Analyzing Snapshot Isolation

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Outline

Intro

- We focus on *Snapshot Isolation* presented in the last talk
- ... same context of DBMS and transactional memory systems

Intro

- DBMS typically offer various guarantees for transaction management
- Each mode exhibits different *anomalies*
- Stronger modes exhibit less anomalies at expense of performance
  - Stronger guarantees incur more overhead on the DBMS side
  - Less allowed behaviors $\rightarrow$ more concurrent transactions expected to abort

Snapshot Isolation

Snapshot Isolation - originally specified as operational model:

- When transaction T begins, $snapshot_T$ is taken
- All reads in T read values from $snapshot_T$
- All writes in T write to transient write-set
- T commits only if passes write conflict check:
  - No object in T's write-set was updated by other transactions since $snapshot_T$ was taken

Outline

1. Introduction
2. Snapshot Isolation
   - Definitions
   - Dependency Graphs
   - Characterization
3. Static Analysis
   - Transaction Chopping
   - Robustness

Definitions

We'll start by formulating a declarative definition of Snapshot Isolation.

We'll use a lot of notation similar to Daniel's talk two weeks ago

- $Obj = \{x, y, \dots\}$ - objects in the data-set
- $Event = \{e, f, \dots\}$ - transaction events
- $Op = \{read(x, n), write(x, n) \mid x \in Obj, n \in \mathbb{Z}\}$
- $op : Event \to Op$

- **Strict partial order** - transitive + irreflexive relation
- **Total order** - strict partial order that orders all pairs

A **transaction** $T, S, \ldots$ is a pair (E, po) where $E \subseteq \mathit{Event}$ is a *finite*, non-empty set of events and **program-order** $po \subseteq E \times E$ is a total order.

A **history** is a pair $\mathcal{H} = (\mathcal{T}, SO)$ where $\mathcal{T}$ is a finite set of transactions with disjoint set of events and the **session-order** $SO \subseteq \mathcal{T} \times \mathcal{T}$ is a union of total orders defined on disjoint subsets of $\mathcal{T}$, which correspond to transactions in different sessions.

We elide treatment of:

- aborted transactions - all transactions in any history are committed.
- infinite computations - histories are always finite.

An **abstract execution** is a tuple $\mathcal{X} = (\mathcal{T}, \mathit{SO}, \mathit{VIS}, \mathit{CO})$, where $(\mathcal{T}, \mathit{SO})$ is a history and the **visibility** and **commit order** $\mathit{VIS}, \mathit{CO} \subseteq \mathcal{T} \times \mathcal{T}$ are such that $\mathit{VIS} \subseteq \mathit{CO}$ and CO is total.

- We'll use $(T, S) \in \mathit{VIS}$ and $T \xrightarrow{\mathit{VIS}} S$ interchangeably for *VIS* and other relations.
- For $\mathcal{H} = (\mathcal{T}, \mathit{SO})$ we'll shorten $(\mathcal{T}, \mathit{SO}, \mathit{VIS}, \mathit{CO})$ to $(\mathcal{H}, \mathit{VIS}, \mathit{CO})$

For the relations defined in abstract execution:

- $T \xrightarrow{VIS} S$ means that T is included in S's snapshot.
- $T \xrightarrow{CO} S$ means that T is commited before S.
- $VIS \subseteq CO$ makes sure that snapshots include only already committed transactions.

For a set A and a total order $R \subseteq A \times A$

- $max_R(A) = \{a \mid \forall b \in A : a = b \vee (b, a) \in R\}$
- $min_R(A) = \{a \mid \forall b \in A : a = b \vee (a, b) \in R\}$
- $R^{-1}(a) = \{b \mid (b, a) \in R\}$
- $R_1 ; R_2 = \{(a, b) \mid \exists c : (a, c) \in R_1 \wedge (c, b) \in R_2\}$
- $R? = R \cup \{(a, a) \mid a \in A\}$
- R^+ a transitive closure of R
- R^* a transitive and reflexive closure of R

For $T = (E, po)$ we'll use:

- $T \vdash write(x, n)$ if T writes to x and n is s.t.

