Structure and dynamics of complex networks

February 4, 2020

Technical infos
Contacts

Contacts:

Theoretical lectures:

Gergely Palla[1] : pallag@hal.elte.hu

Practical/Lab sessions

Dániel Ábel[1,2] : ait@danielabel.hu

[1] MTA-ELTE Statistical and Biological Research Group, Department of Biological Physics, Eötvös University

[2] Maven Seven Network Research Ltd.

Technical infos

Schedule

Networks Schedule 2020 Spring		
1. week	Feb. 4. Tue	Feb. 6. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
2. week	Feb. 11. Tue.	Feb. 13. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
3. week	Feb. 18. Tue.	Feb. 20. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
4. week	Feb. 25.Tue.	Feb. 27. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
5. week	Mar. 3. Tue.	Mar. 5. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
6. week	Mar. 10. Tue.	Mar. 12. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
7. week	Mar. 17. Tue.	Mar. 19. Thu.
	Theory: Gergely Palla	Mid term test
Break	Mar. 24. Tue.	Mar. 26. Thu.
8. week	Mar. 31. Tue.	Apr. 2. Thu.
	Theory: Illés Farkas	Lab. session: Dániel Ábel
9. week	Apr. 7. Tue.	Apr. 9. Thu.
	Theory: Illés Farkas	Theory: Gergely Palla
10. week	Apr. 14. Tue.	Apr. 16. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
11. week	Apr 21. Tue.	Apr. 23. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
12. week	Apr. 28. Tue.	Apr. 30. Thu.
	Theory: Gergely Palla	Lab. session: Dániel Ábel
13. week	May 5. Tue.	May 7. Thu.
	Lab. session: Dániel Ábel	Theory: András Vicsek
14. week	May 12. Tue.	May 14. Thu.
	Lab. session: Dániel Ábel	Term project presentations

Gergely Palla: Introduction to complex network theory

Dániel Ábel: Developing practical skills for network analysis

Illés J. Farkas: Financial applications of network theory

András Vicsek: Network theory in organisational consulting

Technical infos
Grading

Grading

- midterm test (60-90 min.), (30%)
- term project → presentation (30%)
- homeworks (lab sessions) (20%)
- activity during the lectures (20%)

Technical infos
Reading material

Online:

- Slides are always uploaded to the AIT website.

Technical infos
Reading material

Online:

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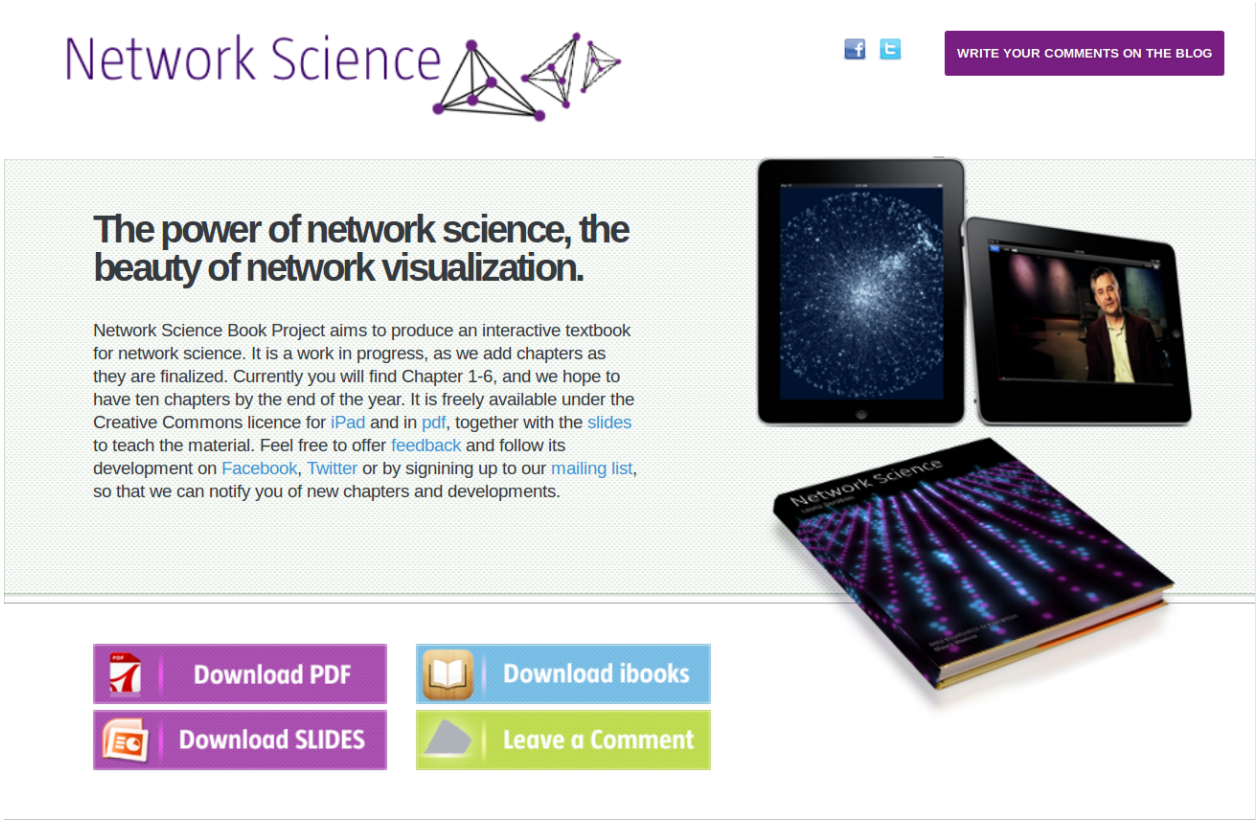
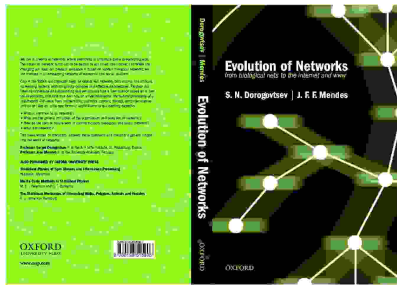
- Albert-László Barabási: Network Science
  `http://networksciencebook.com`



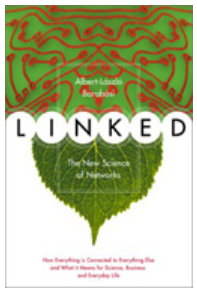
Technical infos
Reading material

Offline:

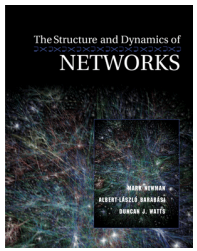
- J. F. F. Mendes and S. N. Dorogovtsev: **Evolution of Networks**, *(Oxford University Press, Oxford, 2003)*



- A.-L. Barabási: **Linked** *(Perseus Books Group, 2002)*



- M. E. J. Newman, A.-L. Barabási, D. J. Watts: **The Structure and Dynamics of Networks** *(Princeton University Press, Princeton, 2006)*



WARNING!!!

Screen shot from theory coming up later...

- At the end of the network generation process $N = t$. The number of links between the neighbors of node l:

$$
\begin{aligned}
n_l &= \frac{1}{2}\sum_{t_i=1}^{N}\sum_{t_j=1}^{N}\mathcal{P}(l-i)\mathcal{P}(l-j)\mathcal{P}(i-j) \\
&= \frac{1}{2}\int_1^N dt_i \int_1^N dt_j \mathcal{P}(l-i)\mathcal{P}(l-j)\mathcal{P}(i-j) \\
&= \frac{m^3}{16}\int_1^N dt_i \int_1^N dt_j (t_l t_i)^{-\frac{1}{2}} (t_l t_j)^{-\frac{1}{2}} (t_i t_j)^{-\frac{1}{2}} \\
&= \frac{m^3}{16 t_l}\int_1^N dt_i \frac{1}{t_i} \int_1^N dt_j \frac{1}{t_j} = \frac{m^3}{16 t_l} (\ln N)^2 .
\end{aligned}
$$

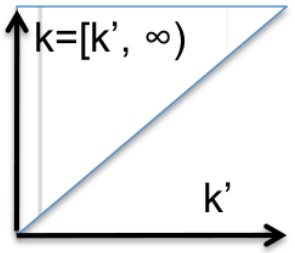
Screen shot from theory coming up later...

- The modified $\langle k' \rangle$ after the removal:

$$\langle k' \rangle = \sum_{k'=0}^{\infty} k' p'(k') = \sum_{k'=0}^{\infty} k' \sum_{k=k'}^{\infty} p(k) \binom{k}{k'} f^{k-k'} (1-f)^{k'} =$$

$$\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} p(k) \frac{k(k-1)!}{(k'-1)!(k-k')!} f^{k-k'} (1-f)^{k'-1} (1-f)$$

The sum is done over the triangle, thus, it can be replaced as $\sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} = \sum_{k=0}^{\infty} \sum_{k'=0}^{k}$



$$\langle k' \rangle = \sum_{k=0}^{\infty} \sum_{k'=0}^{k} p(k) \frac{k(k-1)!}{(k'-1)!(k-k')!} f^{k-k'} (1-f)^{k'-1} (1-f) =$$

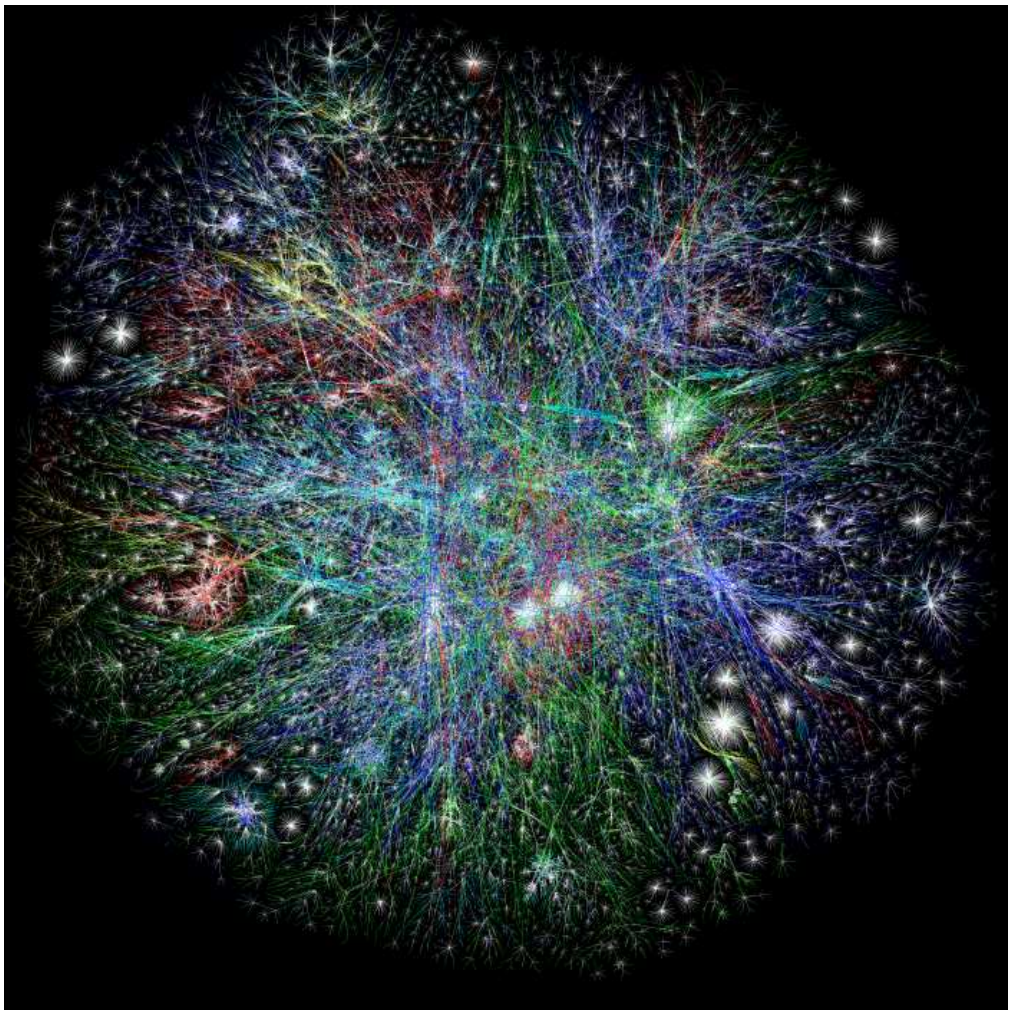
$$(1-f) \sum_{k=0}^{\infty} kp(k) \underbrace{\sum_{k'=0}^{k} \binom{k-1}{k'-1} f^{k-k'} (1-f)^{k'-1}}_{1} = (1-f) \langle k \rangle$$

