Python scripting for network analysis — III

Lab session on March 5th

Homework review

Homework: ring & shortcuts network

We construct the following network:

1. ring of N nodes with first and second neighbor connections
2. add $N/2$ shortcuts (additional edges) at random

Question: what is the average clustering coefficient of this network as $N \to \infty$?

1. Generate network
2. Measure quantity (average clustering)
3. Estimate for $N \to \infty$

Generate network

1. Start with ring
2. Add shortcuts

In [2]:
```
import networkx
import matplotlib.pyplot as plt

def ring_network(N):
    g = networkx.Graph()
    for i in range(N):
        g.add_edge(i, (i+1)%N)
        g.add_edge(i, (i+2)%N)
    return g
```

In [3]:
```
import random
def add_shortcuts_first_try(graph):
    nodes = list(graph.nodes())
    for i in range(int(len(nodes)/2)):
        node_a = random.choice(nodes)
        node_b = random.choice(nodes)
        graph.add_edge(node_a, node_b)
    return graph
```

In [4]:
```
for i in range(10):
    g = add_shortcuts_first_try(ring_network(10))
    print(g.number_of_nodes(), g.number_of_edges())
```

```
10 24
10 22
10 21
10 23
10 22
10 23
10 24
10 22
10 24
10 21
```

In [5]:

```python
import random
def add_shortcuts(graph):
    if graph.number_of_nodes() < 6:
        raise ValueError("network is too small!")
    nodes = list(graph.nodes())
    for i in range(int(len(nodes)/2)):
        node_a = random.choice(nodes)
        node_b = random.choice(nodes)
        while graph.has_edge(node_a, node_b) or node_a == node_b:
            node_a = random.choice(nodes)
            node_b = random.choice(nodes)
        graph.add_edge(node_a, node_b)
    return graph

for i in range(10):
    g = add_shortcuts(ring_network(10))
    print(g.number_of_nodes(), g.number_of_edges())
```

```
10 25
10 25
10 25
10 25
10 25
10 25
10 25
10 25
10 25
10 25
```

Estimating $N \to \infty$

In [6]:
```
networkx.average_clustering(add_shortcuts(ring_network(10)))
```

Out[6]:
```
0.5809523809523809
```

In [7]:
```
networkx.average_clustering(add_shortcuts(ring_network(100)))
```

Out[7]:
```
0.35037301587301584
```

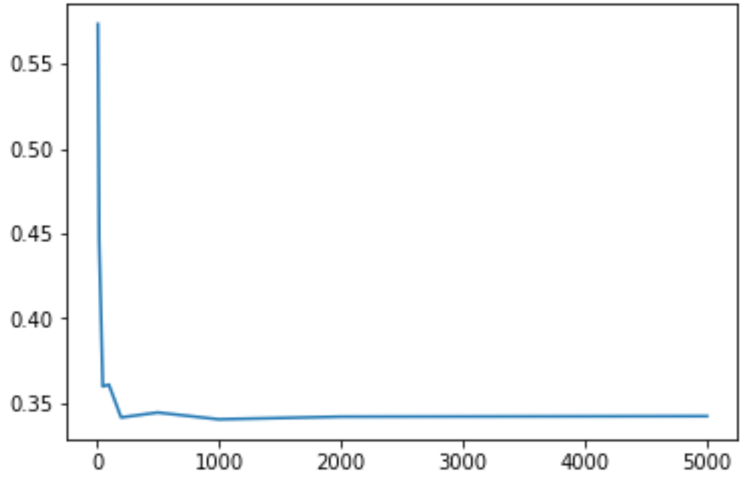
In [8]:
```
networkx.average_clustering(add_shortcuts(ring_network(100000)))
```

Out[8]:
```
0.34191382900428413
```

Make convincing!

In [9]:
```
N_values = [10,20,50,100,200,500, 1000, 2000,5000]
values = [networkx.average_clustering(add_shortcuts(ring_network(N))) for N in N_values]
plt.plot(N_values, values)
```

Out[9]:
```
[<matplotlib.lines.Line2D at 0x7fd911baeb00>]
```



In [10]:
```
values
```

Out[10]:
```
[0.5733333333333335,
 0.446904761904762,
 0.3600000000000002,
 0.3610714285714285,
 0.3417619047619043,
 0.3446460317460319,
 0.34065000000000134,
 0.3421999999999986,
 0.34251857142856473]
```

State the result!

