Structure and dynamics of complex networks

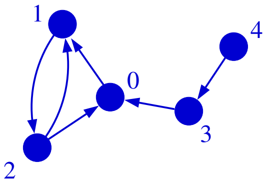
February 11, 2020

Adjacency matrix
Definition

Adjacency matrix

- each row and column corresponds to a node,

$$A_{ij} = \begin{cases} 1 & \text{if } i \to j \\ 0 & \text{otherwise} \end{cases}$$



$\longleftrightarrow$

$$\begin{array}{c c} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Adjacency matrix
Properties

- if the graph is undirected,

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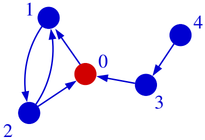

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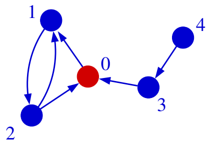

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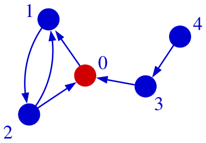

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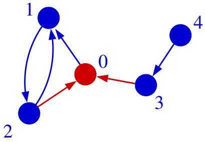

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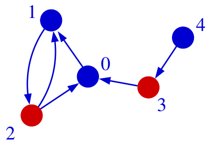

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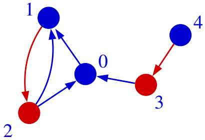

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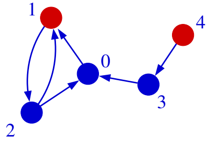

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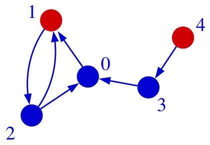

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- To proceed **forward** on the links, we need to multiply from the **left**.

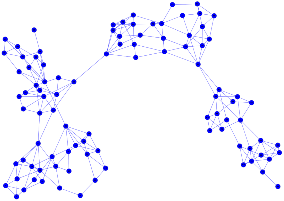
Adjacency matrix

Drawback

The adjacency matrix can be very useful in a number of problems, however, storing a network on the computer via its adjacency matrix is usually not a good idea...

Adjacency matrix
Drawback

- The A_{ij} for real networks contains mostly zeros...



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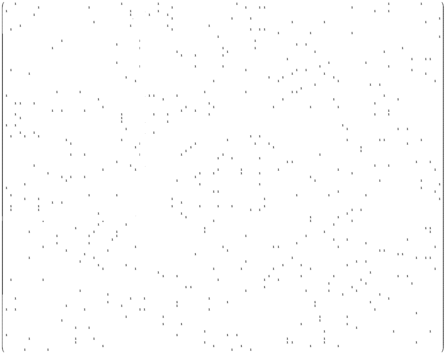

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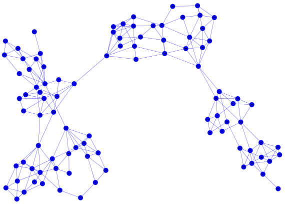

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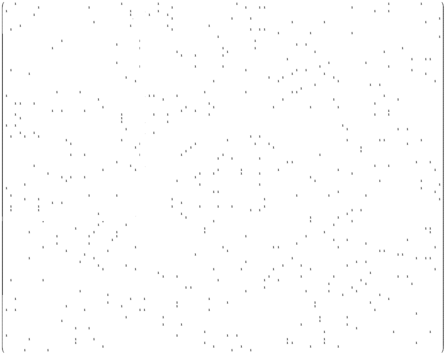

- This is called sparseness.

Adjacency matrix
Drawback

- Storing a lot of zeros does not seem to be a good idea...

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→ What to do?

Adjacency matrix
Drawback

- Storing a lot of zeros does not seem to be a good idea...

$\rightarrow$ What to do?

- Use a link list instead.

Sparseness

How would you give sparseness a mathematical definition?

Sparseness

Sparseness

- A network (graph) is **sparse** in the $N \to \infty$ limit (N denotes the number of nodes) **if the number of links** $L \sim N$.
- A network (graph) is **dense** in the $N \to \infty$ limit (N denotes the number of nodes) **if the number of links** $L \sim N^2$.

What about the average number of connections per node?