Introduction

BASIC CONCEPTS

What are networks?

Networks...
(The Internet)



Networks...
(Social)

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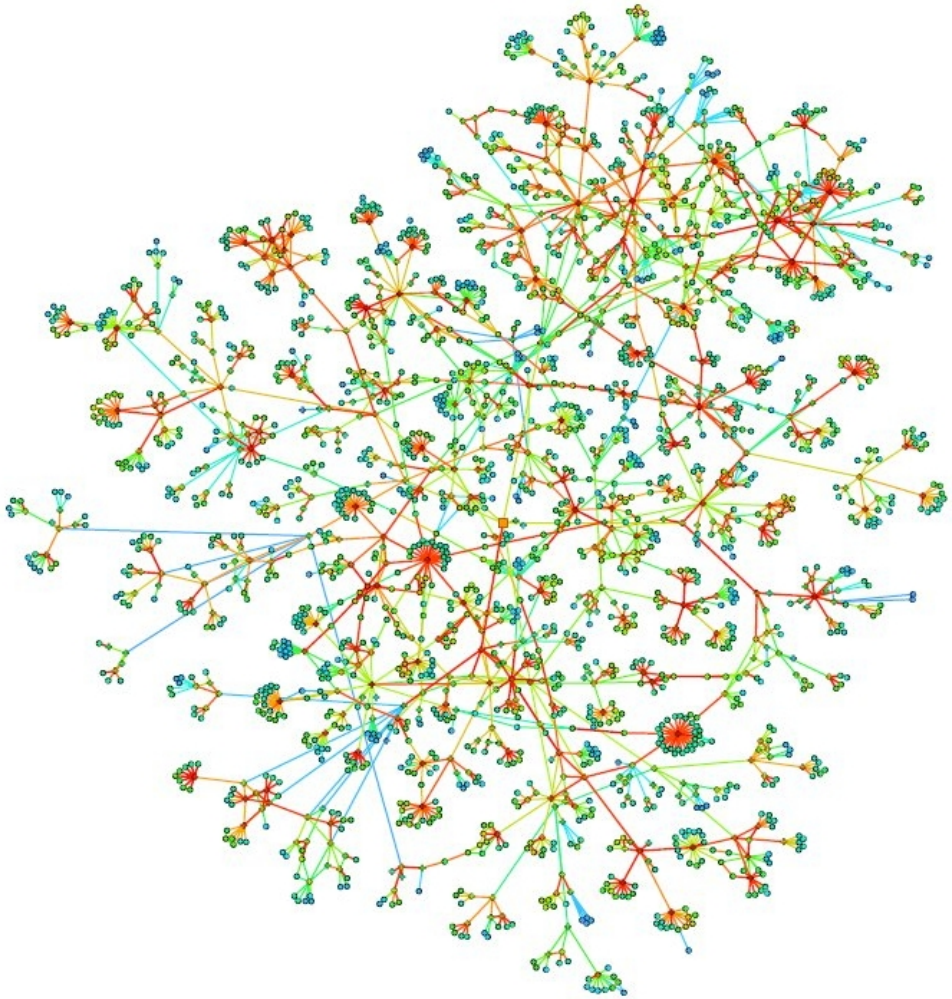
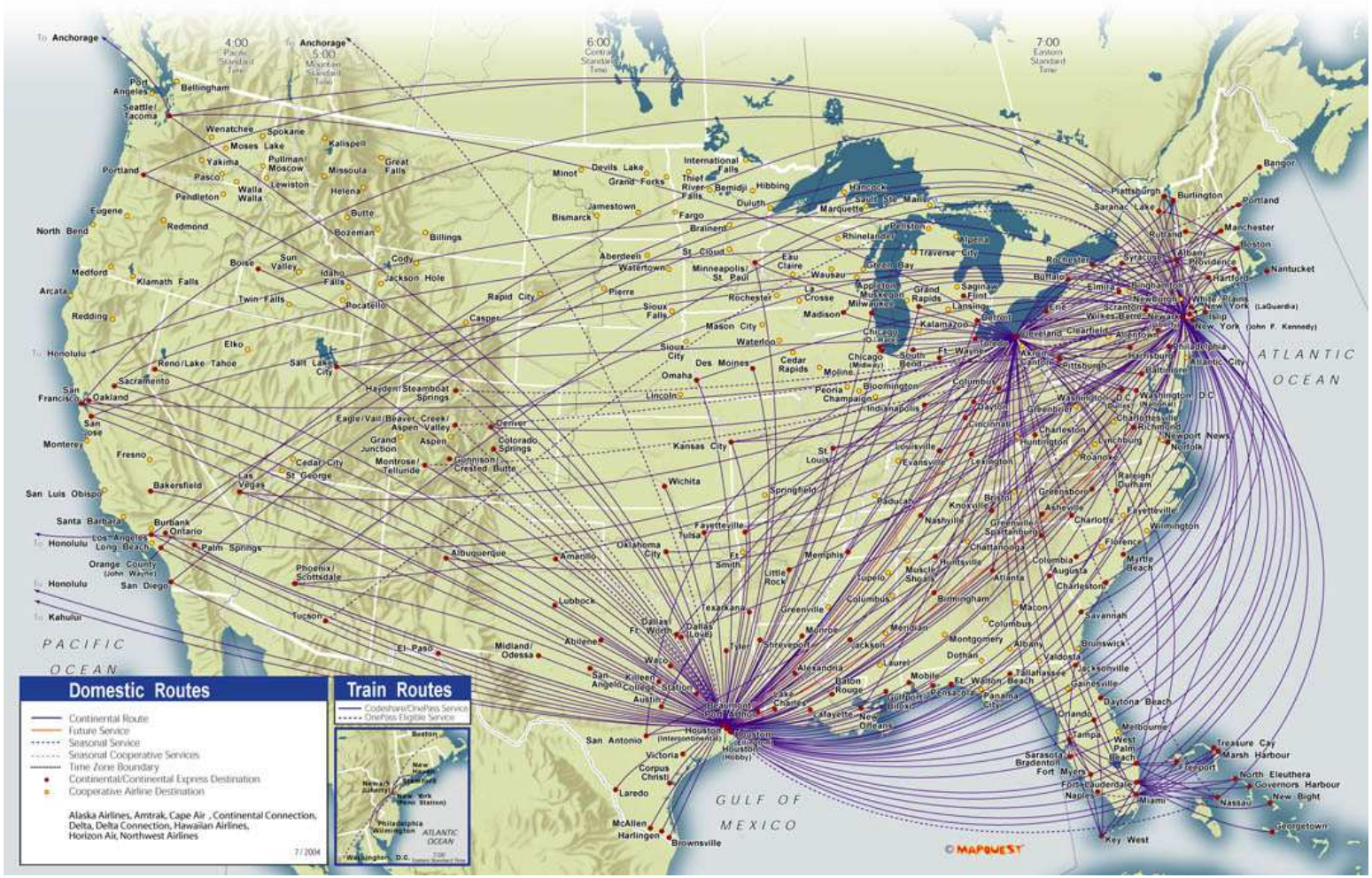
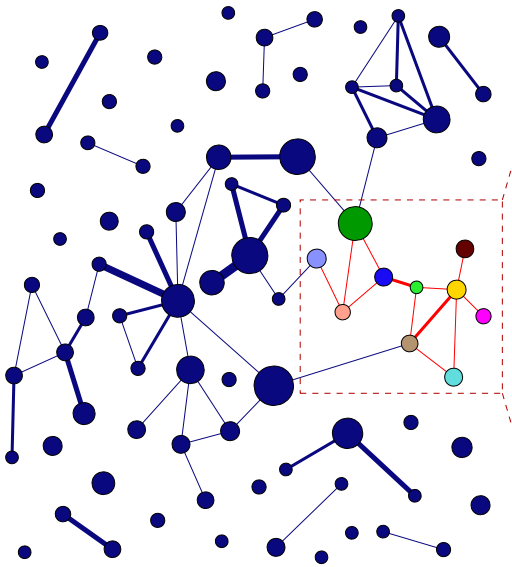
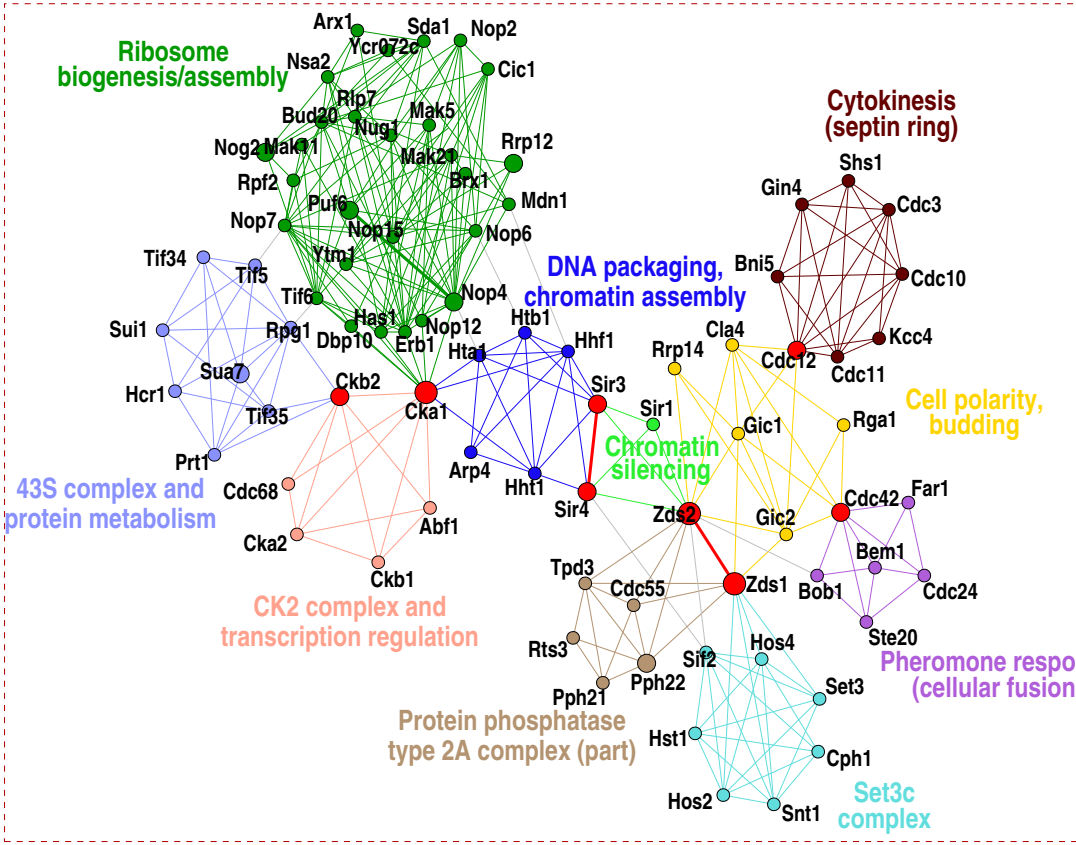