- in this case, for example: "the average clustering as $N \to \infty$ is between 0.340 and 0.342"
- specifying interval allows showing the uncertainty of the estimate

Does our result match the theoretical estimate?

- theoretical estimate was 0.3
- this is outside the range of our estimate ("between 0.340 and 0.342"), thus the two results don't match

What causes the discrepancy?

- Is it the approximation done for the theoretical estimate? (that we ignore triangles created by the shortcut edges)

In [11]:
```
sum(networkx.triangles(ring_network(100)).values())/3.0
```

Out[11]: 100.0

In [12]:
```
sum(networkx.triangles(add_shortcuts(ring_network(100))).values())/3.0
```

Out[12]: 105.0

In [13]:
```
def additional_triangles2(N):
    return sum(networkx.triangles(add_shortcuts(ring_network(N))).values())/3.0 -
sum(networkx.triangles(ring_network(N)).values())/3.0
```

In [14]:
```
additional_triangles2(100)
```

Out[14]: 5.0

In [15]:
```
additional_triangles2(100000)
```

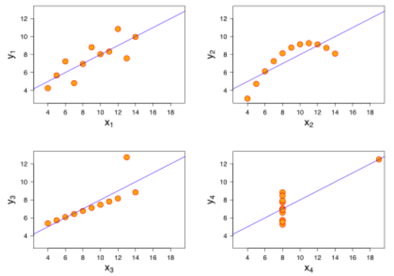
Out[15]: 6.0

What causes the discrepancy?

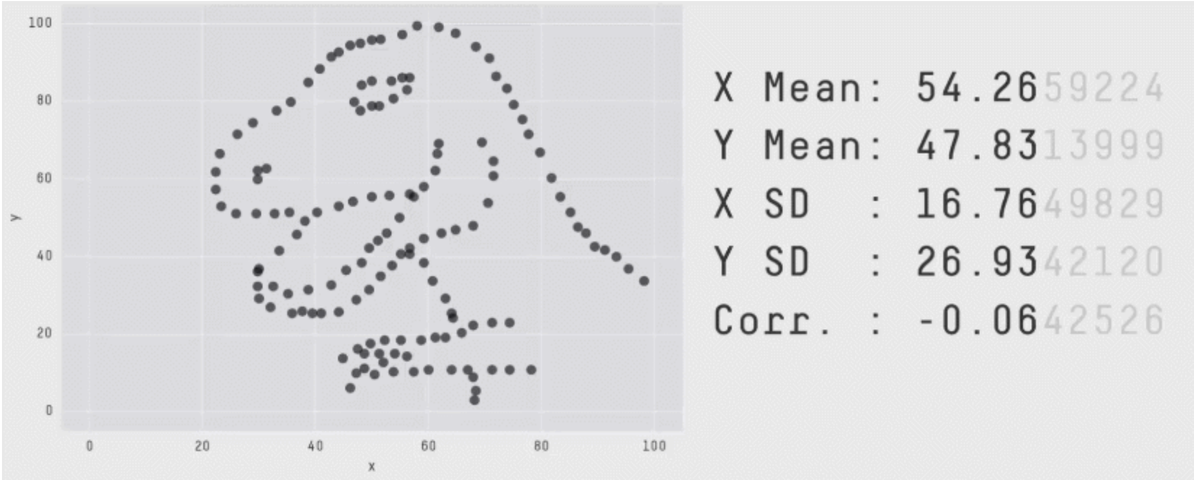
- We are calculating an average, which is doing something strange. A important lession about statistical quantities:

Anscombe's quartet

- 11 (x,y) datapoints
- Mean & variance of x match exactly
- Mean & variance of y, correlation between x & y, fitted line (linear regression) match to at least 2 decimal places



