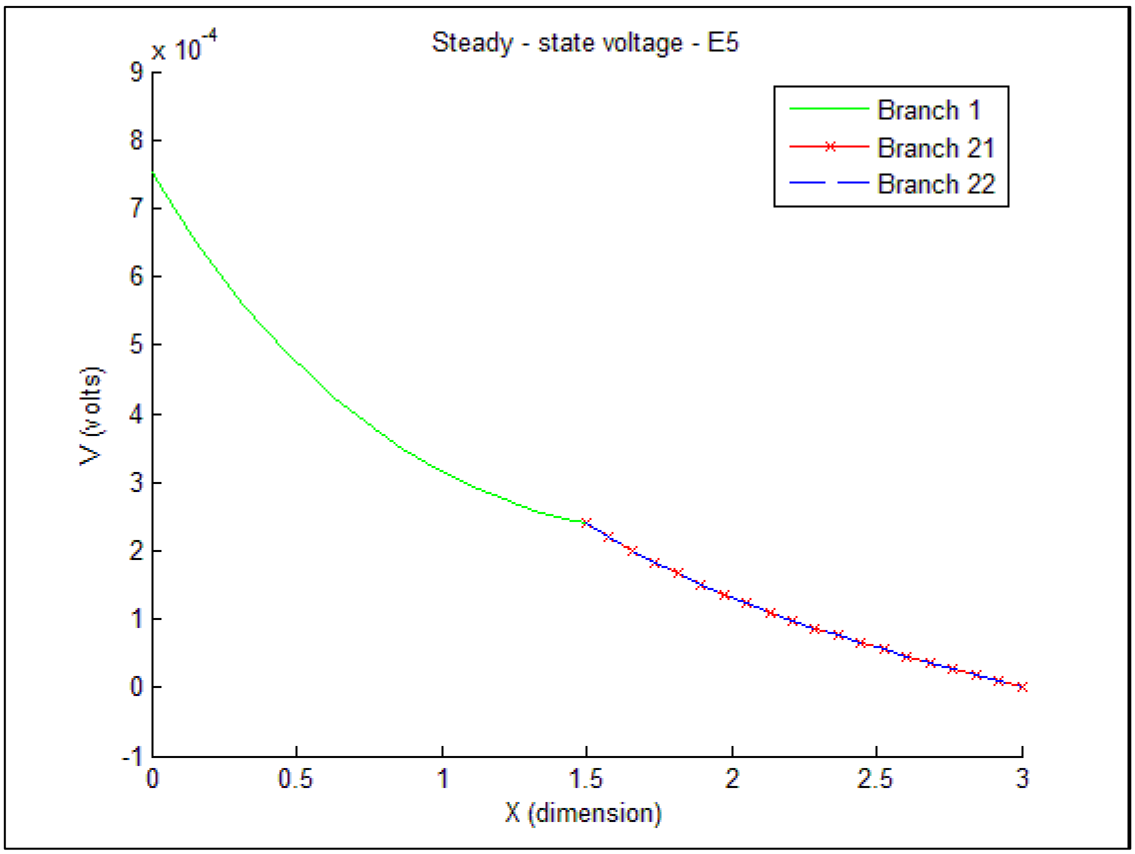
$$\boldsymbol{A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0}$$

$$\boldsymbol{-\frac{A_1e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0}$$

- **Question 3**
  - Answer obtained through MatLAB file

$$x = \begin{pmatrix} 0.0007 \\ 0.0000 \\ 0.0011 \\ -0.0000 \\ 0.0011 \\ -0.0000 \end{pmatrix}$$

