$$\frac{\partial y(z,\tau)}{\partial \tau} - \frac{\partial^2 y(z,\tau)}{\partial z^2} = 0, (z,\tau) \in [\ln(a), \ln(b) + cT] \times [0, \alpha T], \quad (2.9)$$

$$y(z_i, 0) = \varphi_i, z_i \in [\ln(a), \ln(b)], i = \overline{1, I}, I \in \mathbb{N}, \quad (2.10)$$

$$y\left(z_j, \tau_j\right) = 0, \left(z_j, \tau_j\right) \in [\ln(a), \ln(b) + cT] \times [0, \alpha T], j = \overline{1, J}, J \in \mathbb{N}, \quad (2.11)$$

$$y(z_k, \tau_k) = y_k, (z_k, \tau_k) \in [ln(a), ln(b) + cT] \times [0, \alpha T], k = \overline{1, K}, K \in \mathbb{N}, \quad (2.12)$$

В позначеннях файлу математика_для_програми:

$$x \equiv z, t \equiv \tau$$

$$S_0^T = S_0 \times [0, \alpha T] = [ln(a), ln(b)] \times [0, \alpha T]$$

$$S_\Gamma^T = S_\Gamma \times [0, \alpha T] = [ln(a), ln(b) + cT] \times [0, \alpha T]$$

$$L(\partial z, \partial \tau) = \frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial z^2}$$

$$L_r^0(\partial \tau) \equiv 1, R_0 = 1, L_0 = I$$

$$L_\rho^\Gamma(\partial z) \equiv 1, R_\Gamma = 1, L_\Gamma = J$$

$$L_i(\partial z, \partial \tau) \equiv 1, I = 1, J_1 = K$$

$$u(x,t) = w(z,\tau) \equiv 0$$

$$S^0 = [\ln(a), \ln(b)] \times (-\infty, 0]$$

$$S^\Gamma = (-\infty, \ln(a)) \times (\ln(b) + cT, +\infty)$$

$$\overline{Y_{\rho l}} = Y_{\rho l}^\Gamma - L_\rho^\Gamma(\partial x) y_\infty(x,t)\big|_{(x,t)=s_l^\Gamma} = \bar{0}$$

$$\overline{Y_{ij}} = Y_{ij} - L_{ij}(\partial x, \partial t) y_\infty(x,t)\big|_{x=x_{ij}}^{t=t_{ij}} = \overline{y_k}$$

Розв’язок даної задачі:

$$y(z,\tau) = y_\infty(z,\tau) + y_0(z,\tau) + y_\Gamma(z,\tau)$$

$$y_\infty(z,\tau) = \int_0^T \int_{\ln(a)}^{\ln(b)} G(z - z', \tau - \tau') w(z', \tau') dz' d\tau' = \int_0^T \int_{\ln(a)}^{\ln(b)} 0 dz' d\tau' = 0$$

$$y_0(z,\tau) = \int_{-\infty}^0 \int_{\ln(a)}^{\ln(b)} G(z - z', \tau - \tau') w_0(z', \tau') dz' d\tau'$$

$$y_\Gamma(z,\tau) = \int_0^T \left(\int_{-\infty}^{\ln(a)} G(z - z', \tau - \tau') w_\Gamma(z', \tau') dz' + \int_{\ln(b)+cT}^{+\infty} G(z - z', \tau - \tau') w_\Gamma(z', \tau') dz' \right) d\tau'$$

$$\Phi = \sum_{j=1}^{J} \left(y(z,\tau)|_{(z,\tau)=(z_j,\tau_j)} \right)^2 + \sum_{k=1}^{K} \left(y(z,\tau)|_{(z,\tau)=(z_k,\tau_k)} - y_k \right)^2 \to \min_{\varphi_i, i=\overline{1,I}}$$

$$\int_{-\infty}^{0} \int_{\ln(a)}^{\ln(b)} G(z-z',\tau-\tau')|_{(z,\tau)=(z_j,\tau_j)} w_0(z',\tau') dz' d\tau'$$
$$+ \int_{0}^{T} \left(\int_{-\infty}^{\ln(a)} G(z-z',\tau-\tau')|_{(z,\tau)=(z_j,\tau_j)} w_\Gamma(z',\tau') dz' \right.$$
$$\left. + \int_{\ln(b)+cT}^{+\infty} G(z-z',\tau-\tau')|_{(z,\tau)=(z_j,\tau_j)} w_\Gamma(z',\tau') dz' \right) d\tau' = \bar{0} \ (j = \overline{1,J})$$

$$\int_{-\infty}^{0} \int_{\ln(a)}^{\ln(b)} G(z-z',\tau-\tau')|_{(z,\tau)=(z_k,\tau_k)} w_0(z',\tau') dz' d\tau'$$
$$+ \int_{0}^{T} \left(\int_{-\infty}^{\ln(a)} G(z-z',\tau-\tau')|_{(z,\tau)=(z_k,\tau_k)} w_\Gamma(z',\tau') dz' \right.$$
$$\left. + \int_{\ln(b)+cT}^{+\infty} G(z-z',\tau-\tau')|_{(z,\tau)=(z_k,\tau_k)} w_\Gamma(z',\tau') dz' \right) d\tau' = \overline{y_k} \ (k = \overline{1,K})$$

