

# Brief tutorial to Dispersion Calculator 2.0 software

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The program is developed to calculate spin-wave dispersion characteristics in magnetic waveguides and films. It is written in Mathematica 10 software which opens an access to easy modification of the calculation core (example is below).

```

γ := 1.76 × 107; ωH := γ (H0 + HA);
ωHMSW := γ (H0 + HA - HDemagW); ωM := γ M0; ωHp := γ (H0 - HDemagT - 4 HA / 3 + 2 HU); (*[24] in Koz'

knm[k-, n-, m-] := √(k2 + (n π / t)2 + (m π / w)2);

knmBV[k-, n-, m-] := √(k2 + (n π / t)2 + (m π / wBV)2);

kzm[k-, m-] := √(k2 + (m π / w)2);

kzmBV[k-, m-] := √(k2 + (m π / wBV)2);

κn[n-] := n π / t;

Fn[k-, n-, m-] := 2 / (kzm[k, m] t) (1 - (-1)n e-kzm[k, m] t);

Pnn[k-, n-, m-] := (kzm[k, m]2 / (knm[k, n, m]2 - knm[k, n, m]4) - (1 + KroneckerDelta[0, n]) Fn[k, n, m]) / (1 + KroneckerDelta[0, n]);

PnnBV[k-, n-, m-] := (kzmBV[k, m]2 / (knmBV[k, n, m]2 - knmBV[k, n, m]4) - (1 + KroneckerDelta[0, n]) Fn[k, n, m]) / (1 + KroneckerDelta[0, n]);

fFVMSW[k-, n-, m-] := If[SimplifiedModes, 10-9 / (2 π γ √(H0 (H0 + M0 (1 - (1 - Exp[-k t]) / (k t))))), 10-9 / (2 π γ ((ωHp + α ωM knm[k, n, m]2) (ωHp + α ωM knm[k, n, m]2 + ωM Pnn[k, n, m]))];

```

In order to start program, press **Shift+Enter** (or Enter on the NumPad). Afterwards another window is opened – see below. This window is used exclusively for the definition of the calculation parameters and calculation settings. Do not forget to close this window prior to restarting program if you change the code of the program.

### Parameters

External magnetic field  $H_0$ , mT

Material

Film thickness  $t$ , nm

Saturation magnetization  $M_0$ , kA/m

Exchange constant  $A$ , pJ/m

Uniaxial anisotropy field  $H_U$ , mT

Cubic anisotropy field  $H_A$ , mT

Angle  $\phi_M$  btw.  $M_0$  and axis <100>, deg

Inhomogeneous linewidth  $\Delta H_0$ , mT

Gilbert damping constant  $\alpha$

Structure

Waveguide width  $w$ ,  $\mu\text{m}$

### Calculation parameters

Max number of width modes  $m_{\text{Max}}$

Max number of thickn. modes  $n_{\text{Max}}$

Wavenumber range

Number of wavenumber points

Minimum wavenumber  $k_{\text{Min}}$ , rad/ $\mu\text{m}$

Maximum wavenumber  $k_{\text{Max}}$ , rad/ $\mu\text{m}$

Additional options

To take into account demagnetization ☒

Use eff. waveguide width for BVMSW ☐

Switch off anisotropy ☒

Switch off exchange interaction ☐

Use life time as for bulk material ☐

Use simplified expressions for fund. modes ☐

### Dispersion Calculator information

The program calculates dispersion characteristics for spin waves using analytical expressions in [1–3]. Dipolar and exchange energy contributions are taken into account.

Approaches and assumptions:

- Spins are fully unpinned at film surfaces.
- Spins are fully pinned at waveguide edges.
- Eq. (24) and (30) in [3] are used with additional account of width modes:  $k_{\text{Total}}^2 = k_y^2 + (m\pi/w)^2$  and the angle dependence on the width mode  $\phi(w, m)$ .
- Considers cubic  $H_A$  and uniaxial out-of-plane  $H_U$  crystallogr. anisotropies for film with (111) orientation.
- Assumes ellipsoid to calculate demagnetization fields.
- Takes into account ellipticity of precession for lifetime [4].
- Defines eff. waveguide width for BVMSW according to [4].