Networks...
(Transportation)

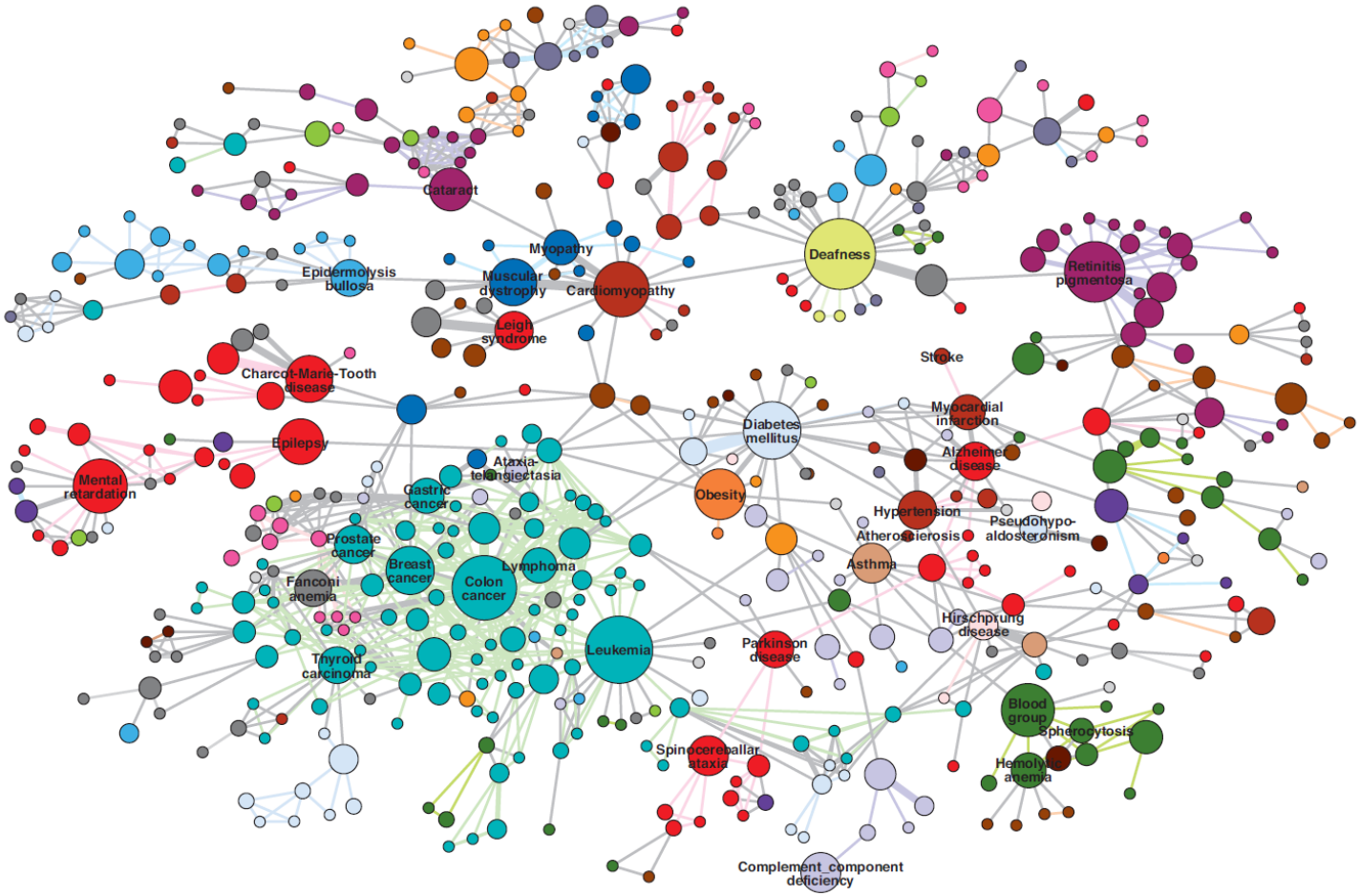


Networks...
(Protein interaction)

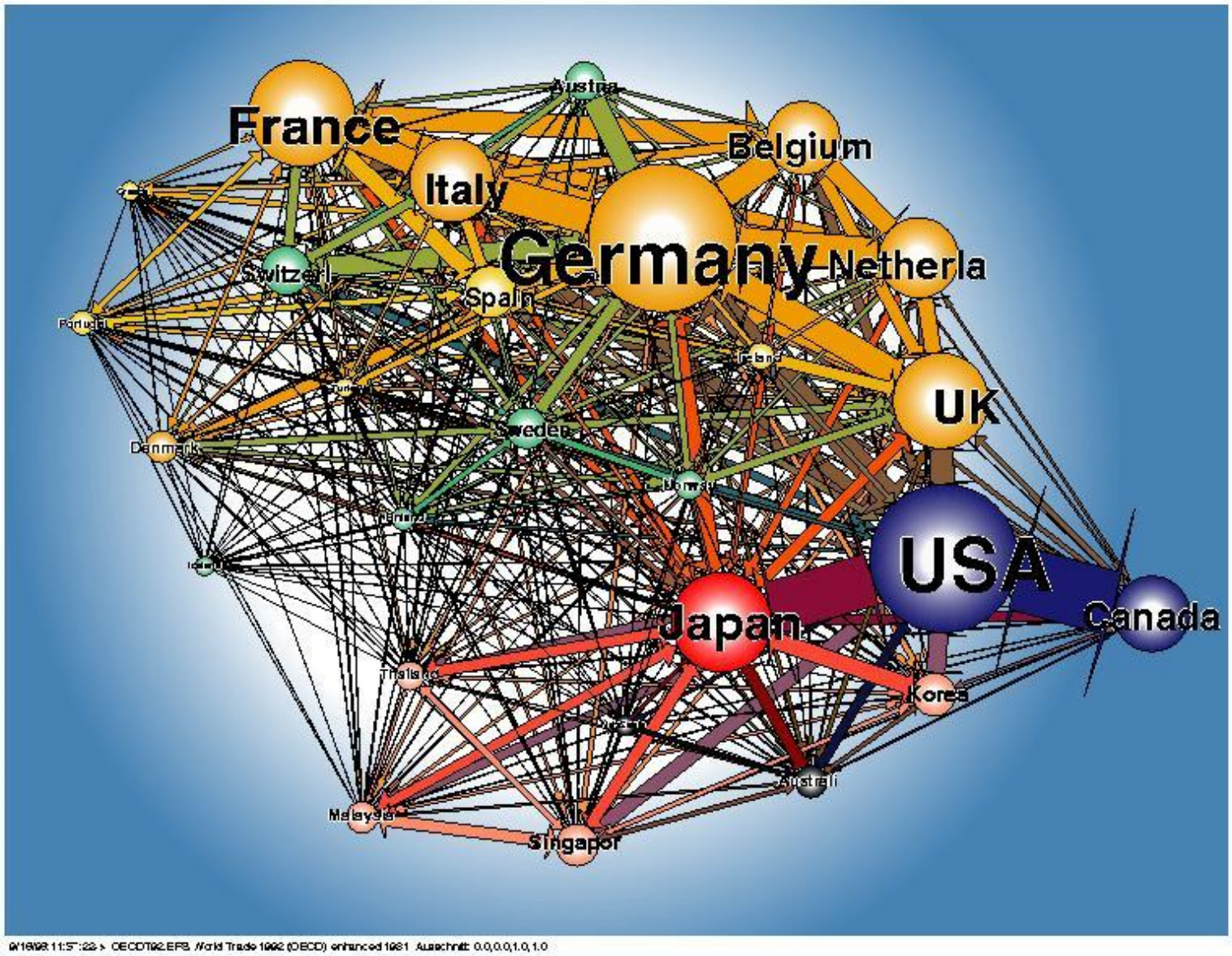




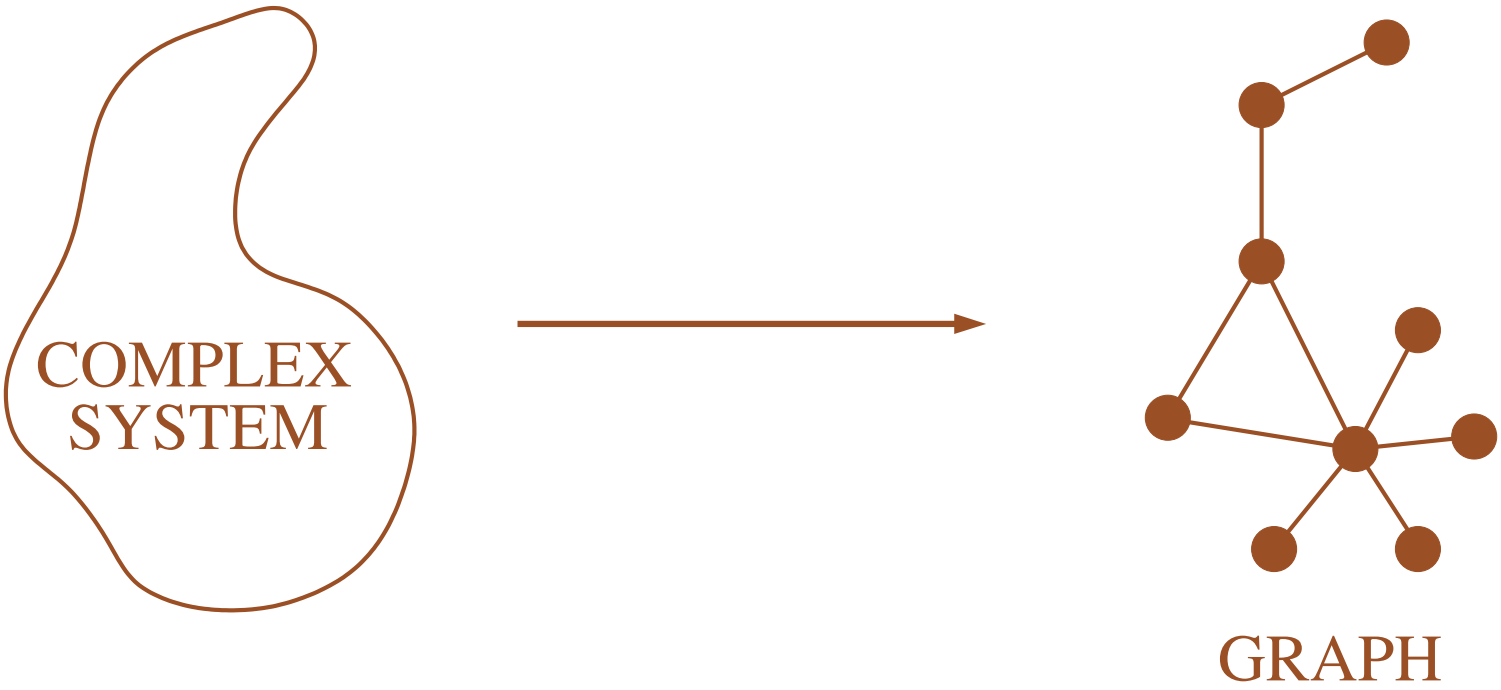
Networks...
(Human diseases)



Networks...
(World trade)



The network approach



What are complex systems?