$$op\,(max_{po}\{e \mid op\,(e) = write\,(x, \_)\}) = write(x, n)$$

- $T \vdash read(x, n)$ if T reads x before writing to is and n is s.t.

$$op\,(min_{po}\{e \mid op\,(e) = \_\,(x, \_)\}) = read(x, n)$$

We'll also use $WriteTx_x = \{T \mid T \vdash write(x, \_)\}$

We'll now try to define *snapshot isolation* and *serializability* in terms of **consistency axioms**:

$$\begin{aligned}
\mathit{ExecSI} &= \{\mathcal{X} \mid \mathcal{X} \models \mathit{INT} \wedge \mathit{EXT} \wedge \mathit{SESSION} \wedge \\
&\qquad \mathit{PREFIX} \wedge \mathit{NOCONFLICT}\} \\
\mathit{ExecSER} &= \{\mathcal{X} \mid \mathcal{X} \models \mathit{INT} \wedge \mathit{EXT} \wedge \mathit{SESSION} \wedge \mathit{TOTALVIS}\} \\
\mathit{HistSI} &= \{\mathcal{H} \mid \exists \mathit{VIS}, \mathit{CO} : (\mathcal{H}, \mathit{VIS}, \mathit{CO}) \in \mathit{ExecSI}\} \\
\mathit{HistSER} &= \{\mathcal{H} \mid \exists \mathit{VIS}, \mathit{CO} : (\mathcal{H}, \mathit{VIS}, \mathit{CO}) \in \mathit{ExecSER}\}
\end{aligned}$$

INT - **internal consistency axiom**: ensures that a read event *e* on object *x* returns the same value a as the last write or read on *x* in the same transaction.

$$\forall (E, po) \in \mathcal{T}. \forall e \in E. \forall x, n :$$
$$op(e) = read(x, n) \wedge \{f \mid op(f) = \_(x, \_) \wedge f \xrightarrow{po} e\} \neq \emptyset \Rightarrow$$
$$op\left(max_{po}\{f \mid op(f) = \_(x, \_) \wedge f \xrightarrow{po} e\}\right) = \_(x, n)$$

EXT - **external consistency axiom**: ensures that if $T \vdash read(x, n)$ then the value is taken from the last visible transaction that wrote to x according to commit order.

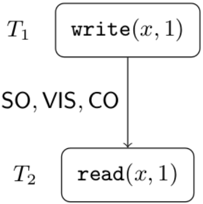
$$\forall T \in \mathcal{T}. \forall x, n :$$

$$T \vdash read(x, n) \Rightarrow max_{CO} \left(VIS^{-1}(T) \cap WriteTx_x \right) \vdash write(x, n)$$

SESSION - **session visibility** requires a snapshot to include all preceding transactions of the same session.

$$SO \subseteq VIS$$

(a) Session guarantees.

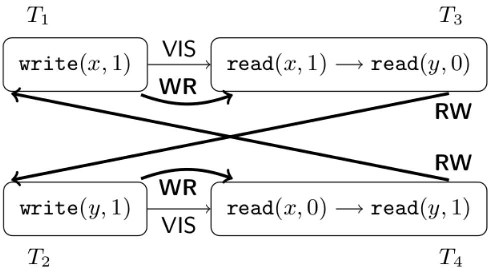


PREFIX - ensures that if snapshot taken by T includes S, then it includes all transactions committed before S as well.

$$CO; VIS \subseteq VIS$$

The **long-fork** anomaly is prevented by *PREFIX* axiom:

(c) Long fork.

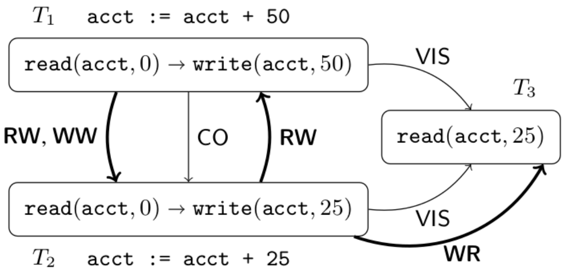


NOCONFLICT - ensures that for any two transactions writing to the same object, one has to be aware of the other.

$$\forall T, S \in \mathcal{T}. \forall x, n.$$
$$(T, S \in \mathit{WriteTx}_x \wedge T \neq S) \Rightarrow \left(T \xrightarrow{\mathit{VIS}} S \vee S \xrightarrow{\mathit{VIS}} T\right)$$

The **lost-update** anomaly is prevented by *NOCONFLICT* axiom:

(b) Lost update.