[1] B.A. Kalinikos, IEE Proc. 127, 4 (1980).  
 [2] B.A. Kalinikos & A.N. Slavin, J. Phys. C 19, 7013 (1986).  
 [3] B.A. Kalinikos, et al., J. Phys. 127, 4 (1990).  
 [4] D.D. Stancil & A. Prabhakar, Spin Waves., Springer 2009.

Developed: D. Bozhko & A. Chumak. Beware of spies!

Below some chosen clarifications about the program are shown.

|  |        |
|--|--------|
| Material   | YIG    |
| Film thickness $t$ , nm  | 100    |
| Saturation magnetization $M_0$ , kA/m                          | 140    |
| Exchange constant $A$ , pJ/m                                   | 3.5    |
| Uniaxial anisotropy field $H_U$ , mT                           | 0      |
| Cubic anisotropy field $H_A$ , mT                              | 4.2    |
| Angle $\phi_M$ btw. $M_0$ and axis $\langle 100 \rangle$ , deg | 0      |
| Inhomogeneous linewidth $\Delta H_0$ , mT                      | 0.16   |
| Gilbert damping constant $\alpha$                              | 0.0002 |

Here are some chosen materials are pre-defined. Change of this box will change all the parameters shown below.

Here you can change the initial pre-defined parameters.

```
(*Materials input:*)
(*Thickness, nm*) tYIG = 100; tPy = 20; tCoFeB = 75; tHeusler = 30;
(*Saturation magnetization, kA/m*) MYIG = 140; MPy = 675; MCoFeB = 1250; MHeusler = 1000;
(*Exchange constant, pJ/m*) AYIG = 3.5; APy = 16; ACoFeB = 15; AHeusler = 13;
(*Uniaxial out-of-plane anisotropy field, mT*) HUIG = 0; HUPy = 0; (*no data*) HUCoFeB = 0.3; HUHeusler = 1;
(*Cubic magnetocrystalline anisotropy field, mT*) HAYIG = 4.2; HAPy = 0; HACoFeB = 0; HAHeusler = 0;
(*Inhomogeneous linewidth, mT*) dHOYIG = 0.16; dHOPy = 0; (*no data*) dH0CoFeB = 0.75; dH0Heusler = 0; (*no data*)
(*Gilbert damping constant*) AlfaYIG = 2e-4; AlfaPy = 1e-3; AlfaCoFeB = 4e-3; AlfaHeusler = 3e-3;
(*Waveguide width, um*) WWidthInit = 2;
```

Check paper [3] to get more details about  $\phi_M$  and exact formulas used for calculations. Have fun Do not hesitate to compare formulas in the paper with formulas in the core of the program.

Damping is used to define spin-wave lifetime and related parameters (not for dispersions).

|                                     |           |
|-------------------------------------|-----------|
| Structure                           | Waveguide |
| Waveguide width $w$ , $\mu\text{m}$ | 2         |

Here you can define if to calculate dispersions for a spin-wave waveguide or for an infinitely large film.

|  |     |
|--|-----|
| Calculation parameters                                   |     |
| Max number of width modes $m_{\text{Max}}$               | 3   |
| Max number of thckn. modes $n_{\text{Max}}$              | 4   |
| Wavenumber range   |     |
| Number of wavenumber points                              | 100 |
| Minimum wavenumber $k_{\text{Min}}$ , rad/ $\mu\text{m}$ | 0   |
| Maximum wavenumber $k_{\text{Max}}$ , rad/ $\mu\text{m}$ | 10  |

Program calculates dispersions for higher-order width and thickness modes

Here everything is clear. Too many points might make the calculation slow.

Additional options

To take into account demagnetization ☒

Use eff. waveguide width for BVMSW ☐

Switch off anisotropy ☒

Switch off exchange interaction ☐

Use life time as for bulk material ☐

Use simplified expressions for fund. modes ☐

Program can take into account demagnetization (for ellipsoid).

Here is the analytic formulas used to calculate demagnetization fields.