Complex systems

Dictionary.com

com·plex [*adj., v.* kuhm-**pleks**, **kom**-pleks; *n.* **kom**-pleks]

Show IPA

adjective

1. composed of many interconnected parts; compound; composite: *a complex highway system.*
2. characterized by a very complicated or involved arrangement of parts, units, etc.: *complex machinery.*
3. so complicated or intricate as to be hard to understand or deal with: *a complex problem.*

WIKIPEDIA
The Free Encyclopedia

Complex system: Composed of interconnected parts that as a whole exhibit one or more properties (behavior among the possible properties) not obvious from the properties of the individual parts...

Complex networks

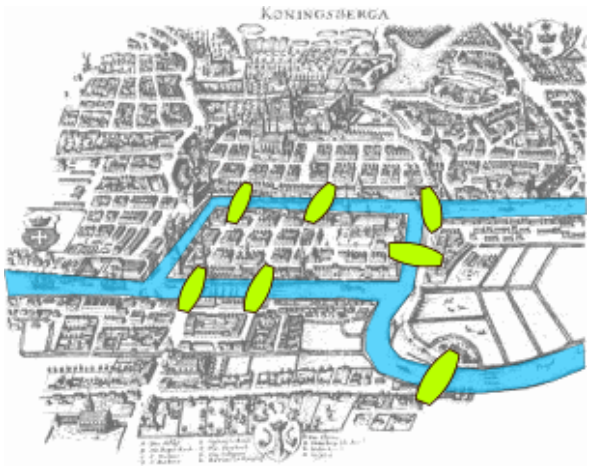
Are real networks complex?

large size 

intricate wiring 

Historical overview of network research

- **Leonhard Euler** (1735):
  The bridges of Königsberg and graph theory.



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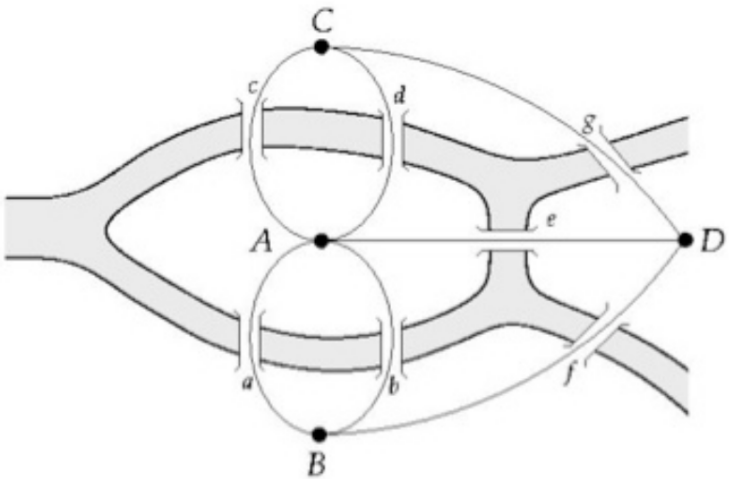

Historical overview of network research

- **Pál Erdős and Alfréd Rényi** (1959): Random graphs.

- **Stanley Milgram** (1967): Six degrees of separation.

Historical overview of network research

- **Duncan Watts and Steven H. Strogatz** (1998): Small world networks.


- **Albert-László Barabási and Réka Albert** (1999): Scale-free networks.


What are the applications of network theory?

Applications of network theory

Quantum Mechanics: 1900

- → electron microscope 1931,
- → transistor 1947,
- → laser 1957,
- → magnetic resonance imaging 1973,

In general, about a **30 year gap** between science and technology.

↕

In case of network science, this drops down to only about **3 years**!

Applications of network theory

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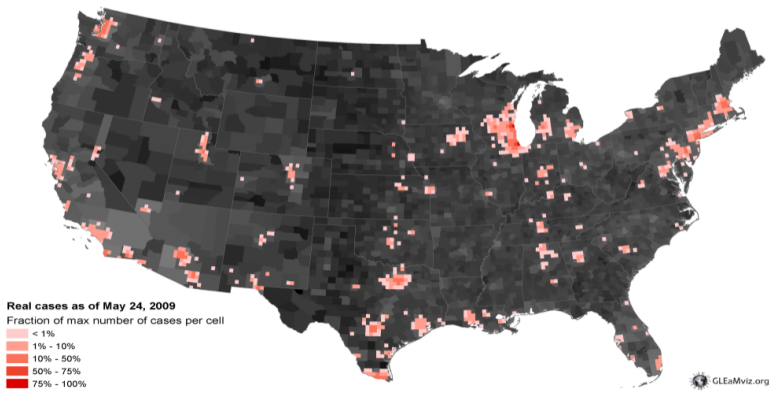
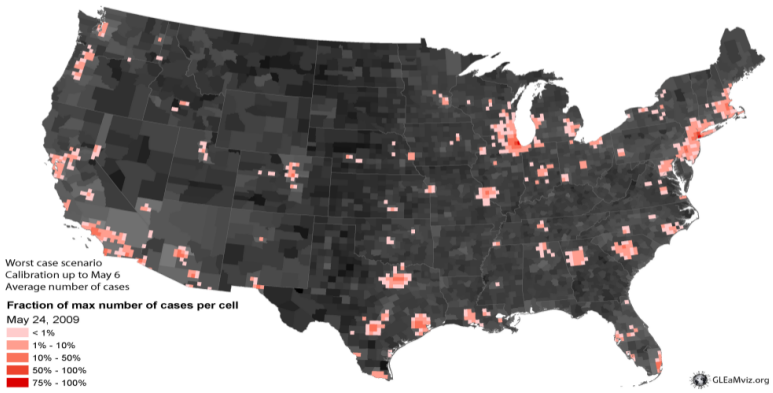
Applications of network theory
Epidemics

Predicting the H1N1 pandemic in 2009:

Real



Projected



Applications of network theory
Epidemics

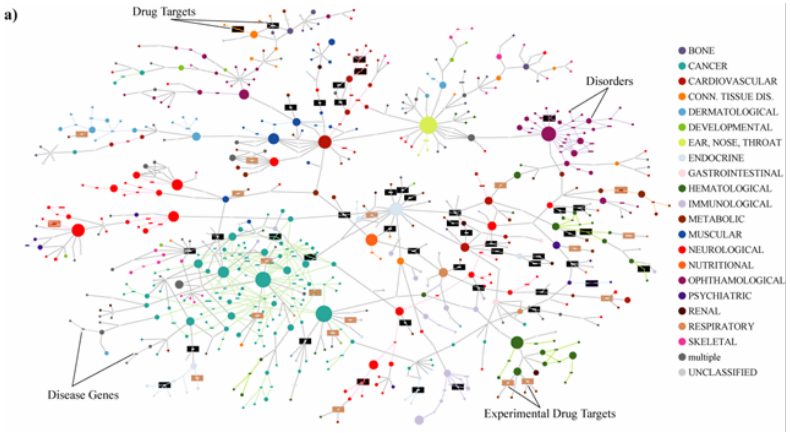
Coronavirus spreading:

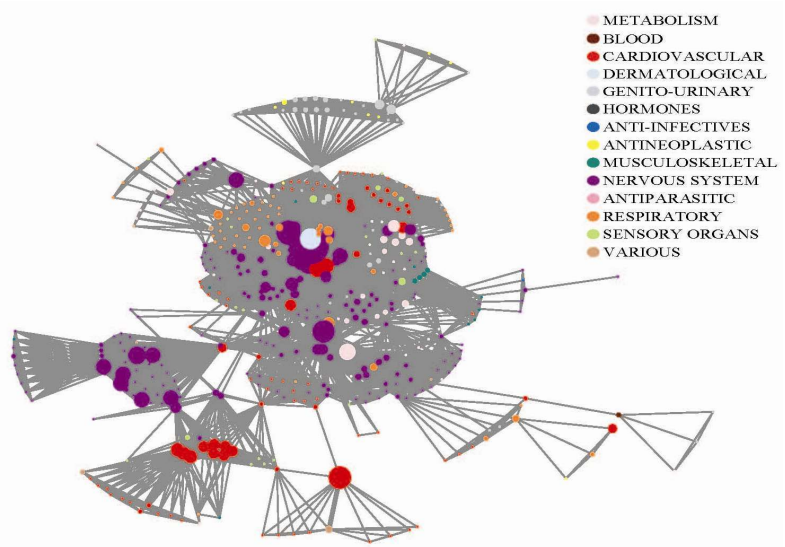
https://gisanddata.maps.arcgis.com/apps/opsdashboard/index.html#/bda7594740fd40299423467b48e9ecf6

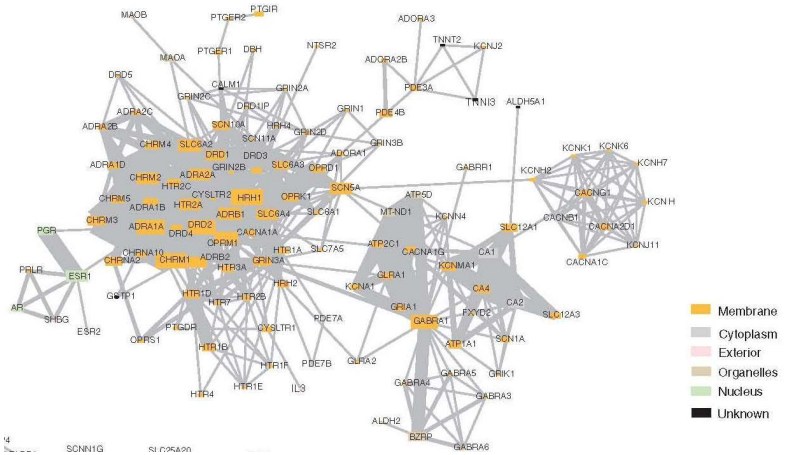
https://public.flourish.studio/visualisation/1292977/?utm_source=showcase&utm_campaign=visualisation/1292977

