Finding Adel

May 24, 2023

1 Delayed-τ model

According to P. Kroupa et al. 2020[2] current star formation rates of galaxies can be described by the 'delayed-τ' model as

$$SFR_{0,del} = \frac{A_{del} x e^{-x}}{\tau}, \text{ where } x = \frac{t_{sf}}{\tau} \quad (1)$$

where τ is the star formation time-scale, t_{sf} is the real time of star formation in a given galaxy and A_{del} a normalization constant.

The average SFR is

$$\overline{SFR_{del}} = \frac{A_{del}}{t_{sf}}[1-(1+x)e^{-x}] \quad (2)$$

and can also be defined by the present day stellar mass

$$\overline{SFR} = \frac{\zeta M_*}{t_{sf}} \quad (3)$$

where ζ accommodates for mass-loss through stella evolution and $\zeta \approx 1.3$

This is a system of 2 equations and 3 variables, since A_{del} has never been calculated

1.1 Constant t_{sf}

The observed ages of galactic discs are $t_{sf} \approx 12$ Gyr[1], so assuming an approximation of $t_{sf} = 12.5$ Gyr, the $\overline{SFR_{del}}$ can be calcuated, from the equation (3).

After that the equation of ratio

$$\frac{\overline{SFR_{del}}}{SFR_{0,del}} = \frac{e^x - x - 1}{x^2} \quad (4)$$

can be solved numerically for x and using the equations (1) and (2) the A_{del} and τ of each galaxy are found.

	A_{tsf}	τ	x_{tsf}
count	578	579	579
mean	2.24715e+12	1.08958e+11	1.853
std	3.93675e+13	1.04132e+12	1.476
min	2.47798e+07	1.93205e+09	0.001
25%	1.40573e+08	4.18098e+09	0.565
50%	6.83764e+08	7.79265e+09	1.604
75%	5.70379e+09	2.21327e+10	2.99
max	9.10088e+14	2.23774e+13	6.47

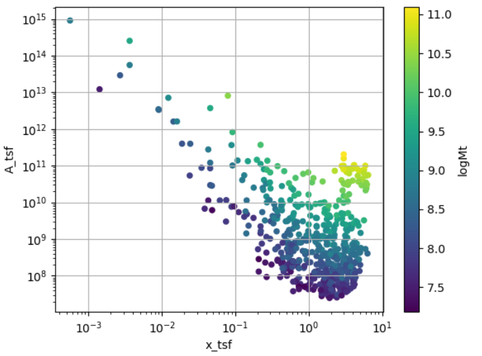

Figure 1: $A_{del} = f(x)$ for constant t_{sf}

$$\log(A_{del}|_{tsf}) = (9.6(4)\times 10^{-1})\cdot\log(M_t) + (8(4)\times 10^{-1}) \quad (5)$$

with correlation $R^2 = 48\%$

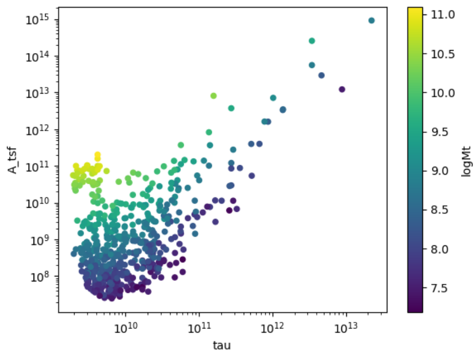

Figure 2: $A_{del} = f(\tau)$ for constant t_{sf}

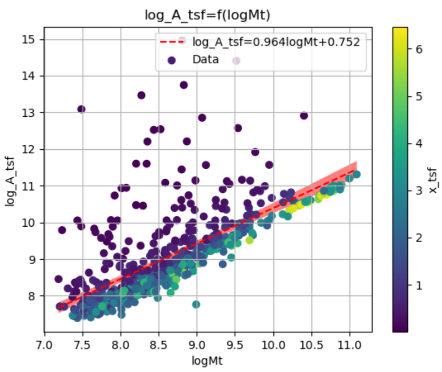

Figure 3: Total Mass M_t - $A_{del}|_{t_{sf}}$

1.2 Constant τ

Assuming for an constant $\tau = 3.5$ Gyr, we cannot use the same $\overline{SFR}$ since it depends on t_{sf}. Using the equations~(3) and (4)