TOTALVIS - requires total order on the visibility relation, giving us *serializability* of transactions.

$$VIS = CO$$

The **write-skew** anomaly allowed by Snapshot Isolation is prevented by *TOTALVIS* axiom:

(d) Write skew. Initially $\texttt{acct1} = \texttt{acct2} = 60$.

```
if (acct1 + acct2 > 100)
    acct1 := acct1 - 100
```

T_1

$\texttt{read}(\texttt{acct1}, 60) \rightarrow \texttt{read}(\texttt{acct2}, 60) \longrightarrow \texttt{write}(\texttt{acct1}, -40)$

RW

RW

```
if (acct1 + acct2 > 100)
    acct2 := acct2 - 100
```

T_2

$\texttt{read}(\texttt{acct1}, 60) \rightarrow \texttt{read}(\texttt{acct2}, 60) \longrightarrow \texttt{write}(\texttt{acct2}, -40)$

Outline

- Our goal now is to characterize SI in terms of dependencies between transactions.
- Then we'll be able to decide whether SI allows a given history by looking for appropriate dependencies.

Let $\mathcal{X} = (\mathcal{T}, SO, VIS, CO)$ be an execution, for $x \in Obj$ we define the following relations on $\mathcal{T}_{\mathcal{X}}$:

- **read-dependency**:

$$T \xrightarrow{WR_{\mathcal{X}}(x)} S \Leftrightarrow$$

$$S \vdash read(x, n) \wedge T = max_{CO}\left(VIS^{-1}(S) \cap WriteTx_x\right)$$

Informally: T $\xrightarrow{WR_{\mathcal{X}}(x)}$ S means that S reads T's write to x.

Let $\mathcal{X} = (\mathcal{T}, SO, VIS, CO)$ be an execution, for $x \in Obj$ we define the following relations on $\mathcal{T}_{\mathcal{X}}$:

- **write-dependency**:

$$T \xrightarrow{WW_{\mathcal{X}}(x)} S \Leftrightarrow T \xrightarrow{CO} S \wedge T, S \in WriteTx_x$$

Informally: T $\xrightarrow{WW_{\mathcal{X}}(x)}$ S means that S overwrites T's write to x.

Let $\mathcal{X} = (\mathcal{T}, SO, VIS, CO)$ be an execution, for $x \in Obj$ we define the following relations on $\mathcal{T}_{\mathcal{X}}$:

- **anti-dependency**:

$$T \xrightarrow{RW_{\mathcal{X}}(x)} S \Leftrightarrow$$

$$T \neq S \land \exists T'. T' \xrightarrow{WR_{\mathcal{X}}(x)} T \land T' \xrightarrow{WW_{\mathcal{X}}(x)} S$$

Informally: T $\xrightarrow{RW_{\mathcal{X}}(x)}$ S means that S overwrites the write to x read by T.

A **dependency graph** is a tuple $\mathcal{G} = (\mathcal{T}, SO, WR, WW, RW)$, where $(\mathcal{T}, SO)$ is a history and:

- WR: $Obj \rightarrow 2^{\mathcal{T} \times \mathcal{T}}$ is such that:
  - $\forall T, S. \forall x. T \xrightarrow{WR(x)} S \Rightarrow$
    $\exists n. T \neq S \wedge T \vdash write(x, n) \wedge S \vdash read(x, n)$
  - $\forall S \in \mathcal{T}. \forall x. S \vdash read(x, \_) \Rightarrow \exists T. T \xrightarrow{WR(x)} S$
  - $\forall T, T', S \in \mathcal{T}. \forall x. \left(T \xrightarrow{WR(x)} S \wedge T' \xrightarrow{WR(x)} S \right) \Rightarrow T = T'$

A **dependency graph** is a tuple $\mathcal{G} = (\mathcal{T}, SO, WR, WW, RW)$, where $(\mathcal{T}, SO)$ is a history and:

- WW: $Obj \to 2^{\mathcal{T} \times \mathcal{T}}$ is such that for every $x \in Obj$, $WW(x)$ is a total order on the set $WriteTx_x$.