$a1 = t/2; b1 = w/2; c1 = LengthInit/2;$

$HDemagTt = \text{If}[\text{structure} = 1, M0, \frac{M0 a1 b1 c1}{2} * NIntegrate[\frac{1}{(a1^2 + x) \sqrt{(a1^2 + x) (b1^2 + x) (c1^2 + x)}}, \{x, 0, \text{Infinity}\}]]];$

$HDemagWt = \text{If}[\text{structure} = 1, 0, \frac{M0 a1 b1 c1}{2} * NIntegrate[\frac{1}{(b1^2 + x) \sqrt{(a1^2 + x) (b1^2 + x) (c1^2 + x)}}, \{x, 0, \text{Infinity}\}]]];$

It is possible to calculate effective waveguide width for BVMSW. See thesis of Thomas Brächer ☺

$$D_{Dip} = \frac{2\pi \left(\frac{w_w}{d_w}\right)}{1 + 2 \ln \left(\frac{w_w}{d_w}\right)}, \quad (2.52)$$

which is completely determined by the ratio of the width and the thickness of the waveguide  $w_w/d_w$ . As a consequence, the magnetization at the edges of the waveguide is not completely pinned. This would be the case for  $\partial \mathbf{m} / \partial y = \infty$ , which corresponds to a vanishing spin-wave amplitude at the edge. Thus, for a wave inside of the waveguide, the width of the waveguide is effectively increased, since the fictive point of a fixed pinning lies outside of the waveguide boundaries. This increase can be described by [72]:

$$w_{\text{eff}} = w_w \left( \frac{D_{Dip}}{D_{Dip} - 2} \right) \quad (2.53)$$

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In order to define the role of crystallographic anisotropy or exchange interaction, it is possible to switch them on/off.

$$\frac{1}{T_k} = \frac{1}{T_0} \frac{\partial \omega}{\partial \omega_0}. \quad (6.21)$$

In general, program takes into account ellipticity of the precession for the calculation of the spin-wave lifetime (see Stancil book).

But it is possible to switch it off in order to use the most straight-forward definition of the lifetime.

$1 / (\gamma * dH0 * 10^{-9} / 2 + 2 * \text{Pi} * \text{alfa} * \text{Abs}[\text{fBVMSW}[k, 0, \# - 1]])$

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[4] D.D. Stancil & A. Prabhakar, Spin Waves..., Springer 2009.

Program can calculate dispersions using simplest possible dispersions for dipolar waves (e.g. like in [YIG Magnonics, J. Phys. D: Appl. Phys. 43, 264002 (2010)]).

Here brief description about the program is given.

### Parameters

External magnetic field  $H_e$ , mT

$H_e \leq 4\pi M_0$  for FVMSW !