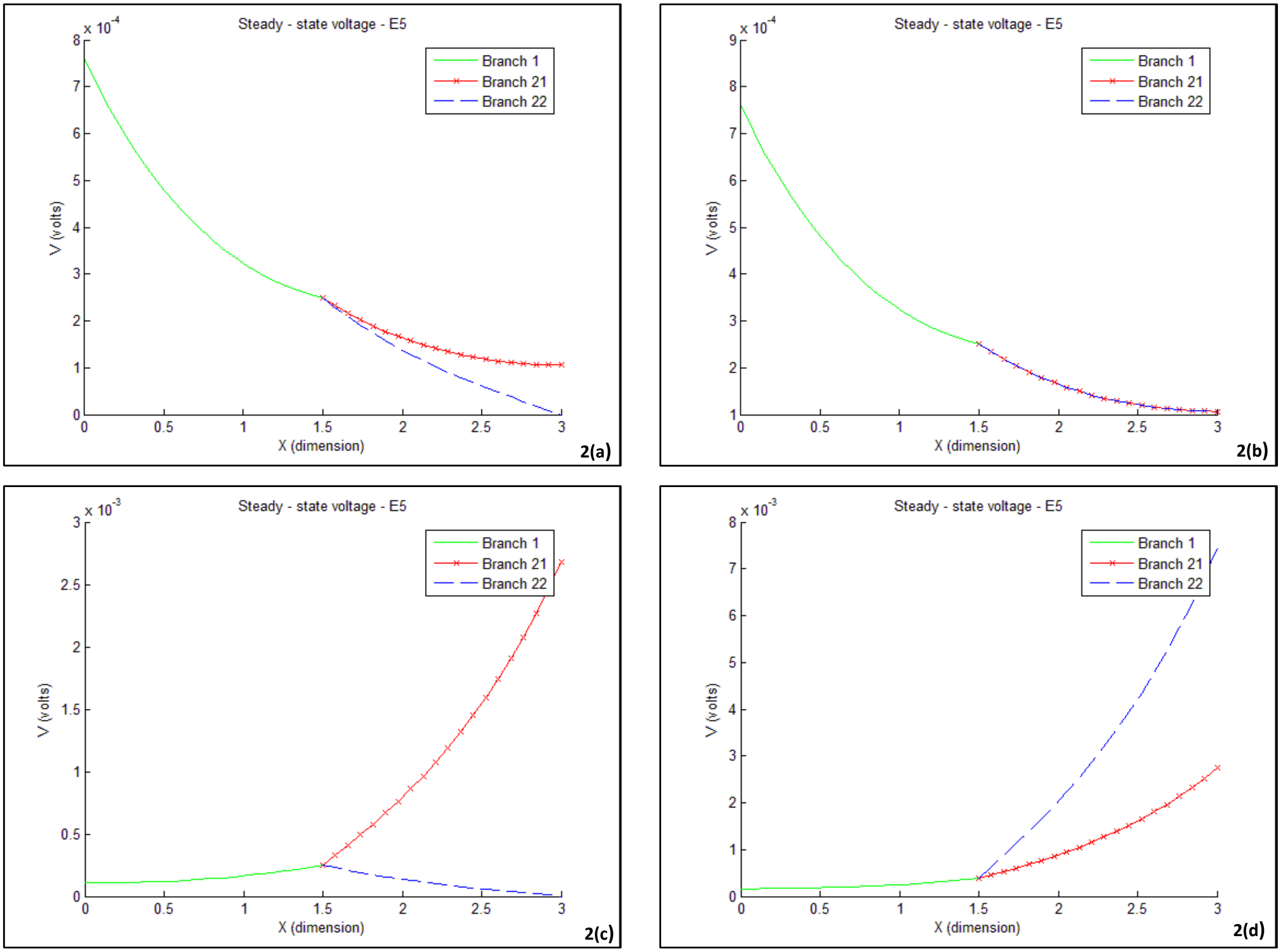
- **Question 4**



- What do you note about the steady state voltage profile in the two daughter branches?

According to the graph we can observe that membrane potential in the parent and daughter branches are same at the node which is represented in the *Equation 5*. When variation of the membrane potential along the distance is considered it is not continuous at node. The gradient (w. r. t. X) of daughter branch membrane potentials are higher at the node compared to that of the parent branch. When daughter branches are compared their membrane potentials are same along the branch and also the gradients (w. r. t. X).

❖ Question 5



- What is the meaning of the positive right hand sides of $dV_{21}/dX|_{X=L_{21}}$ and $dV_{22}/dX|_{X=L_{22}}$ in 2(c) and 2(d)?

