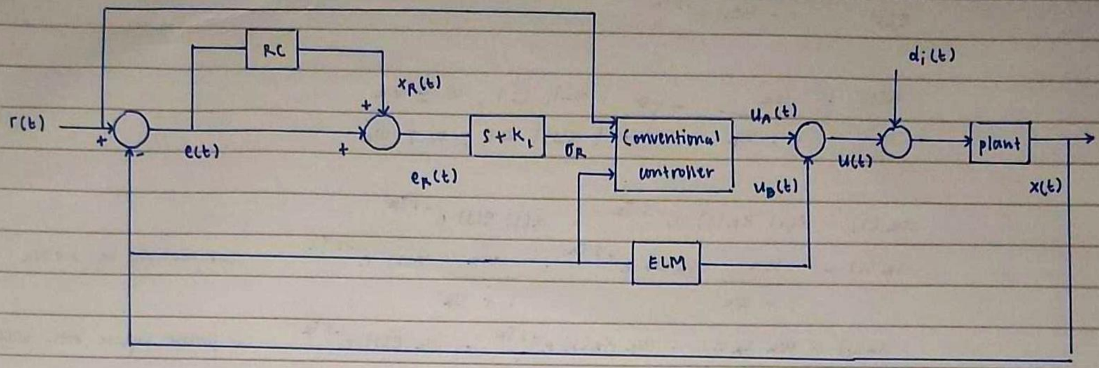
ELM + RC Trial 1

* Block diagram



* Open loop plant

$u(t)$: control input
$x(t)$: position output
$\dot{x}(t)$: velocity output
$\ddot{x}(t)$: acceleration output
$d_i(t)$: input disturbance

$$P(s) = \frac{\Theta(s)}{V(s)} = \frac{K}{s(T+1)}$$

Let $v(t) = u(s)$, and $\theta(t) = x(t)$, then

$$\frac{X(s)}{U(s)} = \frac{K}{Ts^2 + s}$$

$$Ts^2 X(s) + s X(s) = K U(s)$$ → inverse laplace

We have

$$T \ddot{x}(t) + \dot{x}(t) = K u(t)$$

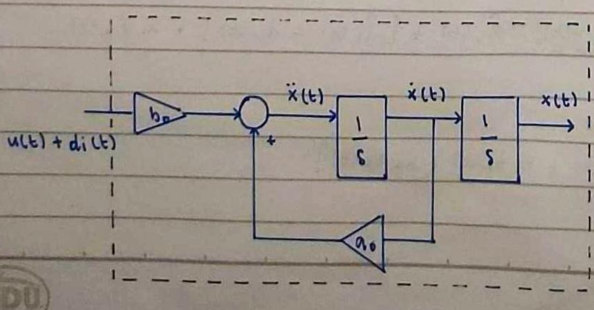
$$\ddot{x}(t) = -\frac{1}{T} \dot{x}(t) + \frac{K}{T} u(t)$$

Let write $a_0 = -\frac{1}{T}$, $b_0 = \frac{K}{T}$, and $d_i(t)$ appears at the input

$$\ddot{x}(t) = -a_0 \dot{x}(t) + b_0 [u(t) + d_i(t)]$$

$$\ddot{x}(t) = -a_0 \dot{x}(t) + b_0 u(t) + b_0 d_i(t)$$

* Open-loop realization



* Consider general modified RC with low-pass filter $\alpha(s)$

$$\frac{X_R(s)}{E(s)} = \frac{\alpha(s)\, e^{-sT_R}}{1-\alpha(s)\, e^{-sT_R}}$$, where $\alpha(s)$ is first order low-pass filter given by :

$$\alpha(s) = \frac{w_R}{s+w_R} \rightarrow \begin{cases} |\alpha(s)| \approx 1, & w \leq w_R \\ |\alpha(s)| < 1, & w > w_R \end{cases}$$

$$X_R(s) - \alpha(s)\, X_R(s)\, e^{-sT_R} = \alpha(s)\, E(s)\, e^{-sT_R}$$

$$X_R(s) - \frac{w_R}{s+w_R} X_R(s)\, e^{-sT_R} = \frac{w_R}{s+w_R} E(s)\, e^{-sT_R}$$ → multiply by $s + w_R$

$$s\, X_R(s) + w_R X_R(s) - w_R X_R(s)\, e^{-sT_R} = w_R\, E(s)\, e^{-sT_R}$$ → inverse laplace both side

$$\dot{x}_R(t) = -w_R\, x_R(t) + w_R\, x_R(t-T_R) + w_R\, e(t-T_r)$$

(state space model of modified RC)