A **dependency graph** is a tuple $\mathcal{G} = (\mathcal{T}, SO, WR, WW, RW)$, where $(\mathcal{T}, SO)$ is a history and:

- RW: $Obj \to 2^{\mathcal{T} \times \mathcal{T}}$ is derived from WR and WW as in the definition of $WR_{\mathcal{X}}(x)$.

Proposition:
For any $\mathcal{X} \in \mathit{ExecSI}$,

$$\mathit{graph}(\mathcal{X}) = (\mathcal{T}_{\mathcal{X}}, \mathit{SO}_{\mathcal{X}}, \mathit{WR}_{\mathcal{X}}, \mathit{WW}_{\mathcal{X}}, \mathit{RW}_{\mathcal{X}})$$

is a dependency graph.
Proof:
By showing $\mathit{graph}(\mathcal{X})$ satisfies all requirements of a dependency graph.

Outline

We'll show that SI is characterized by dependency graphs that contain only cycles with at least two adjacent anti-dependency edges.

Theorem:
Let

$$GraphSER = \{\mathcal{G} \mid (\mathcal{T}_{\mathcal{G}} \models INT) \quad ((SO_{\mathcal{G}} \cup WR_{\mathcal{G}} \cup WW_{\mathcal{G}} \cup RW_{\mathcal{G}}) \text{ is acyclic})\}$$

Then

$$HistSER = \{\mathcal{H} \mid \exists WR, WW, RW. \quad (\mathcal{H}, WR, WW, RW) \in GraphSER\}$$

In other words, execution is serializable if it can be extended into an acyclic dependency graph.

Example:

(d) Write skew. Initially acct1 = acct2 = 60.

```
if (acct1 + acct2 > 100)
    acct1 := acct1 - 100
```

T_1: read(acct1, 60) → read(acct2, 60) ⟶ write(acct1, −40)

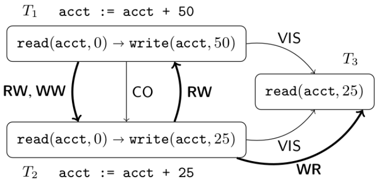
RW ↓ ↑ RW

T_2: read(acct1, 60) → read(acct2, 60) ⟶ write(acct2, −40)

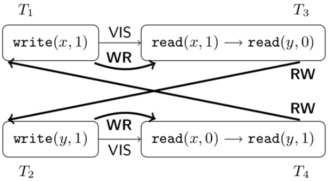
```
if (acct1 + acct2 > 100)
    acct2 := acct2 - 100
```

- Prohibited under **serializability**, and has a dependency graph cycle $T_1 \xrightarrow{RW} T_2 \xrightarrow{RW} T_1$.
- However, allowed under **SI**

In contrast, following is *not* allowed under SI:



(b) Lost update.



(c) Long fork.

contain cycles without adjacent anti dependencies.

Theorem:
Let

$$GraphSI = \{\mathcal{G} \mid (\mathcal{T}_{\mathcal{G}} \models INT) \wedge (((SO_{\mathcal{G}} \cup WR_{\mathcal{G}} \cup WW_{\mathcal{G}})\,;\, RW_{\mathcal{G}}?) \textit{ is acyclic})\}$$

Then

$$HistSI = \{\mathcal{H} \mid \exists WR, WW, RW.(\mathcal{H}, WR, WW, RW) \in GraphSI\}$$

In other words, $\mathcal{H} \in HistSI$ if $\mathcal{H}$ can be extended to a dependency graph satisfying *GraphSI*

To prove it we'll show a stronger result:

1. **Soundness:** $\forall \mathcal{G} \in \mathit{GraphSI}. \exists \mathcal{X} \in \mathit{ExecSI}. \mathit{graph}(\mathcal{X}) = \mathcal{G}$
2. **Completeness:** $\forall \mathcal{X} \in \mathit{ExecSI}. \mathit{graph}(\mathcal{X}) \in \mathit{GraphSI}$

The *completeness* closely follows from existing results[1]. We will focus on the *soundness*.