$$\frac{\overline{SFR_{del}}}{SFR_{0,del}} = \frac{e^x - x - 1}{x^2} \Leftrightarrow \frac{e^x - x - 1}{x} = \frac{\zeta M_*}{SFR \cdot \tau}$$

using this equation x and A_{del} can be calculated numerically.

	A_τ	x_τ	t_{sf}
count	579	579	579
mean	4.58667e+09	2.54057	8.89201e+09
std	1.49896e+10	0.956554	3.34794e+09
min	9.87003e+06	0.406787	1.42376e+09
25%	6.50497e+07	1.87165	6.55079e+09
50%	2.36667e+08	2.43871	8.5355e+09
75%	1.11526e+09	3.07972	1.0779e+10
max	1.0577e+11	5.77102	2.01986e+10

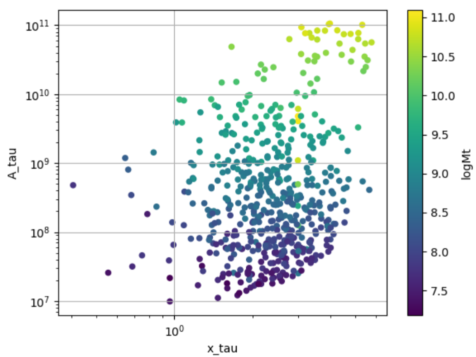

Figure 4: $A_{del} = f(x)$ for constant τ

log($A_{del}|_\tau$) $= (8.74(12) \times 10^{-1})\cdot$log($M_t$) + (1.31(10) $\times 10^0$)
with correlation $R^2 = 90\%$

(6)

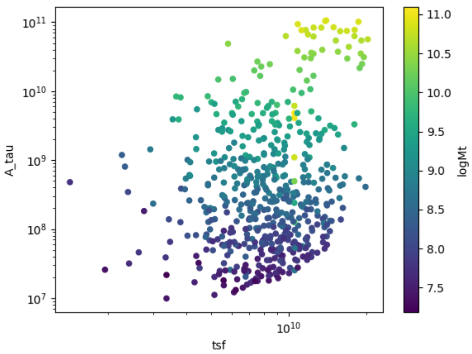

Figure 5: $A_{del} = f(t_{sf})$ for constant τ

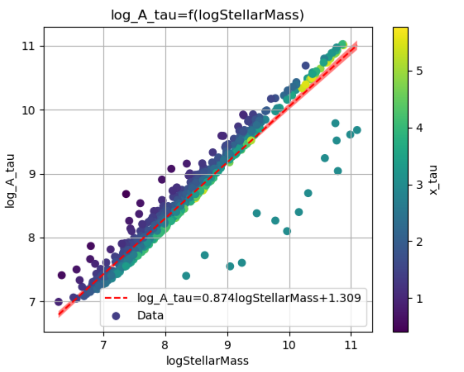

Figure 6: Total Mass M_t - $A_{del}|_\tau$

1.3 Comparing the two results

1.3.1 Comparing the x's

Comparing the two different results for x, we see that the $x|_\tau$ has a lower σ

	$x\|_\tau$	$x\|_{tsf}$
count	5.79E+02	5.79E+02
mean	2.54E+00	1.85E+00
std	9.57E-01	1.48E+00
min	4.07E-01	5.59E-04
25%	1.87E+00	5.65E-01
50%	2.44E+00	1.60E+00
75%	3.08E+00	2.99E+00
max	5.77E+00	6.47E+00