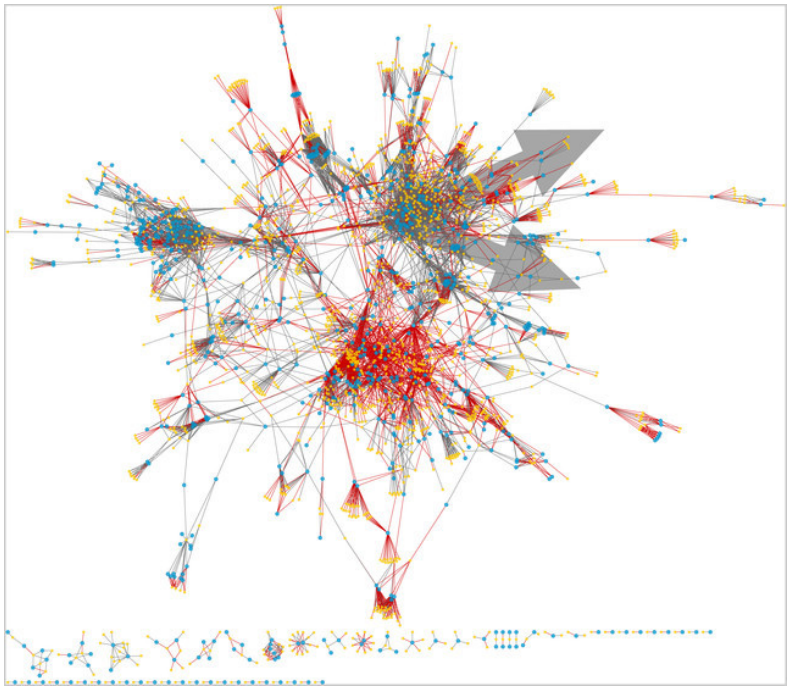
https://www.mobs-lab.org/2019ncov.html

Applications of network theory
Drug design









Applications of network theory
Fighting terrorism

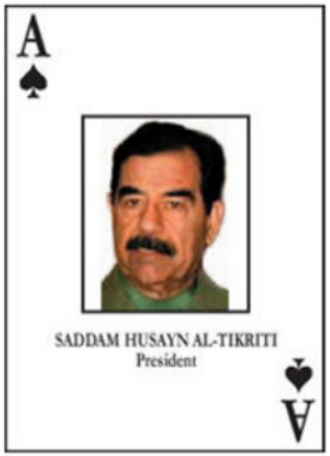
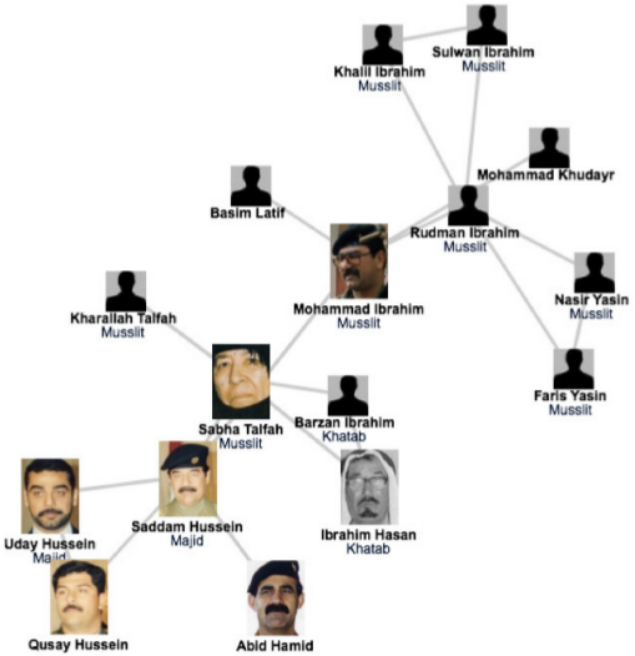


Image 1.2a

The network of Saddam Hussein.

Ace of Spades. One of the 55 cards the US military has handed out to the coalition forces in Iraq, each listing a top official to be captured following the country's 2003 invasion. The card shows the ace of spades, with the image of Saddam Hussein, Iraq's deposed president and dictator, the top prize of the hunt.



(From the book of A.-L. Barabási).

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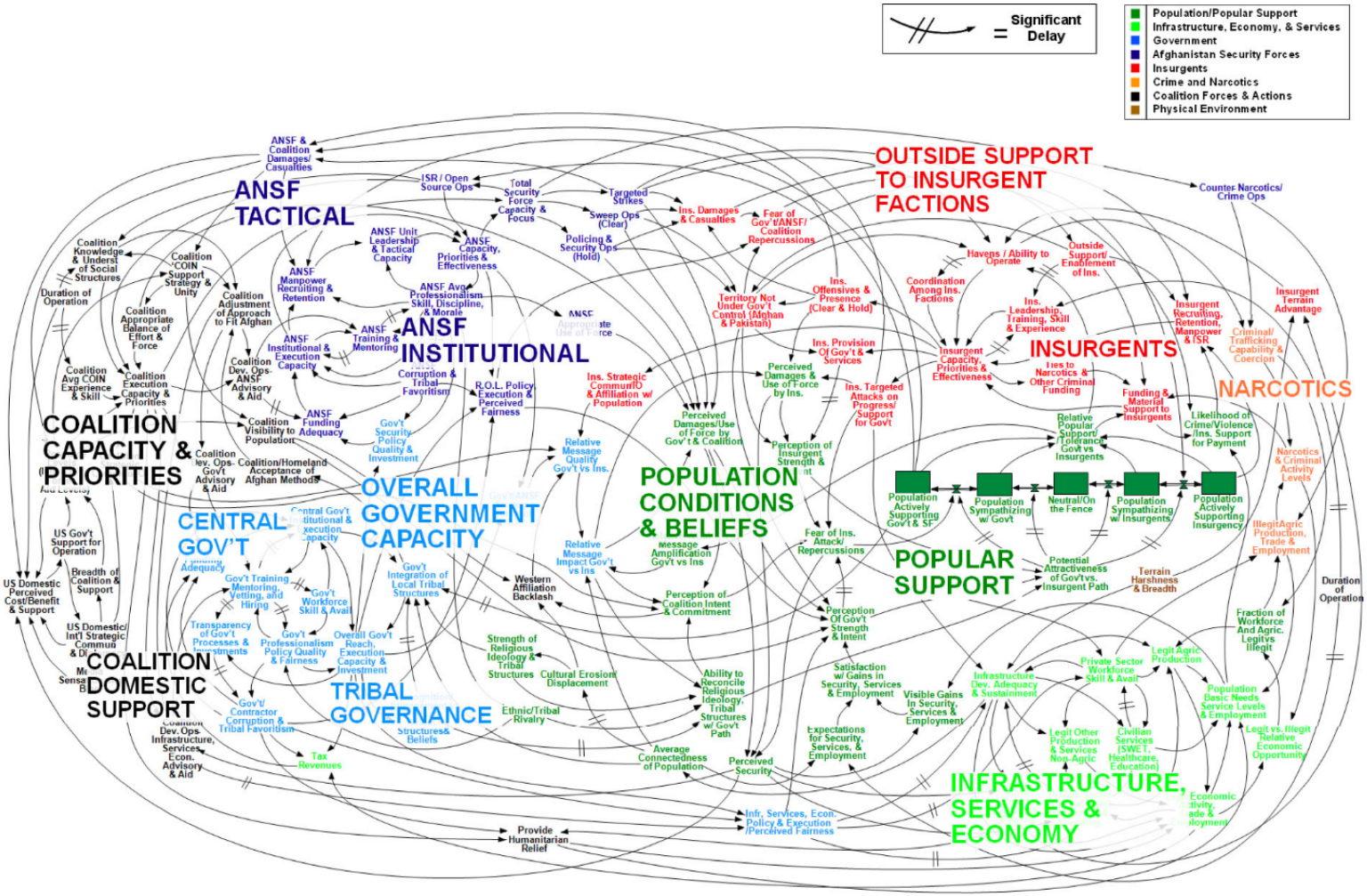

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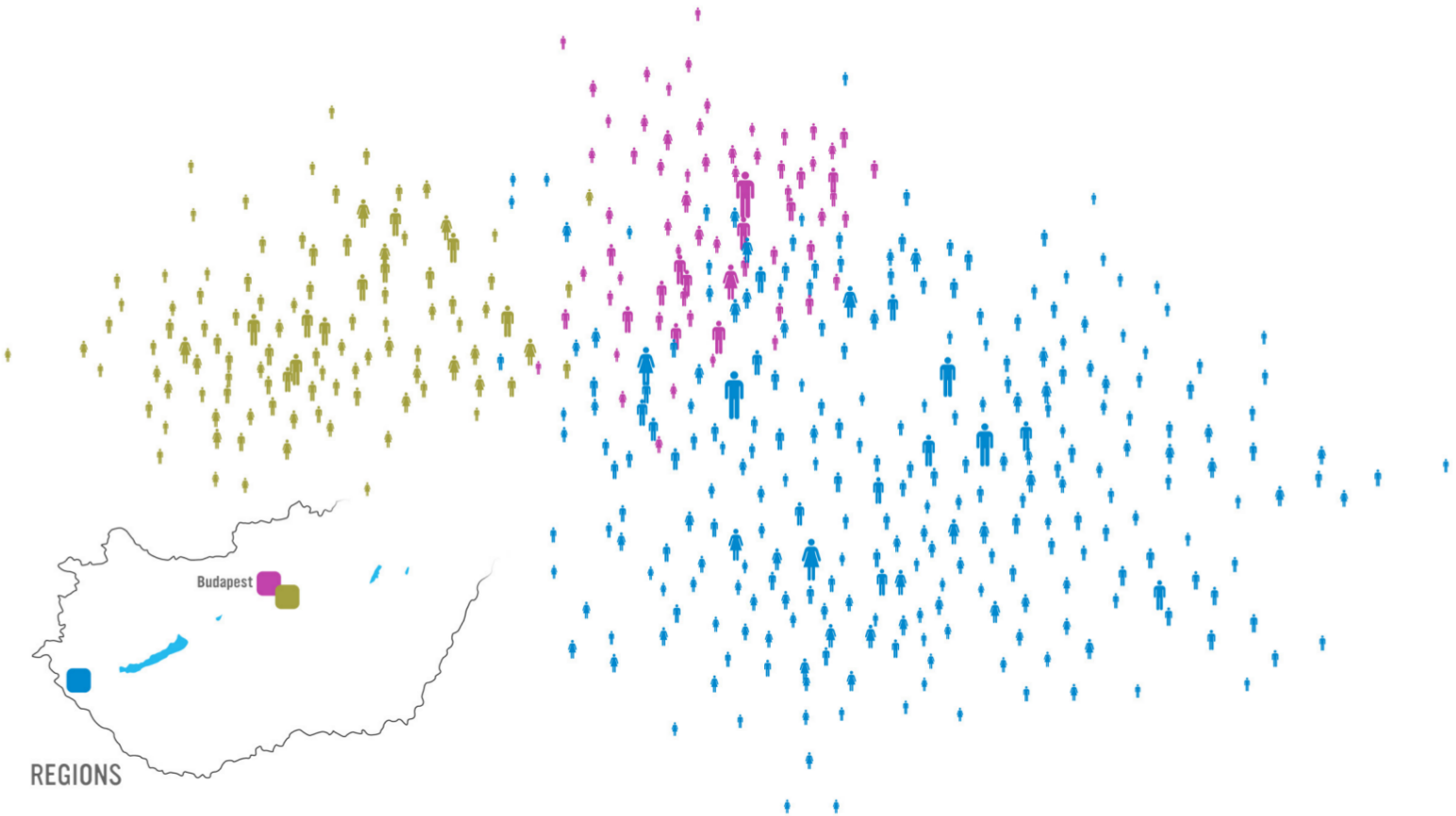
Applications of network theory
Organizational consulting