[1] Making Snapshot Isolation Serializable, 2005, A. Fekete et al

Proof sketch:

- Construct a basic **pre-execution** from $\mathcal{G}$
- Iteratively extend it until satisfies execution definition

A tuple $\mathcal{P} = (\mathcal{T}, \mathit{SO}, \mathit{VIS}, \mathit{CO})$ is a **pre-execution** if it satisfies all the conditions of being an *execution*, except *CO* is a strong partial order that may not be total. We let *PreExecSI* be the set of pre-executions satisfying the SI axioms:

$$\begin{aligned}\mathit{PreExecSI} = \{\mathcal{P} \mid \mathcal{P} \models & \mathit{INT} \wedge \mathit{EXT} \wedge \mathit{SESSION} \wedge \\ & \mathit{PREFIX} \wedge \mathit{NOCONFLICT}\}\end{aligned}$$

- For a given dep. graph $\mathcal{G} = (\mathcal{H}, \mathit{WR}, \mathit{WW}, \mathit{RW})$, let $\mathcal{P} = (\mathcal{H}, \mathit{VIS}, \mathit{CO})$ a respective pre-execution.
- To conform with $\mathcal{G}$ we require that *VIS*, *CO* hold:

$$\mathit{SO} \cup \mathit{WR} \cup \mathit{WW} \subseteq \mathit{VIS} \tag{1}$$
$$\mathit{CO}; \mathit{VIS} \subseteq \mathit{VIS} \tag{2}$$
$$\mathit{VIS} \subseteq \mathit{CO} \tag{3}$$
$$\mathit{CO}; \mathit{CO} \subseteq \mathit{CO} \tag{4}$$
$$\mathit{VIS}; \mathit{RW} \subseteq \mathit{CO} \tag{5}$$

Lemma:
Let $\mathcal{G} = (\mathcal{T}, SO, WR, WW, RW)$ be a dependency graph, for any relation $R \subseteq \mathcal{T} \times \mathcal{T}$, the relations

$$\begin{aligned} VIS = {} & (((SO \cup WR \cup WW); RW?) \cup R)^{*}; \\ & (SO \cup WR \cup WW) \\ CO = {} & (((SO \cup WR \cup WW); RW?) \cup R)^{+} \end{aligned}$$

are a solution to the system of inequalities in the previous requirements. They also are the smallest solution to the system for which $R \subseteq CO$.

Back to the proof:
Let $\mathcal{G} = (\mathcal{T}, SO, WR, WW, RW) \in GraphSI$

- Define $\mathcal{P}_0$ derived from the last lemma by fixing $R_0 = \emptyset$.
- Construct $\{\mathcal{P}_i = (\mathcal{T}, SO, VIS_i, CO_i)\}_{i=0}^{n}$ series of pre-executions.
- While CO_i is not total:
  - Pick arbitrary pair T, S not ordered by CO_i
  - $R_{i+1} = R_i \cup \{(T, S)\}$
  - Use the lemma with $R = R_{i+1}$ to derive VIS_{i+1}, CO_{i+1} (and thus $\mathcal{P}_{i+1}$)
- Let $\mathcal{X} = \mathcal{P}_n$ as CO_n is now total. $\square$

Outline

Transaction Chopping under SI:

- We'll derive a static analysis that checks if transactions can be chopped into smaller sessions
- The analysis must check that any execution with chopped transaction does not exhibit new behaviors.

For history $\mathcal{H}$, let

$$\approx_{\mathcal{H}} = SO_{\mathcal{H}} \cup SO_{\mathcal{H}}^{-1} \cup \{(T, T) \mid T \in \mathcal{T}_{\mathcal{H}}\}$$

be the equivalence relations grouping transactions from the same session.