Even better: Datasaurus

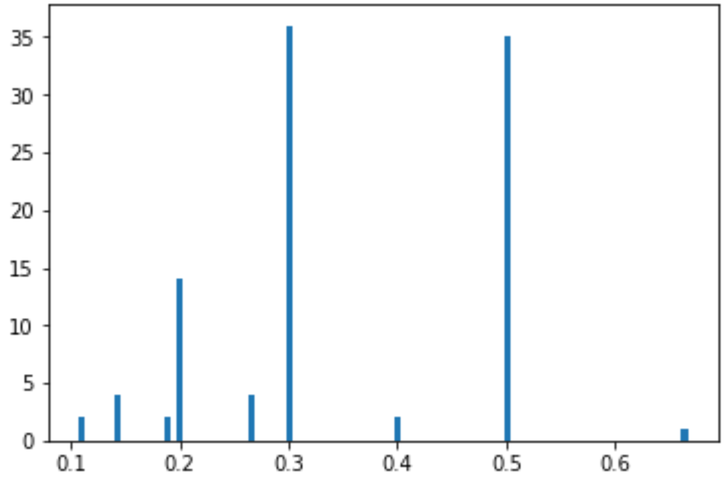


https://www.autodeskresearch.com/publications/samestats
(https://www.autodeskresearch.com/publications/samestats)

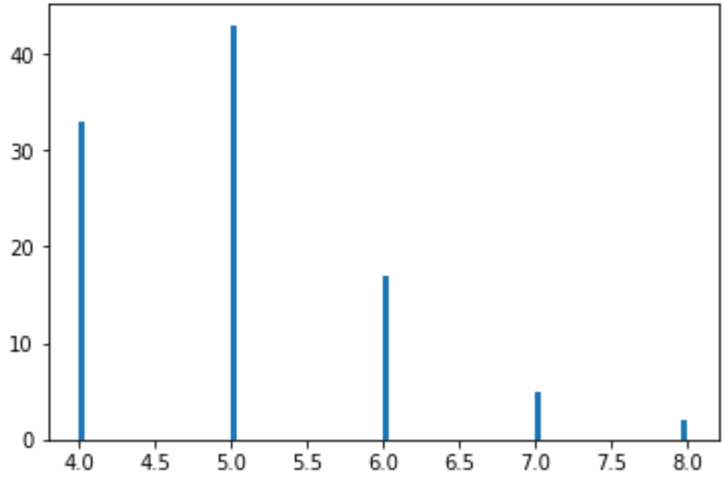
Let's look at what we are averaging!

In [16]:

```
_ = plt.hist(list(networkx.clustering(add_shortcuts(ring_network(100))).values()),
bins=100)
```



In [17]:
```
_ = plt.hist(list(dict(networkx.degree(add_shortcuts(ring_network(100)))).values
()), bins=100)
```



In [18]:
```
_ = plt.hist(list(dict(networkx.degree(add_shortcuts(ring_network(10000)))).values
()), bins=100)
```

Are we sure this explains the discrepancy?

- There is some difference between the two methods, but there might be others
- How can we check? → Homework assignment

Calculating at a fixed N

- Given N, what is the average clustering coefficient of these networks?

One possible way to calculate: generate a single network, and use that:

In [19]:
```
networkx.average_clustering(add_shortcuts(ring_network(10000)))
```

Out[19]:
```
0.3421695238095362
```

Obviously, we shouldn't use a single network for the estimate, but re-run it a few times:

In [20]:
```
networkx.average_clustering(add_shortcuts(ring_network(10000)))
```

Out[20]:
```
0.3423485064935197
```

In [21]:
```
networkx.average_clustering(add_shortcuts(ring_network(10000)))
```

Out[21]:
```
0.341968686868699
```

Even better, we can calculate an average of these:

In [22]:
```
values = [networkx.average_clustering(add_shortcuts(ring_network(10000))) for i in
range(10)]
```

In [23]:
```
def avg(values):
    return sum(values) / len(values)
```

In [24]:
```
avg([networkx.average_clustering(add_shortcuts(ring_network(10000))) for i in rang
e(10)])
```

Out[24]:
```
0.34188237373738656
```

And re-run this average calculation, as well:

In [25]:
```
avg([networkx.average_clustering(add_shortcuts(ring_network(10000))) for i in rang
e(10)])
```

Out[25]:
```
0.34203490620491894
```

In [26]:
```
avg([networkx.average_clustering(add_shortcuts(ring_network(10000))) for i in rang
e(10)])
```