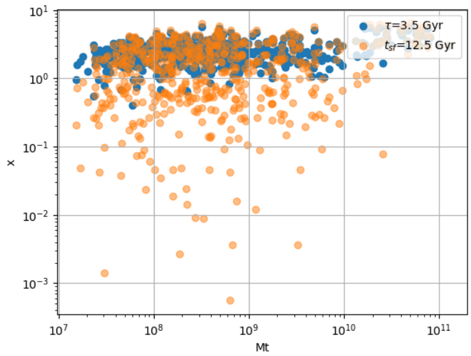

Figure 7: Comparing the two x's, According to their total masses

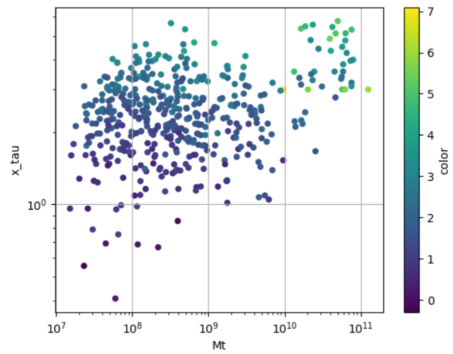

Figure 8: $x|_\tau = f(M_t)$, with their color index

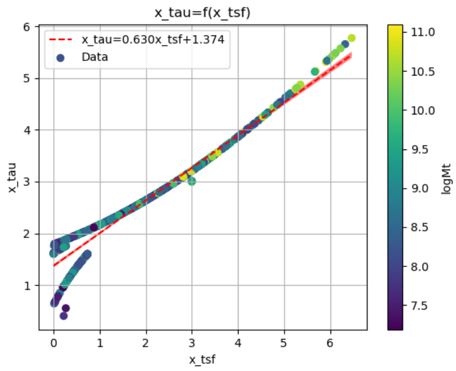

Figure 9: Comparing the two x, according to their total mass

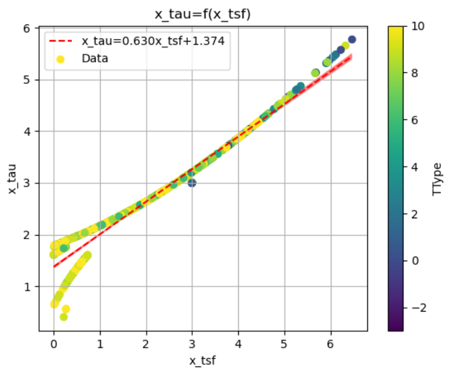

Figure 10: Comparing the two x, according to their type

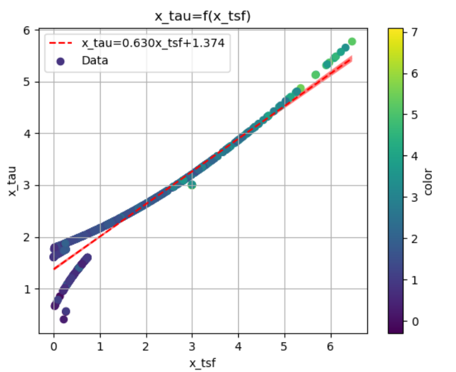

Figure 11: Comparing the two x, according to their color index

The two results are interrelated through the equation:

$$x|_\tau = (6.30(6) \times 10^{-1}) \cdot x|_{tsf} + (1.374(15) \times 10^0) \quad \text{with correlation } R^2 = 94\% \tag{7}$$

and from the plots the following conclusions can be drawn:

1. The galaxies with a higher total mass deviate less from the linear fit and are older.
2. The younger galaxies are mainly later types of galaxies
3. For lower x's, the galaxies have a lower color index which indicates that they are younger. So the values are inline with the experimental values.