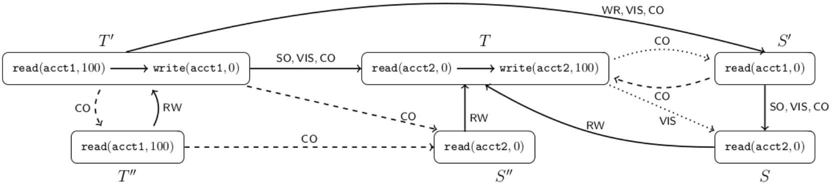
Let $\boxed{T}_{\mathcal{H}} = (E, po)$ where $E = (\bigcup\{E_S\ midS \approx_{\mathcal{H}} T\})$ and

$$po = \{(e, f) \mid \left(\exists S.e, f \in E_S \wedge e \xrightarrow{po_S} f \wedge S \approx_{\mathcal{H}} T\right) \vee \\ \left(\exists S, S'.e \in E_S \wedge f \in E_{S'} \wedge S \xrightarrow{SO_{\mathcal{H}}} S' \wedge S' \approx_{\mathcal{H}} T\right)\}$$

Informally $\boxed{T}_{\mathcal{H}}$ is the result of splicing all transactions in session of T into the same transaction.

- For history $\mathcal{H}$, let $\mathit{splice}(\mathcal{H}) = \left(\{\boxed{T}_{\mathcal{H}} \mid T \in \mathcal{T}_{\mathcal{H}}\}, \emptyset\right)$ history resulting from splicing all sessions in a history.
- $\mathcal{G} \in \mathit{GraphSI}$ is **spliceable** if exists a dependency graph $\mathcal{G}' \in \mathit{GraphSI}$ such that $\mathcal{H}_{\mathcal{G}'} = \mathit{splice}(\mathcal{H}_{\mathcal{G}})$.
- For graph $\mathcal{G}$ we let $\approx_{\mathcal{G}} = \approx_{\mathcal{H}_{\mathcal{G}}}$.

Example, let graph $\mathcal{G}$:



T', T: `session transfer { tx { acct1 = acct1 - 100 }; tx { acct2 = acct2 + 100 } }`
T'': `session lookup1 { tx { return acct1 } }`
S'': `session lookup2 { tx { return acct2 } }`
S', S: `session lookupAll { tx { var1 = acct1 }; tx { var2 = acct2 }; return var1 + var2 }`

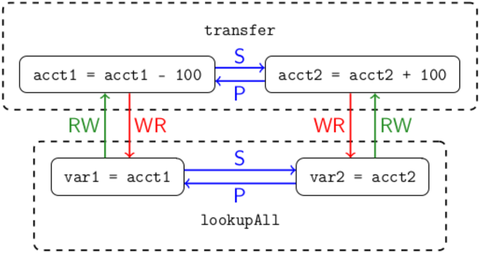
The above graph is not splice-able: $\boxed{S}_{\mathcal{G}}$ observes write by $\boxed{T}_{\mathcal{G}}$ to *acct*1 but not to *acct*2.

Given $\mathcal{G}$ let **dynamic chopping graph** $DCG(\mathcal{G})$ obtained from $\mathcal{G}$ by

- Removing $WR_{\mathcal{G}}, WW_{\mathcal{G}}, RW_{\mathcal{G}}$ edges between transactions related by $\approx_{\mathcal{G}}$
- Adding **predecessor** edges $SO_{\mathcal{G}}^{-1}$
- We'll call SO **successor** edges
- And call $(WR_{\mathcal{G}} \cup WW_{\mathcal{G}} \cup RW_{\mathcal{G}}) \setminus \approx_{\mathcal{G}}$ **conflict** edges

A cycle in $DCG(\mathcal{G})$ is **critical** if:

- Does not contain 2 occurrences of the same vertex
- Contains 3 consecutive edges in form of *conflict,predecessor,conflict*
- Any 2 anti dependency edges ($RW_{\mathcal{G}} \setminus \approx_{\mathcal{G}}$) are separated by at least one read ($WR_{\mathcal{G}} \setminus \approx_{\mathcal{G}}$) or write ($WW_{\mathcal{G}} \setminus \approx_{\mathcal{G}}$) dependency edges.



