Finding Adel

April 6, 2023

According to P. Kroupa et al. 2020[2] current star formation rates of galaxies can be described by the 'delayed-τ' model as

$$SFR_{0,del} = \frac{A_{del} x e^{-x}}{\tau}, \text{ where } x = \frac{t_{sf}}{\tau} \quad (1)$$

where τ is the star formation time-scale, t_{sf} is the real time of star formation in a given galaxy and A_{del} a normalization constant.

For a galaxy that forms stars from the time t_1 to t_2=the pressent day (if the galaxy is still forming stars) the SFH can be written as $SFR(t) = \overline{SFR} + \Delta SFR(t)$, such that the average SFR of the galaxy can be witten as

$$\overline{SFR} = \frac{1}{t_{sf}} \left(\int_{t_1}^{t_2} \overline{SFR} dt + \int_{t_1}^{t_2} \Delta SFR(t) \right) \quad (2)$$

with temporal deviations from $\overline{SFR}$ satisfying:

$$\int_{t_1}^{t_2} \Delta SFR dt = 0 M_{\odot}$$

So from the equations (1) and (2) we get the averge SFR:

$$\overline{SFR_{del}} = \frac{A_{del}}{t_{sf}} [1 - (1 + x) e^{-x}] \quad (3)$$

and can also be defined by the present day stellar mass

$$\overline{SFR} = \frac{\zeta M_*}{t_{sf}} \quad (4)$$

where ζ accommodates for mass-loss through stellar evolution and $\zeta \approx 1.3$.

From the equations (2) and (3) we can derive that the A_{del} must be independent from the time/ constant throughout the life of the galaxy.

Also from the equation (1) we can find the units of the A_{del}

$$\begin{aligned} [SFR_{0,del}] &= \frac{[A_{del}]\,[x]\,[e^{-x}]}{[\tau]} \\ \frac{[M_*]}{[time]} &= \frac{[A_{del}] \cdot 1 \cdot 1}{[time]} \\ [M_*] &= [A_{del}] \end{aligned} \quad (5)$$

So we can expect that the A_{del} can be expressed as

$$A_{del} = c \cdot M \Leftrightarrow \log(A_{del}) = log(M) + log(c) \quad (6)$$

where c is a constant and M the stellar mass (M_*), the gas mass (M_g) or the total mass of galaxy($M_t = M_* + M_g$).

The equations (1) and (3) create a system of 2 equations and 3 variables (the SFR and the stellar masses are given), since A_{del} has never been calculated

1 Constant t_{sf}

The observed ages of galactic discs are $t_{sf} \approx 12$ Gyr[1], so we assume an approximation of $t_{sf} = 12.5$ Gyr. Having one out of the three variables as a constant we can calculate the $A_{del}|_{t_{sf}}$ and $\tau|_{t_{sf}}$ and plot them (figures 1 and 2).

2 Constant τ

Now assuming for a constant $\tau = 3.5$ Gyr the system can again be solved numerically for each galaxy and the values of $A_{del}|_{\tau}$ and $t_{sf}|_{\tau}$ can be found and plotted (figures 3 and 4).