1.3.2 Comparing the normalization constants

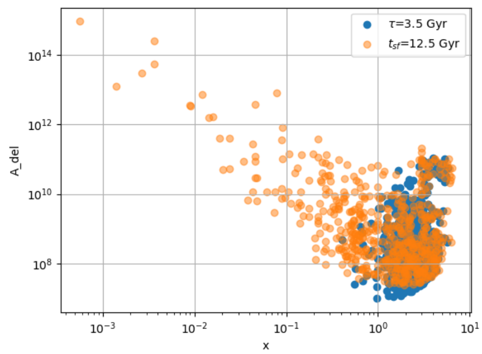

Figure 12: Comparing the two A_{del}

For high x and high masses the two A_{del}s have a high correlation. Specifically:

1. For high x the $A_{del}|_\tau - A_{del}|_{t_{sf}}$ plot follows a $y = x$ trend, which means that for older stars and stars with a low star formation timescale τ, the normalization constant is the same despite the method used to calculate it.
2. The same is true for more massive galaxies, since they deviate less from the $y = x$ line

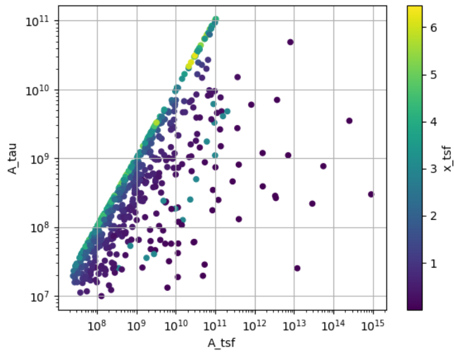

Figure 13: Comparison of the 2 A_{del}s according to their x

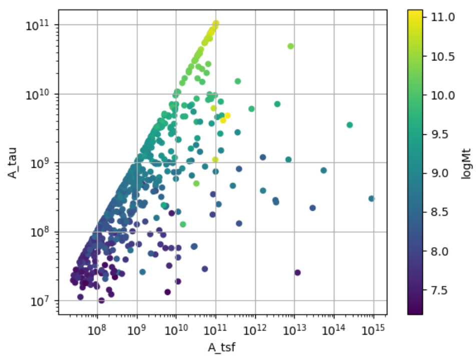

Figure 14: Comparison of the 2 A_{del}s according to their total masses

1.4 Int SFR to find the A_{del}

If we integrate equation (1) we get:

$$\begin{aligned}
\int_0^{t_{sf}} SFR_{del} dt_{sf} &= \int_0^{t_{sf}} \frac{A_{del} t_{sf} e^{-t_{sf}/\tau}}{\tau^2} dt_{sf} \\
\zeta \cdot M_* &= -A_{del} \frac{(t_{sf}\tau + \tau^2) e^{\left(-\frac{t_{sf}}{\tau}\right)}}{\tau^2} + A_{del} \\
\zeta \cdot M_* &= -A_{del} \frac{\tau^2 (x+1) e^{-x}}{\tau^2} + A_{del} \\
\zeta \cdot M_* &= A_{del}(1 - (x+1)e^{-x}) \\
A_{del} &= \zeta \cdot M_* \frac{e^x}{e^x - x - 1}
\end{aligned} \tag{8}$$

The integral $\int SFRdt =$ The total mass that is turned into stars. But during the evolution of the stars, the stars spew mass to Interstellar space, so the galaxies lose mass during this process. So the observed Stellar Mass M_* is smaller than the total mass turned into Stellar Mass.

The constant ζ accommodates for this mass-loss and, as discussed earlier, we can assume a conservative value of 1.3 for the galaxies in the LCV.

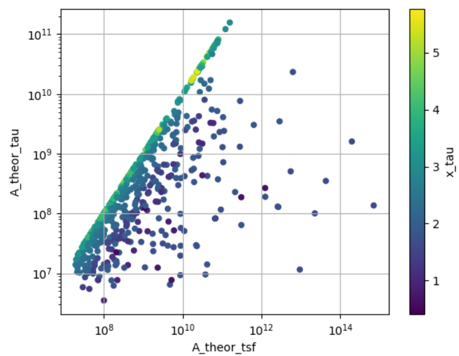

Figure 15: Comparison of the 2 A_{del}s according to their total masses

From the plots we get correlations of $R^2 = 91\%$ and $A_{tsf} = (8.97(12) \times 10^{-1}) \cdot A_{tsf\,theor} + (1.02(10) \times 10^0)$ so the theoretical values fit the experimental.

