Exercise. *Covariance matrix:*

$$\Sigma = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$$

Calculate the following quantities!

1. $\text{Corr}(X, Y)$

2. $\text{Mahal}(A, B)$, *where* $A = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \end{bmatrix}$

Solution.   *1.* $\text{Corr}(X, Y) = \frac{\text{Cov}(X,Y)}{\sigma(X)\sigma(Y)} = \frac{0.2}{\sqrt{0.3}\sqrt{0.3}} = \frac{2}{3}$.

$\left(\text{Other notation: } r_{jk} = \frac{q_{jk}}{s_j s_k} \right)$

2. $\text{Mahal}(A, B) = (A - B)\Sigma^{-1}(A - B)^T$, *where*

$$\Sigma^{-1} = \frac{1}{\det \Sigma} \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix} = \frac{1}{0.05} \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}$$

$$(A-B)\Sigma^{-1}(A-B)^T = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 5 & -5 \end{bmatrix} \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} = 5$$