Networks theoretical practice problems no.2.

1. Suppose we generate a Watts-Strogatz random graph made of $N = 1000$ nodes, where every node is connected to its first and second neighbours along the ring (this means that $q = 2$), and we set the random rewiring probability to $\beta = 10^{-4}$. What do you expect, is this going to result in a small world network?

2. Suppose we modify the B-A model in the following way: each node is given a uniform fitness value $a \in [0, m]$, and the probability for an already existing node i to gain a new link is proportional to $k_i - a$:

$$\mathcal{P}_i \sim k_i - a.$$

→ Derive the decay exponent for $p(k)$ in the mean-field approximation for the **large k regime**.

Hint: since we are interested only in the tail of $p(k)$ where $k >> 1$, in order to be able to solve the differential equation in a similar way to the original B-A model, neglect a beside k_i after writing down the differential equation.

3. Is preferential attachment really necessary in the B-A model for achieving a scale-free degree distribution? Let us find out, by modifying the model in such a way that the new nodes are choosing simply uniformly at random from the already existing nodes when connecting into the network. Derive the $p(k)$ in this model following the same steps we went through in the slides for the original B-A model.