From the equations (1), (2) and (8), the $SFR_{0,del}$ and the $\overline{SFR_{del}}$ are given by the equa-

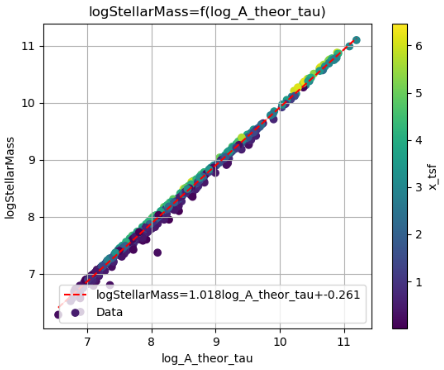

Figure 16: Comparison of the A_{del} according to their Stellar Mass

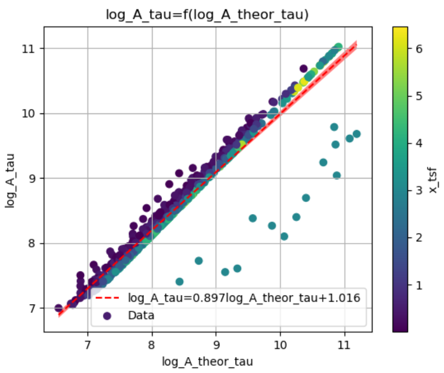

Figure 17: Comparison of the 2 $\$A_{del}|_{\tau=const.}\$$s (theoretical and experimental)

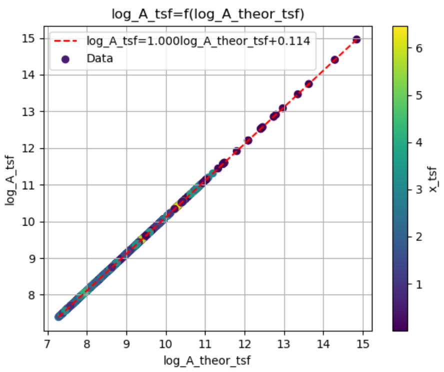

Figure 18: Comparison of the 2 $\$A_{del}|_{tsf=const.}\$$s (theoretical and experimental)

tions:

$$
\begin{aligned}
SFR_{0,del} &= \zeta M_* \frac{e^x}{e^x - x - 1}\frac{xe^{-x}}{\tau} \\
&= \zeta M_* \frac{x}{\tau(e^x - x - 1)}
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\overline{SFR_{del}} &= \zeta M_* \frac{e^x}{e^x - x - 1}\frac{1}{t_{sf}}[1-(1+x)e^{-x}] \\
&= \zeta M_* \frac{e^x}{e^x - x - 1}\frac{1}{t_{sf}}\frac{e^x - x - 1}{e^x} \\
&= \zeta \frac{M_*}{t_{sf}}
\end{aligned} \tag{10}
$$

The new $\overline{SFR_{del}}$ is the same with the $\overline{SFR}$ of the equation (3).

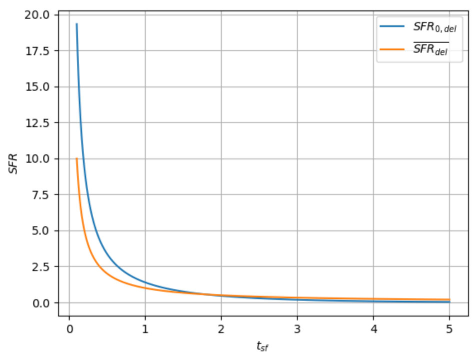

Figure 19: The $SFR_{0,del}$ and $\overline{SFR_{del}}$ for constant $\tau = 1$ and $\zeta M_* = 1$

1.5 Calculating the t_{sf} and τ for each galaxy

Having found an expression for the A_{del}, we have eliminated on out of the 3 variables and now the t_{sf} and τ of each galaxy can be calculated.