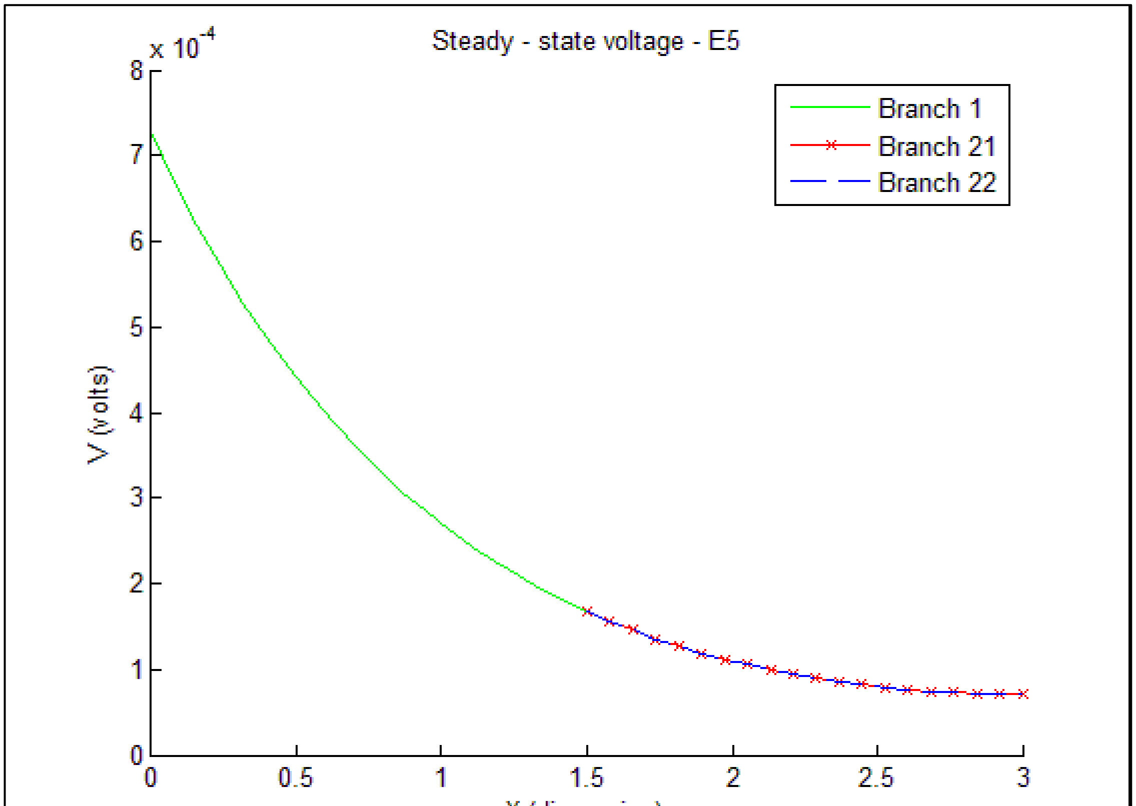
At the terminal of the daughter branches, gradient of membrane potentials are positive and that of the parent is zero. Therefore membrane potential has increased from the beginning of parent branch towards node and then increases along the daughter branch with terminals having positive membrane potential gradient. When potential differences are considered current will flow from daughter branch with higher membrane potential to the branch towards branch with lower potentials.

In $2(c)$, L_{21} has a higher gradient, parent branch and other daughter branch and has a zero gradient. Therefore membrane potential increase along the daughter branch L_{21} and membrane potential decreases along other branches with increasing X. Current flows from terminal of L_{21} towards node then along the parent and other daughter branch.

