1.1. Give a proof of the transitivity of implication, by showing that we can derive $A \Rightarrow C$ from the assumptions $A \Rightarrow B$ and $B \Rightarrow C$.

$$\dfrac{\dfrac{\dfrac{[A]^1 \quad A \Rightarrow B \quad B \Rightarrow C}{B \quad B \Rightarrow C}(\Rightarrow \mathrm{E})}{C}(\Rightarrow \mathrm{E})}{A \Rightarrow C}(\Rightarrow \mathrm{I})_1$$

1.2. Give a proof of $((A \vee B) \Rightarrow C) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[A]^1}{A \vee B}(\vee \mathrm{I}_1) \quad [(A \vee B) \Rightarrow C]^3}{C}(\Rightarrow \mathrm{E})}{A \Rightarrow C}(\Rightarrow \mathrm{I})_1 \quad \dfrac{\dfrac{\dfrac{[B]^2}{(A \vee B)}(\vee \mathrm{I}_2) \quad [(A \vee B) \Rightarrow C]^3}{C}(\Rightarrow \mathrm{E})}{B \Rightarrow C}(\Rightarrow \mathrm{I})_2}{(A \Rightarrow C) \wedge (B \Rightarrow C)}(\wedge \mathrm{I})}{((A \vee B) \Rightarrow C) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow C))}(\Rightarrow \mathrm{I})_3$$

1.3. Give a proof of $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$.

$$\dfrac{\dfrac{\dfrac{\dfrac{[A \wedge B]^1}{B}(\wedge \mathrm{E}_2) \quad \dfrac{\dfrac{[A \wedge B]^1}{A}(\wedge \mathrm{E}_1) \quad [A \Rightarrow (B \Rightarrow C)]^2}{B \Rightarrow C}(\Rightarrow \mathrm{E})}{C}(\Rightarrow \mathrm{E})}{(A \wedge B) \Rightarrow C}(\Rightarrow \mathrm{I})_1}{(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)}(\Rightarrow \mathrm{I})_2$$

1.4. Give proofs of $(A \Rightarrow B) \Rightarrow (B \Rightarrow A)$ and $A \Rightarrow \neg\neg A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[B]^1 \quad \dfrac{\begin{array}{c}[B]^1\\ \vdots\\ A\end{array} \quad \begin{array}{c}[B]^1\\ \vdots\\ \neg A\end{array}}{\neg B}(\neg \mathrm{I})}{A}(\neg \mathrm{E}) \quad \dfrac{\dfrac{[B]^1 \quad \dfrac{\begin{array}{c}[B]^1\\ \vdots\\ A\end{array} \quad \begin{array}{c}[B]^1\\ \vdots\\ \neg A\end{array}}{\neg B}(\neg \mathrm{I})}{A}(\neg \mathrm{E}) \quad [A \Rightarrow B]^2}{B}(\Rightarrow \mathrm{E})}{A \wedge B}(\wedge \mathrm{I})}{A}(\wedge \mathrm{E}_1)}{B \Rightarrow A}(\Rightarrow \mathrm{I})_1}{(A \Rightarrow B) \Rightarrow (B \Rightarrow A)}(\Rightarrow \mathrm{I})_2$$

$$\dfrac{\dfrac{\begin{array}{c}\dfrac{\begin{array}{cc}[A]^1 & [A]^1\\ \vdots & \vdots \\ B & \neg B\end{array}}{\neg A}\,(\neg \mathrm{I})\\ \vdots \\ B\end{array} \qquad \begin{array}{c}\dfrac{\begin{array}{cc}[A]^1 & [A]^1\\ \vdots & \vdots \\ B & \neg B\end{array}}{\neg A}\,(\neg \mathrm{I})\\ \vdots \\ \neg B\end{array}}{\neg\neg A}\,(\neg \mathrm{I})}{A \Rightarrow \neg\neg A}\,(\Rightarrow \mathrm{I})_1$$