Graphene Basics

The energy-momentum dispersion in graphene is

$$E = pv_F = \hbar v_F k \tag{1}$$

We fill the graphene with electrons up to the Fermi level $E_F = \hbar v_F k_F$. There is one state per $(2\pi/L)^2$ area in k-space, so the number of electrons is

$$N = \frac{4 \times (\text{circle in k-space with radius } k_F)}{(2\pi/L)^2} = \frac{4(\pi k_F^2)}{(2\pi/L)^2} \tag{2}$$

where the factor of 4 comes from spin and valley degeneracies. The density of electrons is then $n = N/A = k_F^2/\pi$ where $A = L^2$ is the area of the graphene. The surface charge density in the graphene is then

$$\sigma = en = ek_F^2/\pi = eE_F^2/(\pi\hbar^2 v_F^2) \tag{3}$$

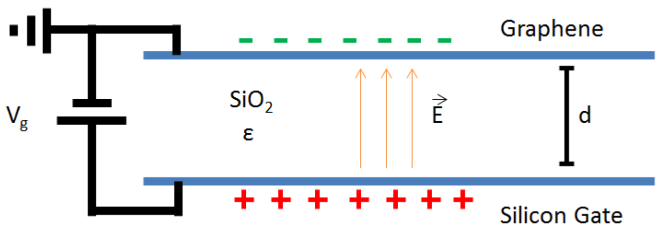
and the density of states is

$$\text{DOS} = \frac{\partial N}{\partial E_F} = \frac{\partial}{\partial E_F}\frac{k_F^2}{\pi} = \frac{\partial}{\partial E_F}\frac{E_F^2}{\pi(\hbar v_F)^2} = \frac{2E_F}{\pi(\hbar v_F)^2} \tag{4}$$

Mobility and conductivity are a bit harder

$$\sigma_{xx} = \frac{ne^2\tau}{m^*} = e^2 \int \frac{d^2k}{(2\pi)^2}\left[\hbar^2 \frac{\partial^2 E}{\partial k_x^2}\right]\tau(k) \tag{5}$$

Note that you can't naively plug in $m^* = 0$ and still hope to get the right answer. Also, this formula won't reproduce the universal minimum conductivity for neutral graphene. Values for the mobility of graphene are about 200,000 $\text{cm}^2/(\text{V}\cdot\text{s})$, an order of magnitude higher than silicon's 20,000 $\text{cm}^2/(\text{V}\cdot\text{s})$.



Now lets derive the expression for the Fermi level in terms of a backgate voltage (assuming no substrate doping, charge impurities, etc..). The electric field in the parallel plate capacitor set up is $\vec{E} \approx V_G/d$. However, by Gauss's Law $\oint \vec{E}\cdot d\vec{A} = Q/\epsilon$

$$\vec{E} = \frac{\sigma}{\epsilon} = \frac{ek_F^2}{\epsilon\pi} = \frac{eE_F^2}{\epsilon\pi(\hbar v_F)^2} \tag{6}$$

Setting this equal to V_G/d, and solving for E_F

$$E_F = \sqrt{\frac{\epsilon\pi(\hbar v_F)^2}{ed}V_G} = \hbar v_F\sqrt{\frac{\pi C}{eA}V_G} \tag{7}$$

where $C = \epsilon A/d$ is the geometric parallel plate capacitance (I've neglected quantum capacitance in the $\vec{E} \approx V_G/d$ approximation).