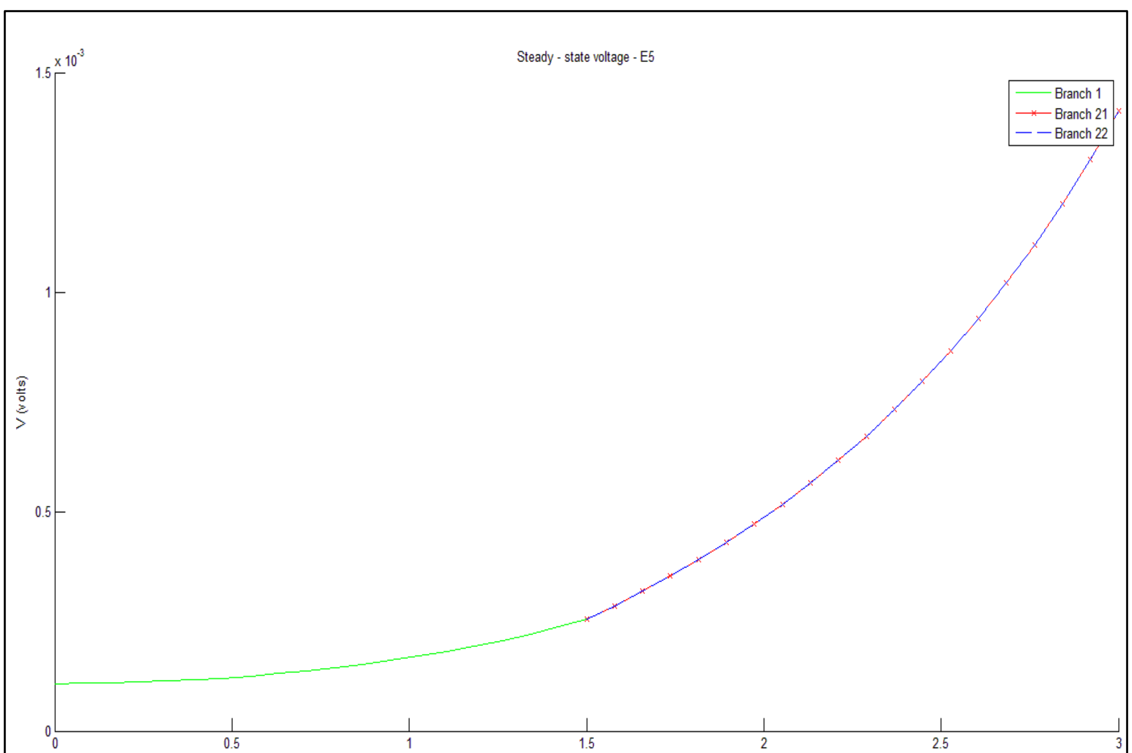
In $2(d)$ both daughter branches has positive gradient at their terminals. Therefore membrane potential increases towards the terminals of the daughter branches. Since parent branch has a zero gradient, from start towards the node along the parent branch membrane potential increases. Current flows from terminal of both daughter branches towards node then along the parent.

To summarize, the positive right hand side of the above equations refer to increasing membrane potential towards the daughter terminals.

❖ Question 6



2(b) modified w.r.t. Question 6



2(d) modified w.r.t. Question

- Recalculate the coefficients of equation (2) and replot the steady-state voltage profile for the boundary conditions of Figure 2(b) and 2(d) for $d_{21} = d_{22} = 47.2470 \times 10^{-4}$ cm. What do you notice?

When gradient of membrane potential at the beginning of the parent branch is non zero and gradient of membrane potential at the terminals of daughter branches are zero, membrane potentials along the daughter branches varies in the same manner, decrease equally away from node and independent of their diameter (compare graphs $2(b), 2(b)\ modified$). That is when current is injected from the parent and flows towards the terminals of the daughter branches. Also we can observe that gradient of the parent branch and gradient of daughter branches near the node tends to be continuous as the diameters of daughter branches are equal to $47.2470 \times 10^{-4}\ cm$. (in graph $2(b) modified$).

When gradient of membrane potential at the beginning of the parent branch is zero and gradient of membrane potential at the terminals of daughter branches positive and equal,(That is when current is injected from the daughter branches and flows along the parent branch) membrane potentials along the daughter branches varies in different manner, achieve different membrane potentials at the terminals (graphs $2(b)$). When diameters of the daughter branches are equal($47.2470 \times 10^{-4}\ cm$) variation of membrane potentials along the daughter branches become same, achieves same membrane potentials at the terminals.

Therefore effect of diameter towards the deviations in each daughter branches are prominent in the case where current in injected from the terminals of the daughter branches, or membrane potential at the terminals of the daughter branches are positive and equal, and that of the parent branch is zero. Also when diameter increases overall voltage profile shifts downwards.