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What about the average number of connections per node?

- sparse case: $\langle k \rangle = \frac{2L}{N} \to \text{const.}$,
- dense case: $\langle k \rangle = \frac{2L}{N} \sim N \to \infty$!

Sparseness
In practice

What does this mean in practice?

(How would you decide whether a real network of finite size is sparse or not?)

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In practice

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If $\langle k \rangle << N$ **by several orders of magnitude smaller**, then the network is considered **sparse**.

Sparseness

Most real networks are sparse!

Network	Size	$\langle k \rangle$	ℓ	ℓ_{rand}	C	C_{rand}	Reference	Nr.
WWW, site level, undir.	153,127	35.21	3.1	3.35	0.1078	0.00023	Adamic 1999	1
Internet, domain level	3015 - 6209	3.52 - 4.11	3.7 - 3.76	6.36 - 6.18	0.18 - 0.3	0.001	Yook *et al.* 2001a, Pastor-Satorras *et al.* 2001	2
Movie actors	225,226	61	3.65	2.99	0.79	0.00027	Watts, Strogatz 1998	3
LANL coauthorship	52,909	9.7	5.9	4.79	0.43	1.8×10^{-4}	Newman 2001a,b	4
MEDLINE coauthorship	1,520,251	18.1	4.6	4.91	0.066	1.1×10^{-5}	Newman 2001a,b	5
SPIRES coauthorship	56,627	173	4.0	2.12	0.726	0.003	Newman 2001a,b,c	6
NCSTRL coauthorship	11,994	3.59	9.7	7.34	0.496	3×10^{-4}	Newman 2001a,b	7
Math coauthorship	70,975	3.9	9.5	8.2	0.59	5.4×10^{-5}	Barabási *et al.* 2001	8
Neurosci. coauthorship	209,293	11.5	6	5.01	0.76	5.5×10^{-5}	Barabási *et al.* 2001	9
E. coli, substrate graph	282	7.35	2.9	3.04	0.32	0.026	Wagner, Fell 2000	10
E. coli, reaction graph	315	28.3	2.62	1.98	0.59	0.09	Wagner, Fell 2000	11
Ythan estuary food web	134	8.7	2.43	2.26	0.22	0.06	Montoya, Solé 2000	12
Silwood park food web	154	4.75	3.40	3.23	0.15	0.03	Montoya, Solé 2000	13
Words, cooccurence	460.902	70.13	2.67	3.03	0.437	0.0001	Cancho, Solé 2001	14
Words, synonyms	22,311	13.48	4.5	3.84	0.7	0.0006	Yook *et al.* 2001	15
Power grid	4,941	2.67	18.7	12.4	0.08	0.005	Watts, Strogatz 1998	16
C. Elegans	282	14	2.65	2.25	0.28	0.05	Watts, Strogatz 1998	17

Sparseness
Implications

What are the consequences of sparseness?

- we use a **link list instead of the adjacency matrix**,
- $\langle k \rangle << N$

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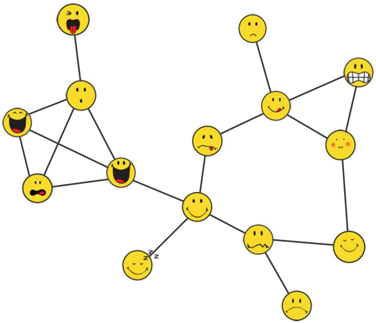
- $\langle k \rangle << N$
- the probability for a connection between a randomly chosen pair?

Sparseness
Implications

What are the consequences of sparseness?

- we use a **link list instead of the adjacency matrix**,
- $\langle k \rangle << N$
- for a randomly chosen pair of nodes, **the probability of being linked is negligible!** (I.e., the probability converges to 0 if $N \to \infty$).