1.5.1 Method 1

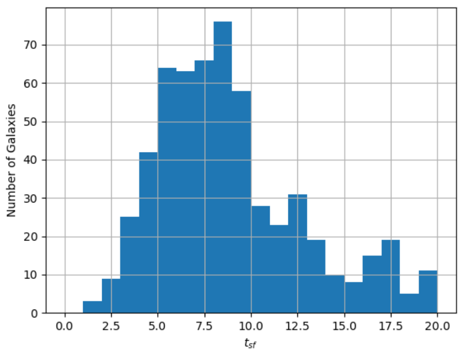

Figure 20: Histogram of t_{sf} from 0 to 20 Gyr

	t_{sf} Gyr	τ Gyr	x
count	579	579	579
mean	9.047	3.429	2.548
std	4.637	1.197	0.849
min	1.307	1.262	0.642
25%	6.066	2.954	1.99
50%	8.238	3.297	2.467
75%	11.007	3.691	2.962
max	62.635	27.605	9.487

1.5.2 Method 2

	$t_{sf,2}$ Gyr	τ_2 Gyr	x
count	579	579	579
mean	27.005	9.848	2.743
std	112.566	41.066	0
min	0.523	0.191	2.738
25%	4.329	1.578	2.743
50%	7.345	2.678	2.743
75%	14.071	5.13	2.743
max	1439.37	525.624	2.743

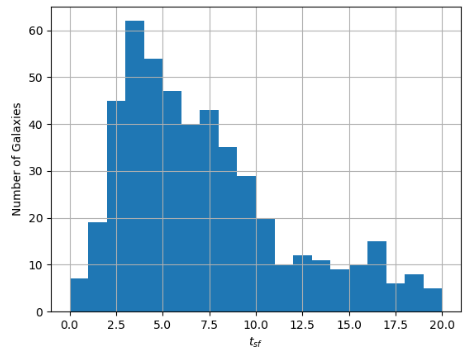

Figure 21: Histogram of t_{sf} from 0 to 20 Gyr

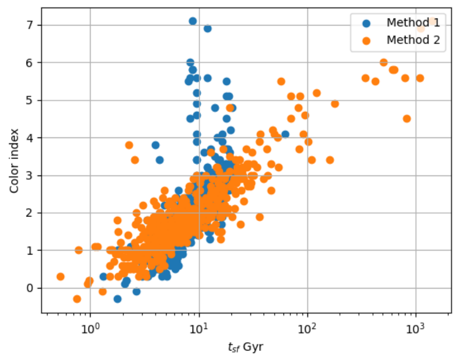

Figure 22: Comparing the two t_{sf}

1.5.3 [?]

- Can we calculate/observe ζ?
  - If not: for galaxies with extreme star-bursting and low-metallicity galaxies $\zeta = 2 - 3$. Can we find those galaxies and approximate the ζ?
- Why couldn't we use (3) to calculate A_{del}
- While in the second method we see a better correlation between the age of the galaxy and the color index, we must have an older universe

References

[1] R. A. Knox, M. R. S. Hawkins, and N. C. Hambly. "A Survey for Cool White Dwarfs and the Age of the Galactic Disc". In: *Monthly Notices of the Royal Astronomical Society* 306.3 (July 1999), pp. 736–752. ISSN: 0035-8711. DOI: 10.1046/j.1365-8711.1999.02625.x. (Visited on 03/13/2023).

[2] P Kroupa et al. "Constraints on the Star Formation Histories of Galaxies in the Local Cosmological Volume". In: *Monthly Notices of the Royal Astronomical Society* 497.1 (Sept. 2020), pp. 37–43. ISSN: 0035-8711. DOI: 10.1093/mnras/staa1851. (Visited on 03/13/2023).