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Applications of network theory
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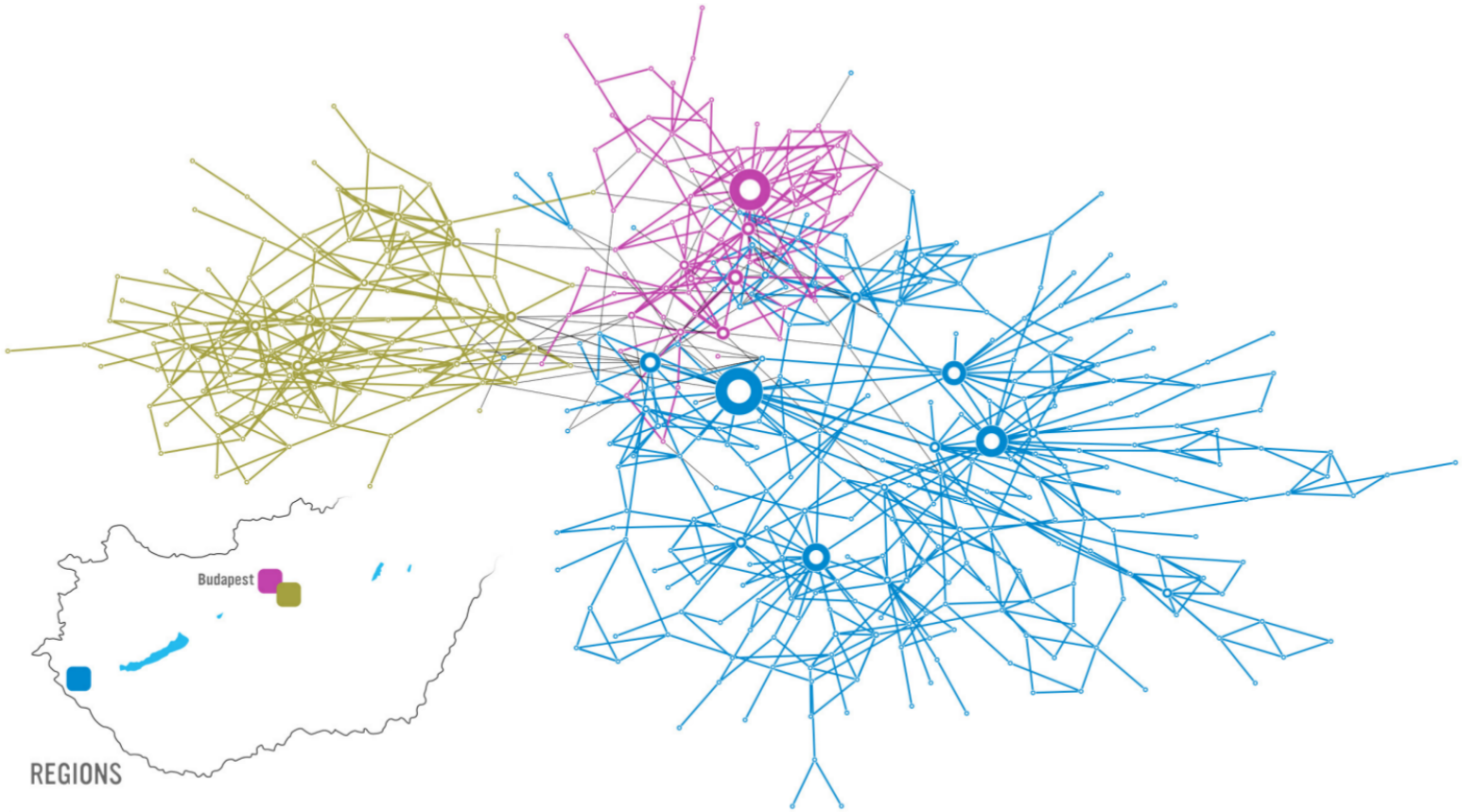

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Applications of network theory
Organizational consulting

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OVERVIEW

Outline

- **Introduction**
- **Basic network characteristics**
- **Advanced network characteristics**
- **Network models**
- **Robustness and spreading**
- **Motifs and communities**

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  - degree,
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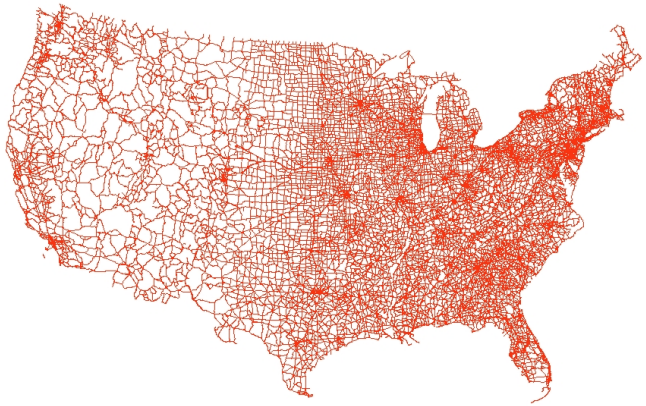

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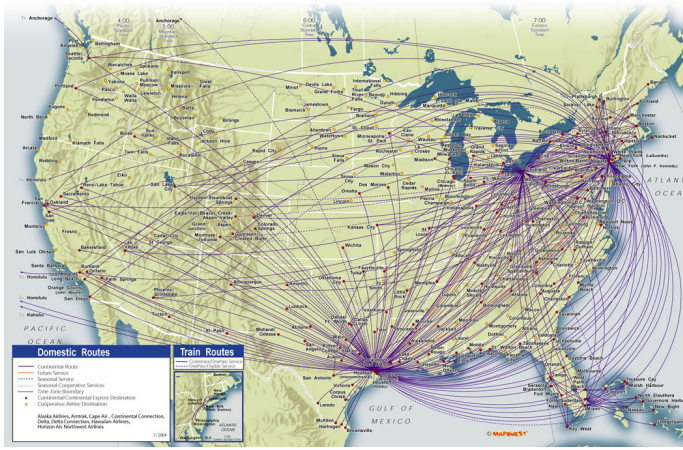

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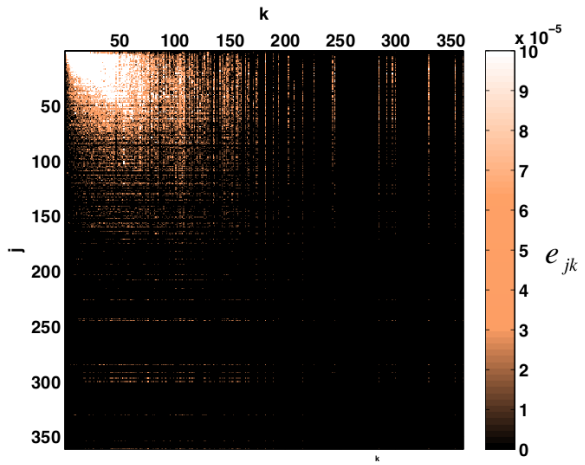
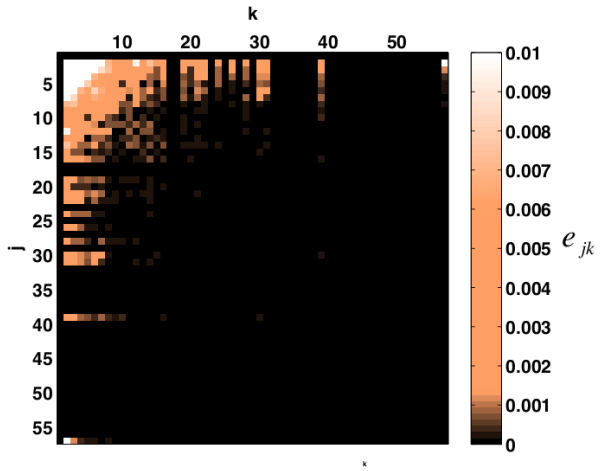
Astrophys. co–authorship



Yeast PPI



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Network models

- **How to generate networks?**
  - Erdős–Rényi model,
  - Watts-Strogatz model,
  - Barabási–Albert model,
  - Configuration model.

- **Why are they important?**
  - Singling out essential features.
  - Explaining relevant phenomena via constructive mechanisms.
  - Generating test beds.

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Robustness

- **Why are some networks more/less robust?**

Robustness

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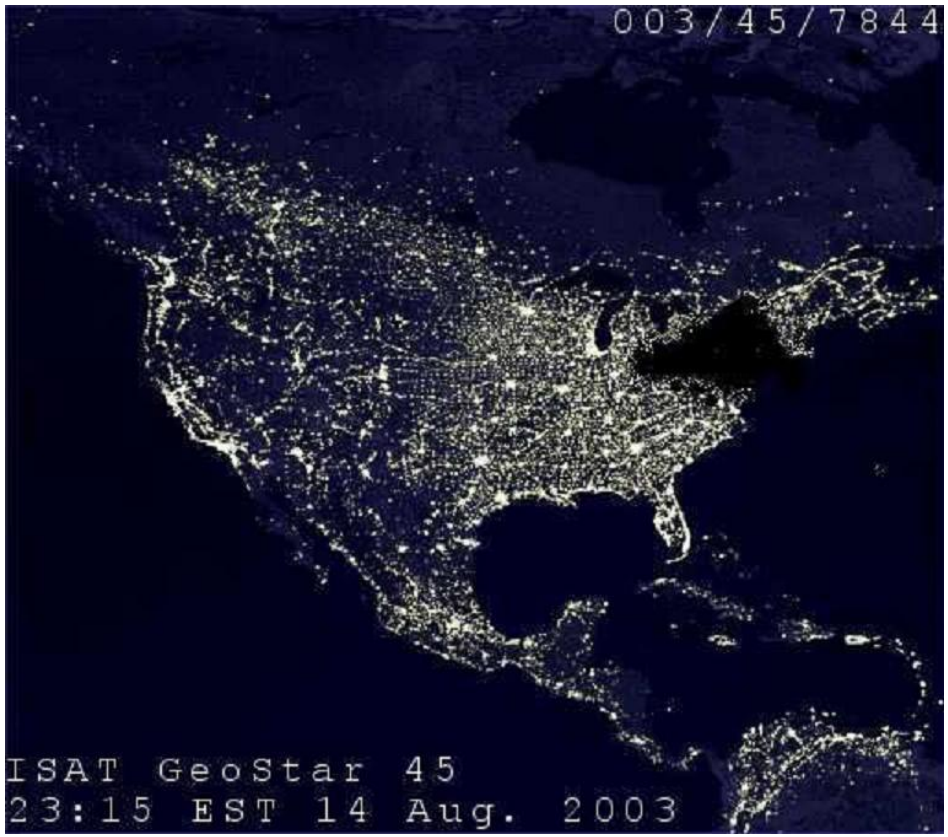
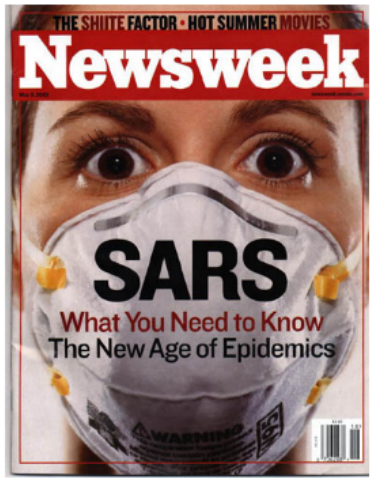


- How to describe in quantitative terms the breakdown of a network under node or link removal?
- How does the network structure affect the robustness?