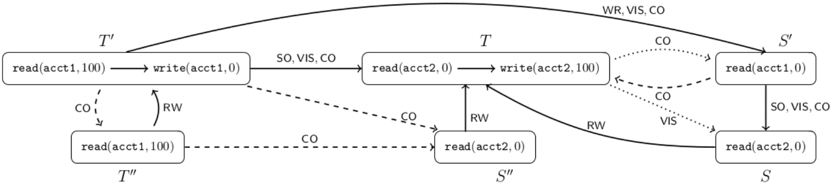
This example contains a critical cycle with

$$. \xrightarrow{S} . \xrightarrow{WR} . \xrightarrow{P} . \xrightarrow{RW} .$$

Theorem:
For $\mathcal{G} \in$ *GraphSI*, if *DCG*($\mathcal{G}$) contains no critical cycles, then $\mathcal{G}$ is splice-able.

We use the last theorem to derive the static analysis.

- Assume set of **programs** $\mathcal{P} = \{P_1, P_2, \dots\}$, each defining code of a session resulting from chopping a single transaction.
- Each P_i is composed of k_i **program pieces**
- W^i_j and R^i_j sets of objects written or read by j-th piece of P_i



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S', S: session lookupAll { tx { var1 = acct1 }; tx { var2 = acct2 }; return var1 + var2 }

For *transfer* session we have 2 program pieces with

- $W_1^1 = R_1^1 = acct1$, $W_2^1 = R_2^1 = acct2$

- History $\mathcal{H}$ **can be produced** by programs $\mathcal{P}$ if there's 1:1 correspondence between every session in $\mathcal{H}$ and program $P_i \in \mathcal{P}$, and the read/write sets of a program agree with the transaction.
- Chopping is defined **correct** if every dependency graph $\mathcal{G} \in \mathit{GraphSI}$, where $\mathcal{H}_{\mathcal{G}}$ can be produced by $\mathcal{P}$ is splice-able.

We check correctness of $\mathcal{P}$ using it's **static chopping graph** $SCG(\mathcal{P})$, and we also construct **successor**, **predecessor**, and **dependency** edges in similar fashion to $DCG(\mathcal{G})$

The edge set of static graphs $SCG(\mathcal{P})$ over-approximate the edges set of the dynamic graphs $DCG(\mathcal{G})$ for corresponding to graphs $\mathcal{G}$ produced by programs $\mathcal{P}$.
Thus we get:
The chopping defined by $\mathcal{P}$ is correct if $SCG(\mathcal{P})$ contains no critical cycles.
FIXME

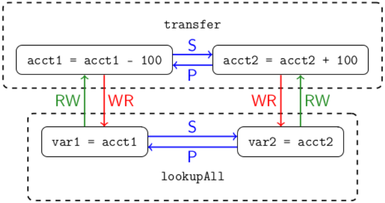

Figure 5: The static chopping graph of the programs {transfer, lookupAll} **from Figure 4. Dashed boxes group program pieces into sessions.**

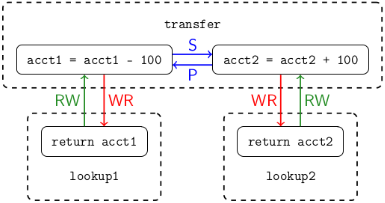

Figure 6: The static chopping graph of the programs {transfer, lookup1, lookup2} **from Figure 4.**

Outline

Robustness against SI

Check if a given application running under **Serializability**, does not produce new histories when runs under **SI**

Theorem:
For any $\mathcal{G}$, we have $\mathcal{G} \in \mathit{GraphSI} \setminus \mathit{GraphSER}$ if $\mathcal{T}_{\mathcal{G}} \models \mathit{INT}$, $\mathcal{G}$ contains a cycle, and all its cycles have at least two adjacent anti-dependency edges.

Thank you!