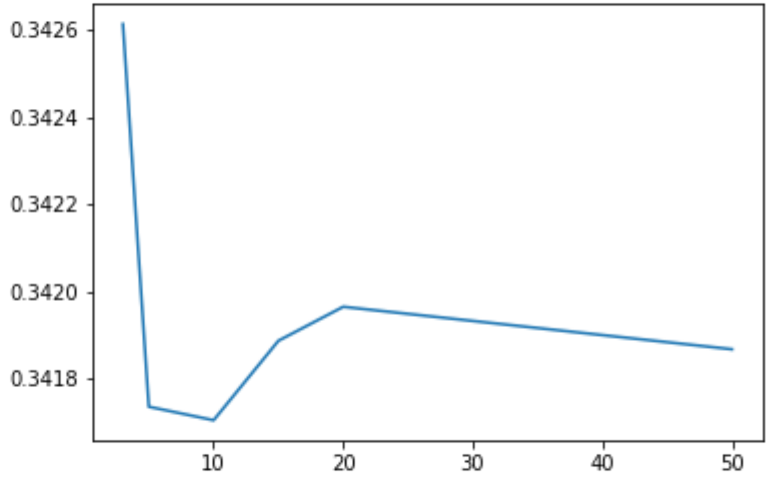
Out[26]:
```
0.3416269784104916
```

Of course, then we have the same problem as above -- do we average these averages? And then average those as well?

A better approach: show how our estimate changes as we use larger and larger number of networks -- this is essentially the same as we did for estimating for $N \rightarrow \infty$.

In [27]:
```
S_values = [3,5,10,15,20,50]
values = [avg([networkx.average_clustering(add_shortcuts(ring_network(10000))) for
i in range(S)]) for S in S_values]
plt.plot(S_values, values)
```

Out[27]:
```
[<matplotlib.lines.Line2D at 0x7fd90b419278>]
```



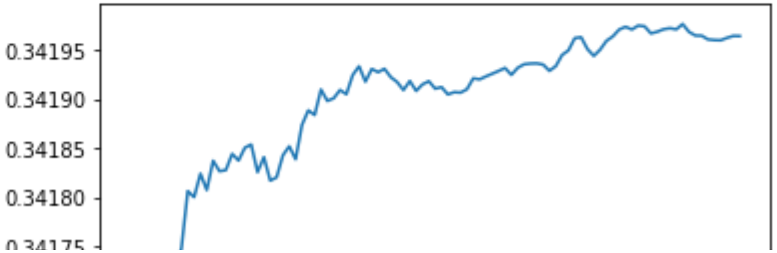
Note that there is an even better way to do this: we don't need to generate a lot of new networks for every datapoint. Instead, we generate a single list of values, and then take successive averages of the first n elements:

In [28]:
```
values = [networkx.average_clustering(add_shortcuts(ring_network(10000))) for i in
range(100)]
```

In [29]:
```
averages = [avg(values[:S]) for S in range(3, len(values))]
```

In [30]:
```
plt.plot(averages)
```

Out[30]:
```
[<matplotlib.lines.Line2D at 0x7fd91181c828>]
```



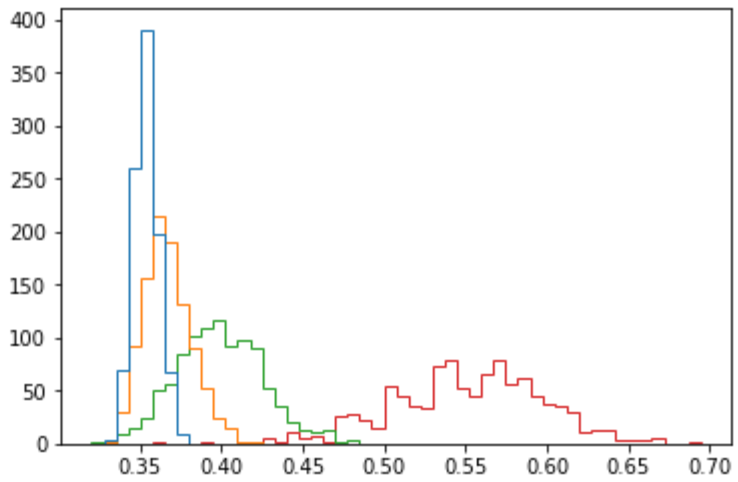
An important aspect about the sample sizes we might need:

- plotting the histogram of the values for different N shows some interesting details:

In [31]:
```
values_100 = [networkx.average_clustering(add_shortcuts(ring_network(100))) for i
in range(1000)]
```

In [32]:
```
values_50 = [networkx.average_clustering(add_shortcuts(ring_network(50))) for i in
range(1000)]
values_25 = [networkx.average_clustering(add_shortcuts(ring_network(25))) for i in
range(1000)]
values_10 = [networkx.average_clustering(add_shortcuts(ring_network(10))) for i in
range(1000)]
```

In [33]:
```
_ = plt.hist([values_100, values_50, values_25, values_10], bins=50, histtype='ste
p')
```



For each N, the histogram shows a bell-shaped curve, which is wider for smaller N and much narrower for larger N. The center of the peak moves to the left as N increases.

Of these, the change in the width is a very general feature: most network quantites behave this way: using a larger network decreases the variance, which also lowers the required sample size (number of networks) needed for achieveing a given accuracy. The movement of the peak is specific to this problem, and this is the same as we have seen when plotting the convergence as a function of N before.

Some interesting features of the random module

We have used or mentioned:

In [7]:
```
import random
random.shuffle()
random.choice()
```

In [ ]:
```
But there is also:
```

In [8]:
```
random.seed()
```

An example for using random.seed():

In [9]:

```
random.seed(42)
print(random.random())
print(random.random())
print(random.random())
```

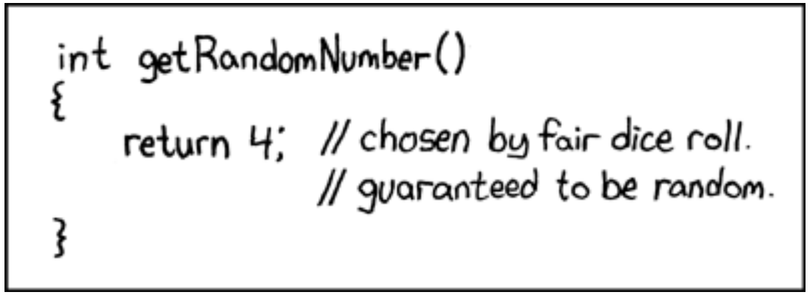
```
0.6394267984578837
0.025010755222666936
0.27502931836911926
```

In [10]:

```
random.seed(42)
print(random.random())
print(random.random())
print(random.random())
```

```
0.6394267984578837
0.025010755222666936
0.27502931836911926
```

Which means that we can generate the same random numbers again -- does this make sense?



From https://xkcd.com/221/ (https://xkcd.com/221/)

John von Neumann: ”Anyone who attempts to generate random numbers by deterministic means is, of course, living in a state of sin.”

Having random.seed()

- is it a problem?
- what is it good for?

- PRNG: Pseudo Random Number Generator

- PRNG has internal state, updating that internal state every time it is used
- Starting point of internal state is set by seed()
- PRNG shouldn't be used for cryptography -- attacker might guess internal state, breaking the encryption
- Why can we use a PRNG for network analysis / simulation?

What do we actually need?

- Want to calculate average over a set of objects
- Can't use all objects (like "networks of size N"), need to use a small sample to approximate
- As long as sample is unbiased, it is fine if it is deterministic

Also: random.getstate(), random.setstate()

- Can't use random.seed() to get the current state of the PRNG -- need .getstate() for that
- Note that random.getstate() returns a much larger state than a hash of an object, which seed() uses to initialize if not a number is passed in
- seed() initializes in a small sub-space of the total possible states of the PRNG

In [12]:
```
random.getstate()
```

Out[12]:
```
(3,
 (2468570525,
  44967195,
  2667364560,
  2449893699,
  1652692239,
  766678126,
  273175325,
  1513475390,
  2407048223,
  2326550691,
  3055735416,
  2487780036,
  476975371,
  81632736,
  1598452444,
  3338301038,
  3898475993,
  1749546629,
  4084786842,
  949316744,
  2086501466,
  4175211502,
  3792229788,
  1718685282,
  2499662139,
  4222931543,
  3063257123,
  910424605,
  1400804300,
  830603822,
  3216023045,
  2756927633,
  3684278863,
  3724968901,
  332416530,
  52016619,
```