Spreading

- **How do ideas, information or infections spread on networks?**



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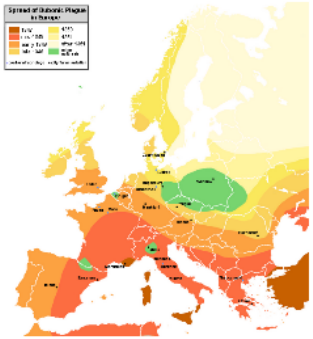
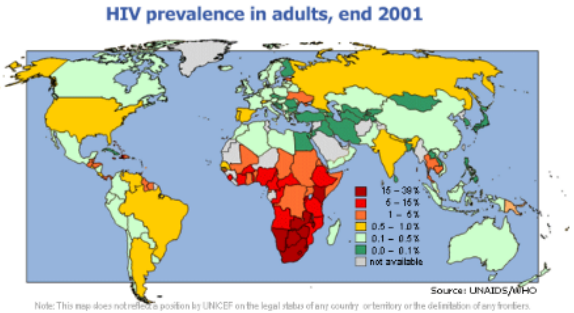
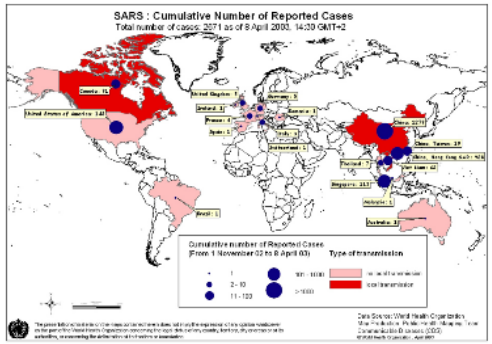
The Great Plague



HIV



SARS



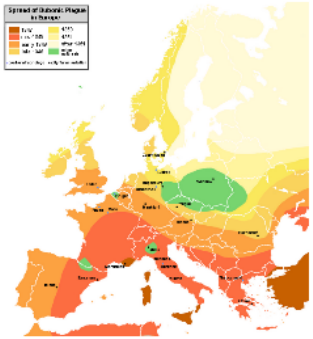
1918 Spanish flu

H1N1 flu

Spreading

- **How do ideas, information or infections spread on networks?**

The Great Plague



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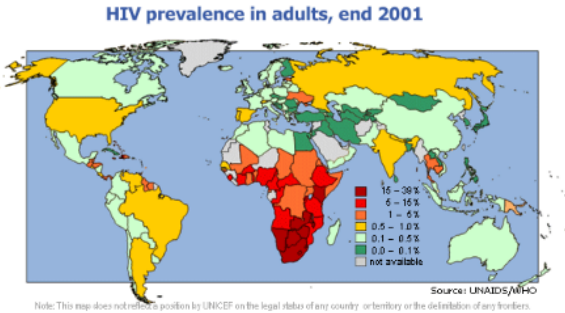

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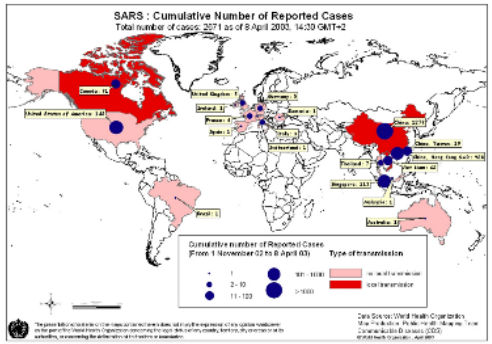

1918 Spanish flu

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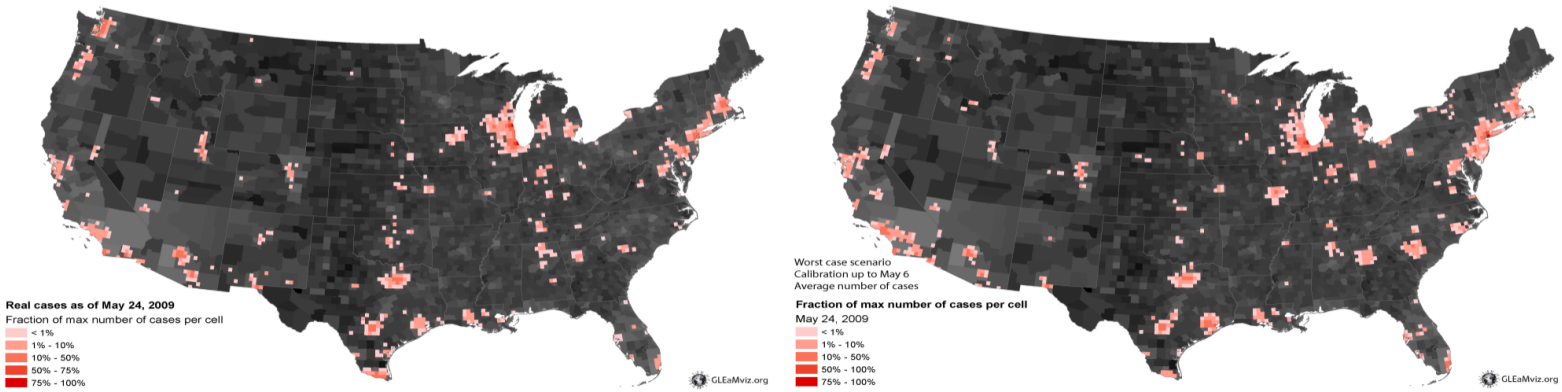
- How does the network structure affect the spreading rate?

Spreading

- **How do ideas, information or infections spread on networks?**

Real **Projected**



- How does the network structure affect the spreading rate?

Outline

- **Introduction**
- **Basic network characteristics**
- **Advanced network characteristics**
- **Network models**
- **Robustness and spreading**
- **Motifs and communities**

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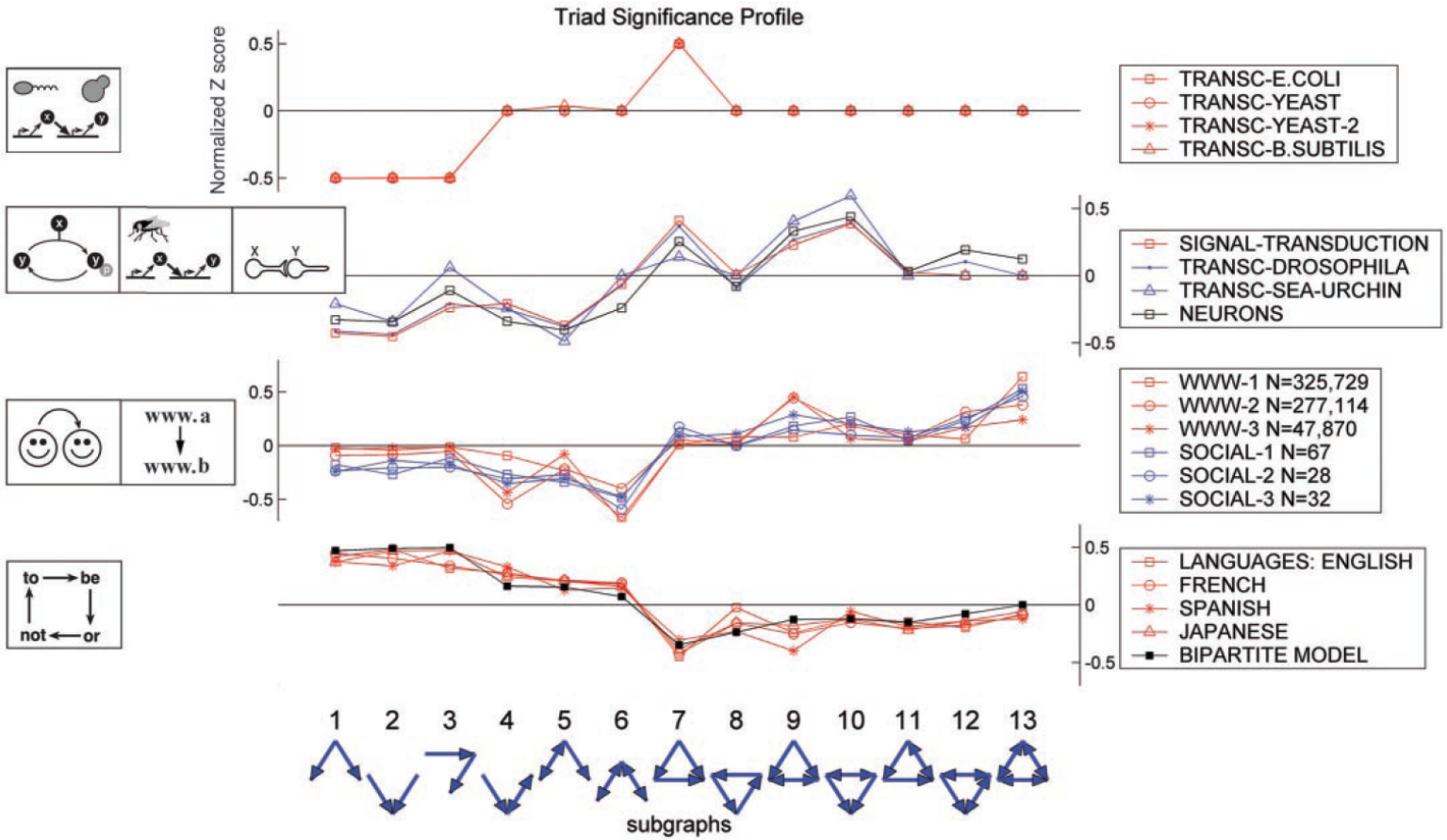
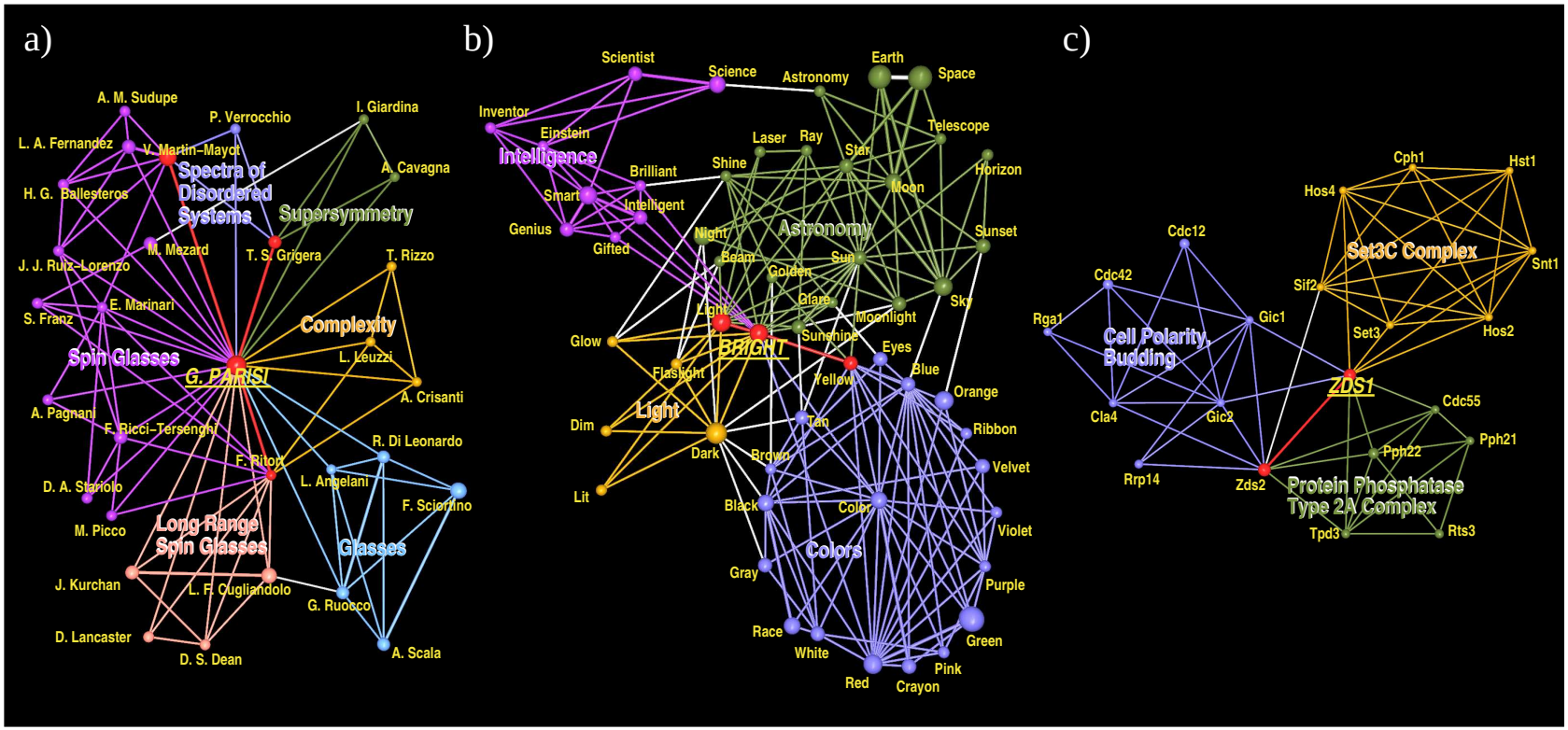
Motifs