Material

Film thickness  $t$ , nm

Saturation magnetization  $M_0$ , kA/m

Exchange constant  $A$ , pJ/m

Uniaxial anisotropy field  $H_u$ , mT

### Calculation parameters

Max number of width modes  $m_{Max}$

Max number of thckn. modes  $n_{Max}$

Wavenumber range

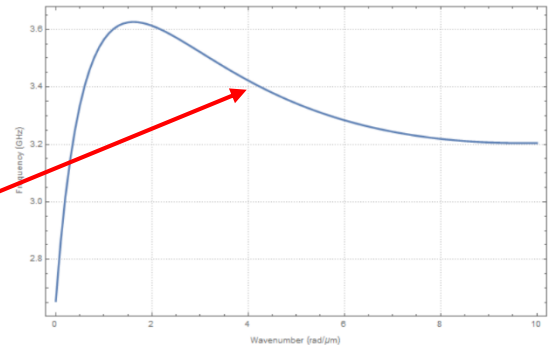
Number of wavenumber points

Minimum wavenumber  $k_{Min}$ , rad/ $\mu$ m

Maximum wavenumber  $k_{Max}$ , rad/ $\mu$ m

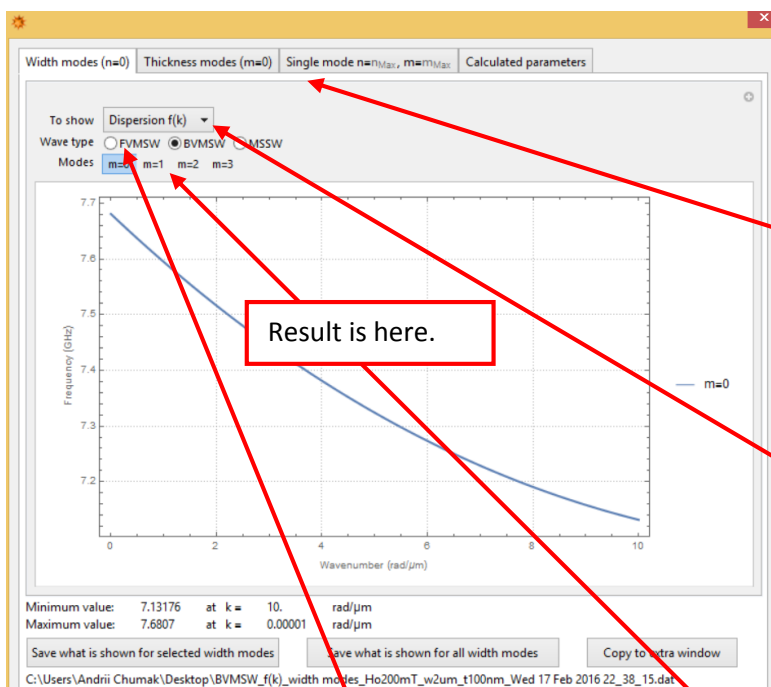
$k_{Max} \cdot d > 1$  !

There are two warning messages in the program. If the left message is shown, dispersions for FVMSW are not calculated properly. If right message is shown, the dispersion for MSSW can be calculated wrong (known drawback of analytical approximation [1-3]).



After all parameters are chosen, press button "Calculate!", and wait.

The second window, which presents results, will be shown afterwards. You can change parameters in the first window and press again "Calculate!". The second window will be re-opened in this case.



Here you can choose what do you want to see:

- width modes ( $m=0, m=1, \dots, m=m_{Max}$ ) for zero thickness mode  $n=0$ ;
- thickness modes ( $n=0, n=1, \dots, n=n_{Max}$ ) for zero width mode  $m=0$ ;
- single non-zero mode with  $m=m_{Max}$  and  $n=n_{Max}$ ;
- or simply some additional calculated parameters in text form.

Here you can choose between:

Dispersion  $f(k)$

Dispersion  $f(k)$

Velocity  $v(k)$

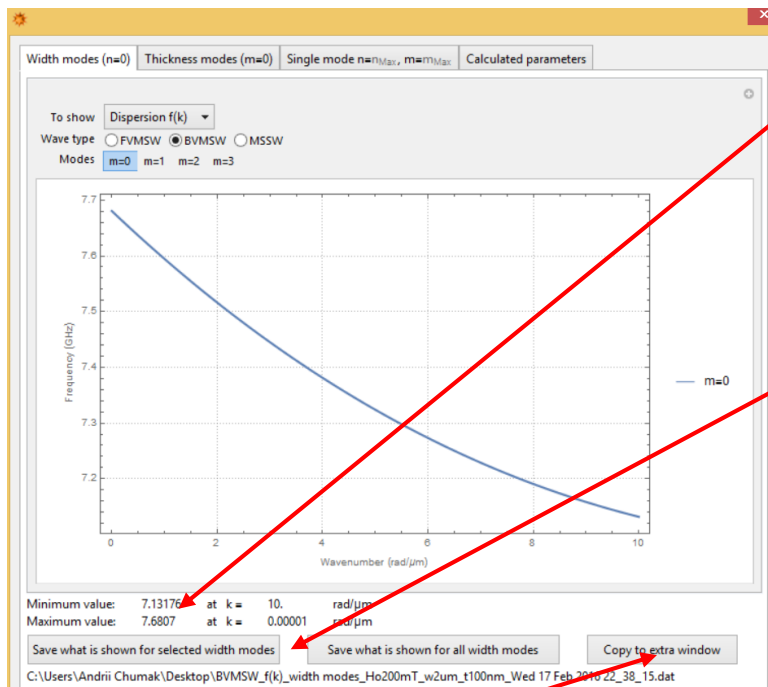
Lifetime  $\tau(k)$

Freepath  $l(k)$

Ratio  $l(k)/\lambda$

Here you can choose between spin-wave mode (BVMSW, MSSW and FVMSW).

Here you can choose the width/thickness modes you want to see (in any possible combination).



Here maximum and minimum values are shown (always).