$$\int_{(\cdot)} A(z,\tau) \bar{w}(z,\tau) dz d\tau = \bar{Y}$$

$$\bar{u}(x,t) = \begin{pmatrix} w(z,\tau) \ \left((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\right) \\ w_\Gamma(z,\tau) \ \left((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b)+cT, +\infty)\right) \end{pmatrix}$$

$$A(z,\tau) = \begin{pmatrix} A_{21}(z,\tau) \ \left((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\right) & A_{22}(z,\tau) \ \left((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b)+cT, +\infty)\right) \\ A_{31}(z,\tau) \ \left((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\right) & A_{32}(z,\tau) \ \left((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b)+cT, +\infty)\right) \end{pmatrix}$$

$$\bar{Y} = \begin{pmatrix} \bar{Y}_\Gamma \\ \bar{Y}_* \end{pmatrix} = \begin{pmatrix} col(\bar{0}, j = \overline{1,J}) \\ col(\overline{y_k}, k = \overline{1,K}) \end{pmatrix}$$

$$A_{2q} = col\left(G(z-z',\tau-\tau')|_{(z,\tau)=(z_j,\tau_j)}, j = \overline{1,J} \right)$$

$$A_{3q} = col\left(\left(G(z-z',\tau-\tau')|_{(z,\tau)=(z_k,\tau_k)}, k = \overline{1,K} \right) \right)$$

$$(z',\tau') \in [\ln(a), \ln(b)] \times (-\infty, 0], \text{при } q = 1,$$

$$(z',\tau') \in (-\infty, \ln(a)) \times (\ln(b)+cT, +\infty), \text{при } q = 2.$$

$$\bar{u}(z,\tau) = A^T(z,\tau) P^+ (\bar{Y} - A_v) + v(z,\tau)$$

$$P = \int_{(\cdot)} A(z,\tau) A^T(z,\tau) dz d\tau$$

$$A_v = \int_{(\cdot)} A(z,\tau) v(z,\tau) dz d\tau$$

$$v(z,\tau) = col\Big(v_0(z,\tau)\ \big((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\big), v_\Gamma(z,\tau)\ \big((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b) + cT, +\infty)\big)\Big) = \begin{pmatrix} v_0(z,\tau)\ \big((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\big) \\ v_\Gamma(z,\tau)\ \big((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b) + cT, +\infty)\big) \end{pmatrix}$$

$$u_0(z,\tau) = A_1(z,\tau)P^+(\bar{Y} - A_v) + v_0(z,\tau)$$

$$u_\Gamma(z,\tau) = A_2(z,\tau)P^+(\bar{Y} - A_v) + v_\Gamma(z,\tau)$$

$$A_1(z,\tau) = (A_{21}^T(z,\tau) \quad A_{31}^T(z,\tau))\big((z,\tau) \in [\ln(a), \ln(b)] \times (-\infty, 0]\big),$$

$$A_2(z,\tau) = (A_{22}^T(z,\tau) \quad A_{32}^T(z,\tau))\ \big((z,\tau) \in (-\infty, \ln(a)) \times (\ln(b) + cT, +\infty)\big),$$

$$P = \begin{pmatrix} P_{21} & P_{22} \\ P_{31} & P_{32} \end{pmatrix}$$

$$A_v = \begin{pmatrix} A_{v_0} \\ A_{v_\Gamma} \end{pmatrix}$$

$$P_{nm} = \int_{-\infty}^{0}\int_{\ln(a)}^{\ln(b)} A_{n1}(z,\tau)A_{m+11}^T(z,\tau)dzd\tau + \int_0^T\left(\int_{-\infty}^{\ln(a)} A_{n2}(z,\tau)A_{m+12}^T(z,\tau)dz + \int_{\ln(b)+cT}^{+\infty} A_{n2}(z,\tau)A_{m+12}^T(z,\tau)dz\right)d\tau\ \ (n = \overline{2,3}, m = \overline{1,2}).$$

$$A_{v_0} = \int_{-\infty}^{0}\int_{\ln(a)}^{\ln(b)} A_{21}(z,\tau)v_0(z,\tau)dzd\tau + \int_0^T\left(\int_{-\infty}^{\ln(a)} A_{22}(z,\tau)v_\Gamma(z,\tau)dz + \int_{\ln(b)+cT}^{+\infty} A_{22}(z,\tau)v_\Gamma(z,\tau)dz\right)d\tau,$$

$$A_{v_\Gamma} = \int_{-\infty}^{0}\int_{\ln(a)}^{\ln(b)} A_{31}(z,\tau)v_0(z,\tau)dzd\tau + \int_0^T\left(\int_{-\infty}^{\ln(a)} A_{32}(z,\tau)v_\Gamma(z,\tau)dz + \int_{\ln(b)+cT}^{+\infty} A_{32}(z,\tau)v_\Gamma(z,\tau)dz\right)d\tau.$$

Точність з якою функція $y(z,\tau)$стану нашої системи задовольняє початково — крайові умови:

$$\varepsilon^2 = \min_{w_0(z,\tau), w_\Gamma(z,\tau)} \Phi = \bar{Y}^T\bar{Y} - \bar{Y}^T P P^+ \bar{Y}.$$