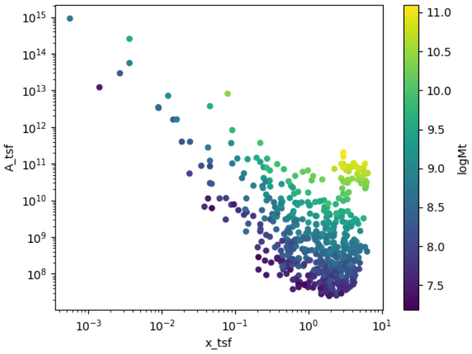

Figure 1: $A_{del} = f(x)$ for constant $\mathrm{t_{sf}}$

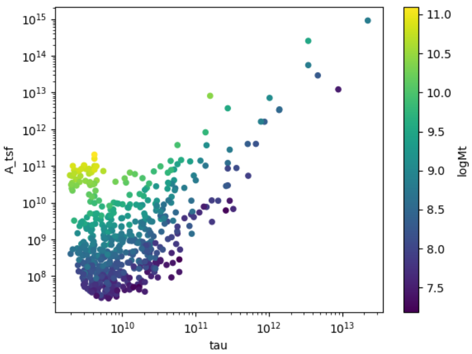

Figure 2: $A_{del} = f(\tau)$ for constant $\mathrm{t_{sf}}$

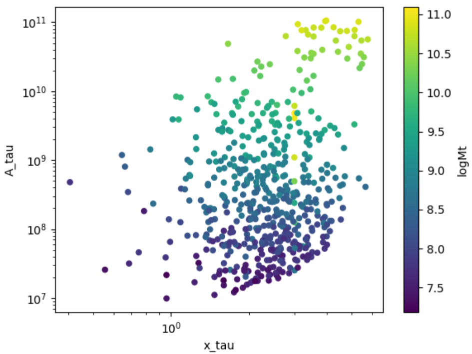

Figure 3: $A_{del} = f(x)$ for constant τ

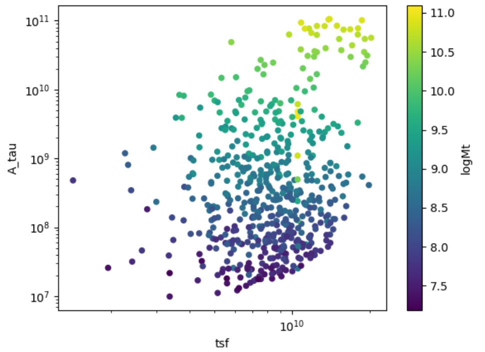

Figure 4: $A_{del} = f(t_{sf})$ for constant τ

3 Finding the $A_{del} = f(\textbf{Mass})$ relation

If we plot the A_{del} with the 3 given masses of each galaxy, both for a constant τ and a constant t_{sf}, we observe that indeed there is a correlation in the form of:

$$\log(A_{del}) = c_1 \log(\text{Mass}) + c_2$$

1. For a constant t_{sf} the correlations are:
   (a) Total mass: $R^2 = 48\%$ (Fig. 5)
   (b) Mass of the Gasses: $R^2 = 43\%$ (Fig. 6)
   (c) Stellar Mass: $R^2 = 44\%$ (Fig. 7)
2. For a constant τ the correlations are:
   (a) Total mass: $R^2 = 91\%$ (Fig. 8)
   (b) Mass of the Gasses: $R^2 = 70\%$ (Fig. 9)
   (c) Stellar Mass: $R^2 = 90\%$ (Fig. 10)

In both cases the best correlation is $A_{del} = f(M_t)$ and for $\tau = $ const. the correlation is excelent.

$$\log(A_{del}|_{tsf}) = (9.6(4)\times 10^{-1})\cdot\log(M_t) + (8(4)\times 10^{-1}) \tag{7}$$

$$\log(A_{del}|_{\tau}) = (1.025(14)\times 10^{0})\cdot\log(M_t) + (-3.0(1.2)\times 10^{-1}) \tag{8}$$

For both equations (7) and (8) $c_1 \approx 1$ so they fit the equation (6).