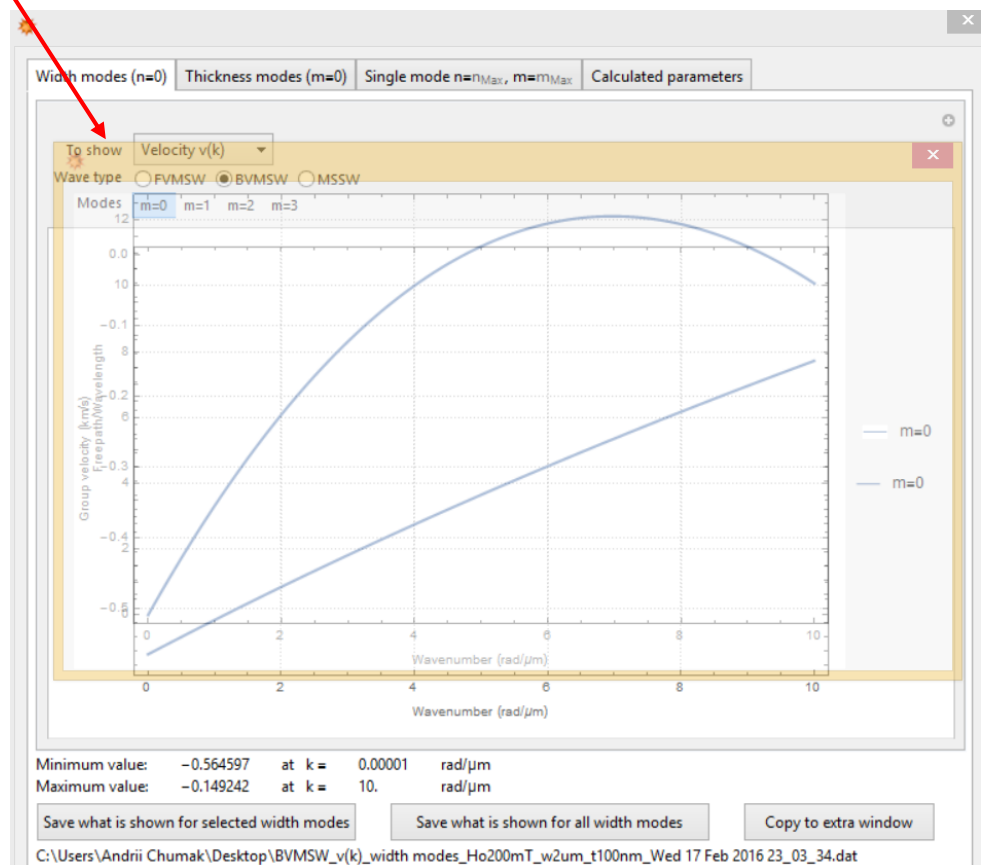
You have two options what to save: only shown modes or all calculated modes.

The file name is generated automatically and is shown below.

It is saved to data file which can be easily opened with Origin.

If you want to compare results for different parameters, etc., you can buffer one figure. It appears in a separate movable half-transparent window and can be used to compare results with new calculations.

| Wavenumber k<br>rad/μm | Frequency<br>GHz  | Frequency<br>GHz  | Frequency<br>GHz  | Frequency<br>GHz |
|------------------------|-------------------|-------------------|-------------------|------------------|
| m=0                    | m=1               | m=2               | m=3               |                  |
| 0.                     | 7.680699430199515 | 7.792631982736609 | 7.88495176169786  |                  |
| 0.1                    | 7.671752671879398 | 7.791866667738289 | 7.884595504020275 |                  |
| 0.2                    | 7.662884418401614 | 7.789592927124968 | 7.883529547672064 |                  |





This is how calculated parameters look like (last tab). The idea is to give some extra values in different unit systems and also to decrease a usage of calculator. ☺