Basic network characteristics

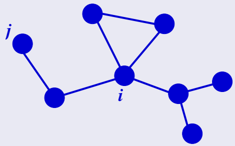


BASIC NODE PROPERTIES

Node degree

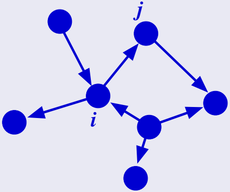
Node degree

- Number of connections, denoted by k_i or d_i.



$$k_i = 4,\ k_j = 1.$$

- Directed networks $\longrightarrow k_i^{\text{in}}$ and k_i^{out}.



$$k_i^{\text{in}} = 2,\ k_j^{\text{out}} = 1.$$

Node degree

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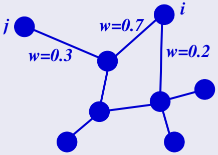
- Weighted networks?



Node degree

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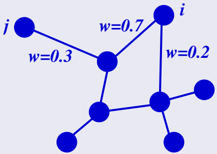
- Weighted networks: node strength, s_i



$$s_i = 0.9, \; s_j = 0.3.$$

Node degree
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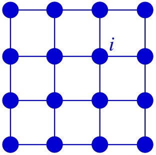
- a 1d chain of nodes?  $k_i = 2$

- a 2d square lattice?



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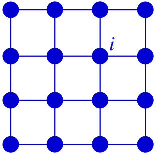

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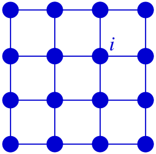

$k_i = 4$
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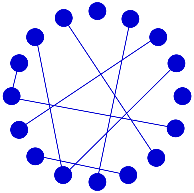

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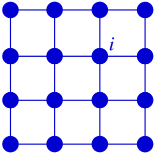

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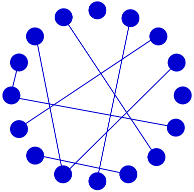
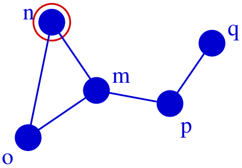


$$\langle k_i \rangle = (N-1)p \simeq Np$$

Node degree and adjacency matrix

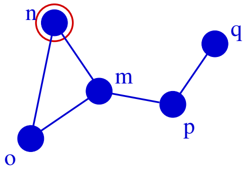


$\longleftrightarrow$

$$\begin{array}{c} \\ \mathrm{m} \\ \mathrm{n} \\ \mathrm{o} \\ \mathrm{p} \\ \mathrm{q} \end{array} \begin{array}{c} \begin{array}{ccccc} \mathrm{m} & \mathrm{n} & \mathrm{o} & \mathrm{p} & \mathrm{q} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$k_{\mathrm{n}} = ?$

Node degree and adjacency matrix



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$$
\begin{aligned}
k_{\mathrm{n}} &= \sum_j A_{jn} \\
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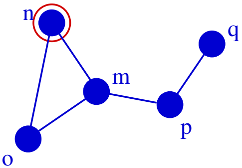
Node degree and adjacency matrix



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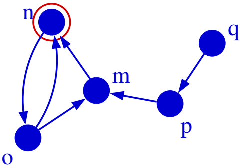

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$$k_{\mathrm{n}}^{\mathrm{in}} = ?$$

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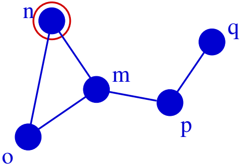
Node degree and adjacency matrix



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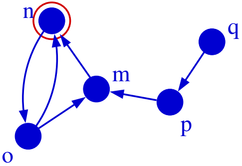

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$$k_{\mathrm{n}}^{\mathrm{in}} = \sum_j A_{jn}$$

$$k_{\mathrm{n}}^{\mathrm{out}} = \sum_j A_{nj}$$

Average degree

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$$\langle k \rangle \equiv \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{2L}{N},$$

- a sort of density.
- directed networks:

$$\langle k_{\text{in}} \rangle \equiv \frac{1}{N} \sum_{i=1}^{N} k_{i,\text{in}} = \frac{L}{N} = \langle k_{\text{out}} \rangle$$

Triangles, transitivity, clustering

- Is the friend of your friend a friend of yours as well?
- Do two of your friends know each other as well? (Are they friends?)

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$\rightarrow$ lot of **triangles** in the network of acquaintances.

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→ lot of **triangles** in the network of acquaintances.

Quantifying local transitivity: **clustering coefficient**.

