The Optical Absorption of Graphene

The Hamiltonian for graphene is

$$H = v_F \vec{\sigma} \cdot (\vec{p} - \frac{e}{c}\vec{A}) \tag{1}$$

so the perturbation is

$$H_{int} = -\frac{v_F e}{c}\vec{\sigma} \cdot \vec{A} \tag{2}$$

If we pick our light to be polarized in the x-direction

$$\vec{A} = \mathrm{Re}[A\exp[-i\omega t]\hat{x}] = \frac{A}{2}\exp[-i\omega t]\hat{x} + \frac{A}{2}\exp[i\omega t]\hat{x} \tag{3}$$

then so the perturbation is

$$H_{int} = -\frac{v_F e A}{2c}\sigma_x \tag{4}$$

and so an optical transition matrix element is

$$\langle\psi_h|H_{int}|\psi_e\rangle = \left[\frac{1}{\sqrt{2}}\begin{pmatrix} \exp[-i\phi/2] \\ -\exp[i\phi/2] \end{pmatrix}\right]^\dagger \left[-\frac{v_F e A}{2c}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]\left[\frac{1}{\sqrt{2}}\begin{pmatrix} \exp[-i\phi/2] \\ \exp[i\phi/2] \end{pmatrix}\right] \tag{5}$$

squaring it gives

$$|\langle\psi_h|H_{int}|\psi_e\rangle|^2 = \frac{v_F^2 e^2 A^2}{4c^2}\sin^2(\phi) \tag{6}$$

The average of $\sin^2\phi$ is $\frac{1}{2}$, and applying Fermi's Golden Rule for the transition rate

$$P = \frac{2\pi}{\hbar}|\langle\psi_h|H_{int}|\psi_e\rangle|^2 D(\hbar\omega/2) \tag{7}$$

where $D(E) = \frac{2E}{\pi\hbar^2 v_F^2}$ is the graphene electronic density of states.

$$P = \frac{e^2 A^2 \omega}{4c^2\hbar^2} \tag{8}$$

Multiply by $\hbar\omega$ for the total energy density absorbed

$$E_{absorbed} = \frac{e^2 A^2 \omega^2}{4c^2\hbar} \tag{9}$$

The incident energy is given by the standard formula (Gaussian Units)

$$E_{incident} = \frac{c}{4\pi}|\text{Electric Field}|^2 = \frac{\omega^2}{4\pi c}A^2 \tag{10}$$

So the optical absorbtion coefficient of graphene is

$$\frac{E_{absorbed}}{E_{incident}} = \pi\frac{e^2}{\hbar c} = \pi\alpha = 2.3\% \tag{11}$$