| Width modes (n=0)  | Thickness modes (m=0) | Single mode n=n <sub>Max</sub> , m=m <sub>Max</sub> | Calculated parameters  |
|--|-----------------------|---|--|
| External magnetic field                                  |                       |   | $H_e = 200. \text{ mT} = 2000. \text{ Oe} = 159.155 \text{ kA/m}$  |
| Saturation magnetization                                 |                       |   | $M_0 = 140. \text{ kA/m} = 1759.29 \text{ G} = 140. \text{ emu/cm}^3$  |
| Effective waveguide width                                |                       |   | $w_{\text{eff}} = 2.25041 \text{ } \mu\text{m}$  |
| Demagnetization field (MSSW conf.)                       |                       |   | $H_d^{\text{MSSW}} = 8.79646 \text{ mT} = 84.446 \text{ Oe} = 7 \text{ kA/m}$  |
| Demagnetization field (FVMSW conf.)                      |                       |   | $H_d^{\text{FVMSW}} = 167.552 \text{ mT} = 1675.52 \text{ Oe} = 133.333 \text{ kA/m}$  |
| Internal magnetic field (MSSW conf.)                     |                       |   | $H_i^{\text{MSSW}} = 191.622 \text{ mT} = 1916.22 \text{ Oe} = 152.488 \text{ kA/m}$   |
| Internal magnetic field (FVMSW conf.)                    |                       |   | $H_i^{\text{FVMSW}} = 32.4484 \text{ mT} = 324.484 \text{ Oe} = 25.8216 \text{ kA/m}$  |
| Exchange constant  |                       |   | $A = 3.5 \text{ pJ/m} = 3.5 \cdot 10^{-7} \text{ erg/cm}$<br>$\Rightarrow \alpha_{\text{ex}} = 2.84205 \cdot 10^{-12} \text{ cm}^2$<br>$\Rightarrow D = 0.5 \cdot 10^{-8} \text{ Oe} \cdot \text{cm}^2$<br>$\Rightarrow \eta = 0.088 \text{ cm}^2/\text{s}$<br>$\Rightarrow q = 0.226163 \cdot 10^{-12} \text{ cm}^2$<br>$\Rightarrow \text{exchange length } \lambda_{\text{ex}} = \sqrt{q} = 4.75566 \text{ nm}$ |
| FMR linewidth @ 10 GHz                                   |                       |   | $\Delta H = 0.3028 \text{ mT} = 3.028 \text{ Oe} = 0.24096 \text{ kA/m}$   |
| Lifetime @ 10 GHz for bulk material                      |                       |   | $\tau = 79.5775 \text{ ns}$  |
| Lifetime @ $k=k_{\text{max}}/2$ for BVMSW (n=0, m=0)     |                       |   | $\tau^{\text{BVMSW}} = 41.5183 \text{ ns} @ k = 5. \text{ rad}/\mu\text{m} (f = 7.32395 \text{ GHz})$  |
| Lifetime @ $k=k_{\text{max}}/2$ for BVMSW (n=0, m=0)     |                       |   | $\tau^{\text{MSSW}} = 41.3247 \text{ ns} @ k = 5. \text{ rad}/\mu\text{m} (f = 7.73847 \text{ GHz})$   |
| Lifetime @ $k=k_{\text{max}}/2$ for BVMSW (n=0, m=0)     |                       |   | $\tau^{\text{FVMSW}} = 59.0961 \text{ ns} @ k = 5. \text{ rad}/\mu\text{m} (f = 1.3719 \text{ GHz})$   |
| Frequency $\omega_M$                                     |                       |   | $\omega_M = 4.928 \text{ GHz}$   |
| Bottom frequency $\omega_H$ (simplest $\gamma H_e$ )     |                       |   | $\omega_H = 5.60225 \text{ GHz}$   |
| Bottom frequency $\omega_H$ (simplest + demag. for MSSW) |                       |   | $\omega_H = 5.36759 \text{ GHz}$   |
| FMR frequency for out-of-plane (simplest)                |                       |   | $\omega_H = 0.674254 \text{ GHz}$  |
| FMR frequency for out-of-plane (simplest + demag.)       |                       |   | $\omega_H = 0.908921 \text{ GHz}$  |
| FMR frequency for out-of-plane (used in calculations)    |                       |   | $\omega_H = 0.908922 \text{ GHz}$  |
| FMR frequency for in-plane magnetized film (simplest)    |                       |   | $\omega_{\text{FMR}} = 7.6807 \text{ GHz}$   |
| FMR frequency for in-plane (simplest + demag.)           |                       |   | $\omega_{\text{FMR}} = 7.43387 \text{ GHz}$  |
| FMR frequency for in-plane (used in calculations)        |                       |   | $\omega_{\text{FMR}} = 7.43387 \text{ GHz}$  |

Enjoy the program! And please let us know if something has to be corrected/improved.

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