Clustering coefficient

Clustering coefficient

- The clustering coefficient of node i:

$$C_i \equiv \frac{2e_i}{k_i(k_i-1)},$$

where e_i stands for the number of links between its neighbors. (Thus, $C_i \in [0,1]$).

- If $k_i < 2$, then $C_i = 0$ by definition.

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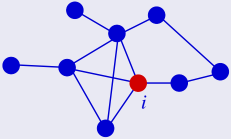
where e_i stands for the number of links between its neighbors. (Thus, $C_i \in [0,1]$).
Example:



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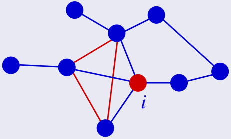

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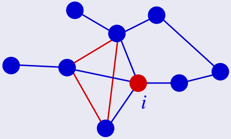

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Clustering coefficient
Directed networks

Clustering coefficient in directed networks

- In a directed network twice as many links are possible between the neighbors of a given node, thus,

$$C_i \equiv \frac{e_i}{k_i(k_i-1)}.$$

Average clustering and transitivity coefficient

Average clustering coefficient

- The average clustering coefficient of the network is

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} C_i,$$

where N denotes the number of nodes in the network.

Clustering coefficient
Illustration

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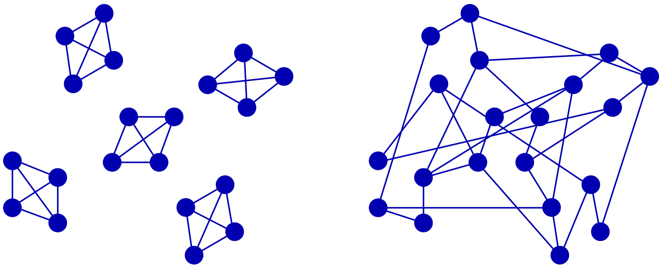

$<k>=3, \; <C>=1$ $\qquad$ $<k>=3, \; <C>=0$

C in real networks

The $\langle C \rangle$ in real networks and their randomized counterparts (with the same N and L):

Network	Size	$\langle k \rangle$	ℓ	ℓ_{rand}	C	C_{rand}	Reference
WWW, site level, undir.	153,127	35.21	3.1	3.35	0.1078	0.00023	Adamic 1999
Internet, domain level	3015 - 6209	3.52 - 4.11	3.7 - 3.76	6.36 - 6.18	0.18 - 0.3	0.001	Yook *et al.* 2001a, Pastor-Satorras *et al.* 2001
Movie actors	225,226	61	3.65	2.99	0.79	0.00027	Watts, Strogatz 1998
LANL coauthorship	52,909	9.7	5.9	4.79	0.43	1.8×10^{-4}	Newman 2001a,b
MEDLINE coauthorship	1,520,251	18.1	4.6	4.91	0.066	1.1×10^{-5}	Newman 2001a,b
SPIRES coauthorship	56,627	173	4.0	2.12	0.726	0.003	Newman 2001a,b,c
NCSTRL coauthorship	11,994	3.59	9.7	7.34	0.496	3×10^{-4}	Newman 2001a,b
Math coauthorship	70,975	3.9	9.5	8.2	0.59	5.4×10^{-5}	Barabási *et al.* 2001
Neurosci. coauthorship	209,293	11.5	6	5.01	0.76	5.5×10^{-5}	Barabási *et al.* 2001
E. coli, substrate graph	282	7.35	2.9	3.04	0.32	0.026	Wagner, Fell 2000
E. coli, reaction graph	315	28.3	2.62	1.98	0.59	0.09	Wagner, Fell 2000
Ythan estuary food web	134	8.7	2.43	2.26	0.22	0.06	Montoya, Solé 2000
Silwood park food web	154	4.75	3.40	3.23	0.15	0.03	Montoya, Solé 2000
Words, cooccurence	460.902	70.13	2.67	3.03	0.437	0.0001	Cancho, Solé 2001
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Power grid	4,941	2.67	18.7	12.4	0.08	0.005	Watts, Strogatz 1998
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MOST REAL NETWORKS ARE HIGHLY CLUSTERED!