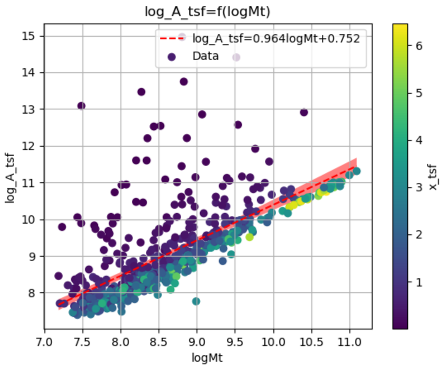

Figure 5: Total Mass M_t - $A_{del}|_{t_{sf}}$

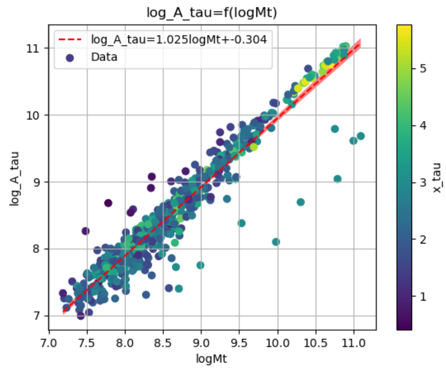

Figure 8: Total Mass M_t - $A_{del}|_{\tau}$

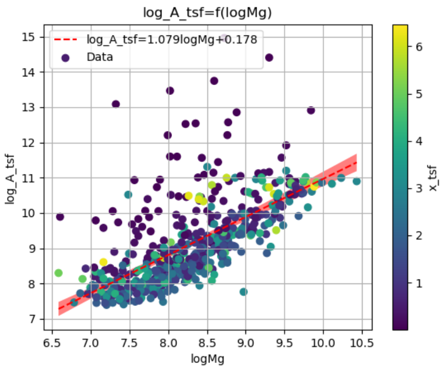

Figure 6: Mass of the gasses M_g - $A_{del}|_{t_{sf}}$

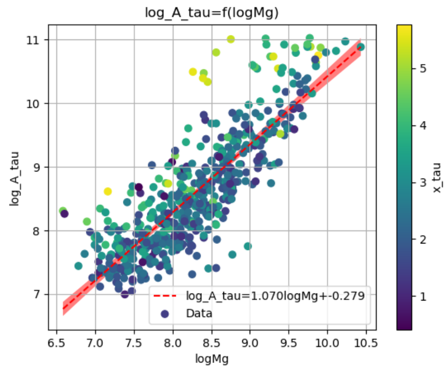

Figure 9: Mass of the gasses M_g - $A_{del}|_{\tau}$

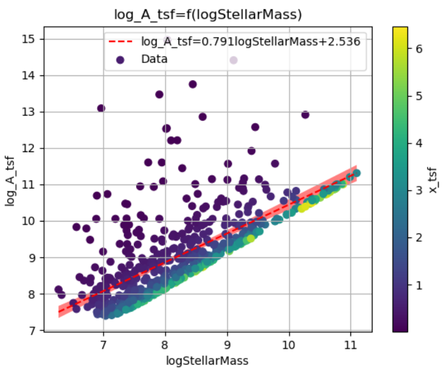

Figure 7: Stellar Mass M_* - $A_{del}|_{t_{sf}}$

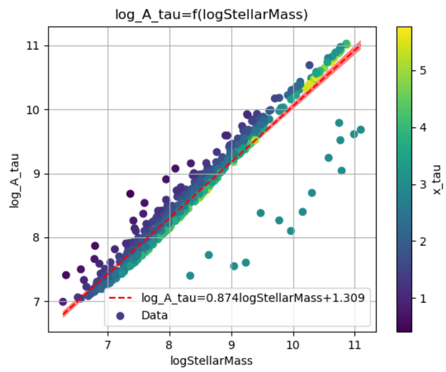

Figure 10: Stellar Mass M_* - $A_{del}|_{\tau}$

References

[1] R. A. Knox, M. R. S. Hawkins, and N. C. Hambly. “A Survey for Cool White Dwarfs and the Age of the Galactic Disc”. In: *Monthly Notices of the Royal Astronomical Society* 306.3 (July 1999), pp. 736–752. ISSN: 0035-8711. DOI: `10.1046/j.1365-8711.1999.02625.x`. (Visited on 03/13/2023).

[2] P Kroupa et al. “Constraints on the Star Formation Histories of Galaxies in the Local Cosmological Volume”. In: *Monthly Notices of the Royal Astronomical Society* 497.1 (Sept. 2020), pp. 37–43. ISSN: 0035-8711. DOI: `10.1093/mnras/staa1851`. (Visited on 03/13/2023).