Tunneling into Bloch States

According to the simple 1D model of tunneling through a square potential barrier, a sample state decays into vacuum as $\psi(z) = \psi(0)e^{-\kappa z}$, where $\kappa = (2m(\phi - E_s)/\hbar^2)^{1/2}$ and ϕ is the sample work function. However, a correction must be made for Bloch states in a crystal. For simplicity, suppose a Bloch state decaying into the vacuum has the form

$$\psi(x, y, z) = e^{i(k_x x + k_y y)} e^{-\gamma z} \tag{1}$$

Then, plugging into the Schrödinger Equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial z^2} \right) \psi = \kappa^2 \psi \tag{2}$$

we get

$$\gamma = \sqrt{\kappa^2 + k_{||}^2} \tag{3}$$

where $k_{||}^2 = k_x^2 + k_y^2$ is the component of the crystal momentum parallel to the surface. Thus, states with a larger parallel component of momentum will decay faster than states with a smaller parallel component of momentum. This is very important in interpreting dI/dV spectroscopy for graphene.