Communities



GRAPHS

Graph vs. network

Graph vs. network

Graph

- vertices,
- edges,
- possibly directed,
- possibly weighted

Network

- nodes,
- links,
- node props.:
  tags, annotations, state variable $x_i(t)$,
- flows,
- epidemics,
- growth,
- rewiring

Graph vs. network

Graph $\longleftrightarrow$ Network $\longleftrightarrow$ Complex system

- vertices,
- edges,
- possibly directed,
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- nodes,
- links,
- node props.: tags, annotations, state variable $x_i(t)$,
- flows,
- epidemics,
- growth,
- rewiring

Graph types

Graph types

Graph types

- simple graph: only single connections are allowed, no self loops.
- multi graph: multiple connections between the same pair of nodes are allowed, no self loops.
- pseudo graph: multiple connections and self loops are also allowed.

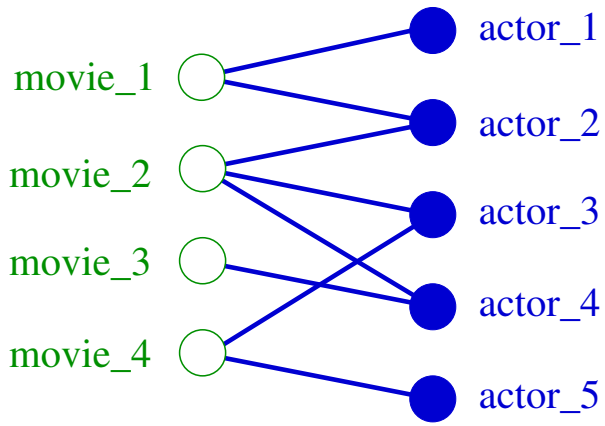
- directed graph: the links are directed.
- undirected graph: links are undirected.
- (mixed graph: contains both directed and undirected links).

- weighted graph: links have weight
- unweighted graph: links are “binary”

- bi-partite graph: two types of nodes, links are allowed only between opposite types.

Bi-partite graphs

E.g., the network of movie actors:



Bi-partite graphs

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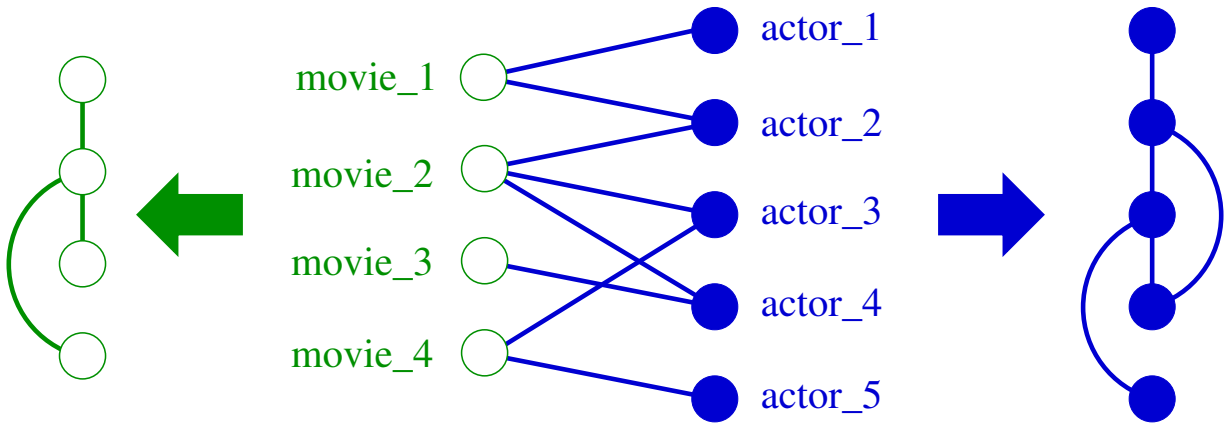
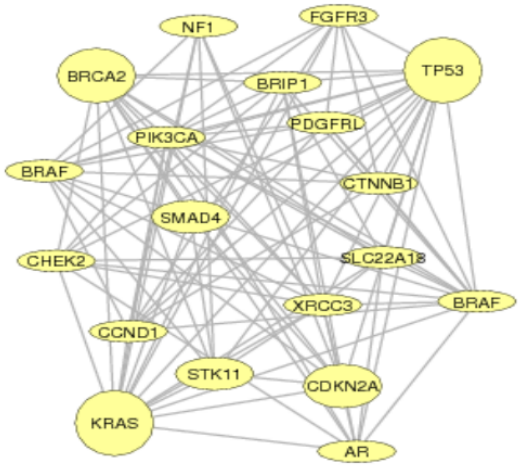
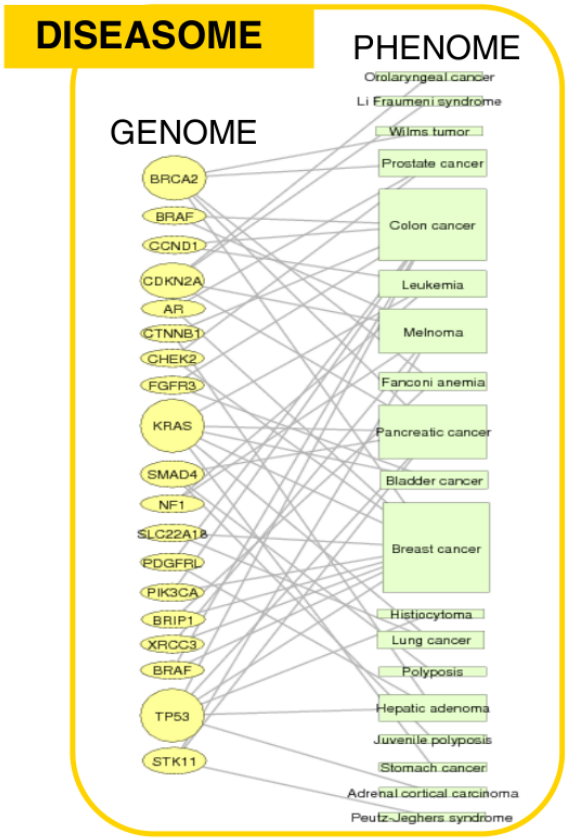
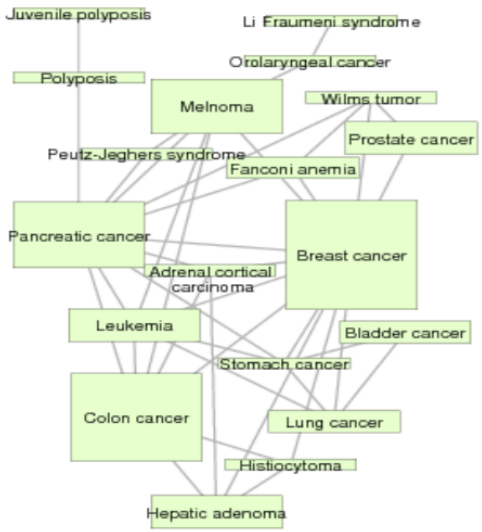


Bi-partite graphs



Gene network





Disease network

Goh, Cusick, Valle, Childs, Vidal & Barabási, PNAS (2007)

Random walk on graphs

- The most basic 'role' or 'function' of a network is to provide indirect access for the nodes to other nodes in the system.
- When we assume the sending of 'messages' or 'packets' over the network from one node to the other, the most simple scenario is to assume that these are performing a **random walk**.

→ Let's examine random walking on a small network!