* consider no saturation at the output of RC
* we define $e_R(t) = e(t) + x_R(t)$
* Tracking error dynamics :

$$e(t) = r(t) - x(t)$$

$$\ddot{e}(t) = \ddot{r}(t) - \ddot{x}(t) = \ddot{r}(t) - (-a_0\, \dot{x}(t) + b_0\, u(t) + b_0\, d_i(t))$$

$$= \ddot{r}(t) + a_0\, \dot{x}(t) - b_0\, u(t) - b_0\, d_i(t)$$

$$\ddot{e}_R(t) = \ddot{e}(t) + x_R''$$

$$= \ddot{r}(t) + a_0\, \dot{x}(t) - b_0\, u(t) - b_0\, d_i(t) + x_R''$$

Consider

$$d_L(t) = b_0\, d_i(t) + x_R'' \longrightarrow \ddot{e}_R(t) = \ddot{r} + a_0 \dot{x} - b_0 u - d_L$$

* The actuated lump disturbance is $d_L(t) = H\beta^*$
* The estimated lump disturbance is $\hat{d}_L(t) = H\hat{\beta}$, where $\hat{\beta}$ is the estimated output weight matrix.
* ~~$\sigma_R(t)$~~ $\sigma_R(t) = (s + k_1)\, e_R(t) = \dot{e}_R(t) + k_1\, e_R(t)$
* $\dot{\sigma}_R(t) = \ddot{e}_R(t) + k_1\, \dot{e}_R(t)$

$$= \ddot{r}(t) + a_0\, \dot{x}(t) - b_0\, u(t) - d_L(t) + k_1\, \dot{e}_R(t)$$

can be rewritten

$$\dot{\sigma}_R(t) = \ddot{r}(t) + a_0\, \dot{x}(t) - b_0\, u(t) - \hat{d}_L(t) + [\hat{d}_L(t) - d_L(t)] + k_1\, \dot{e}_R(t)$$

* control law : $u(t) = u_A + u_B$

$$u_A(t) = \frac{1}{b_0}\left[+a_0\, \dot{x}(t) + \ddot{r}(t) + k_1\, \dot{e}_R(t) + k_2\, \sigma_R(t) \right]$$

$$u_B(t) = \frac{1}{b_0}\left[-\hat{d}_L(t) \right]$$

* estimated output weight matrix $\hat{\beta}$ is updated by :

$$\dot{\hat{\beta}}^T = -\eta \sigma_R H \rightarrow \eta > 0 \text{ positive constant}$$

* substituting the control law $u(t) = u_A + u_B$

$$\dot{\sigma}_R(t) = \ddot{r}(t) + a_0 \dot{x}(t) - b_0 [u_A + u_B] - \hat{d}_L + \hat{d}_L - d_L + k_1 \dot{e}_R(t)$$

$$= \ddot{r}(t) + a_0 \dot{x}(t) - b_0 \left[\frac{1}{b_0} \left(+ a_0 \dot{x}(t) + \ddot{r}(t) + k_1 \dot{e}_R(t) + k_2 \sigma_R(t) \right) + \frac{1}{b_0} \left(- \hat{d}_L(t) \right) \right] - \hat{d}_L(t) + \hat{d}_L(t) - d_L + k_1 \dot{e}_R(t)$$

$$= \cancel{\ddot{r}(t)} + \cancel{a_0 \dot{x}(t)} - \cancel{a_0 \dot{x}(t)} - \cancel{\ddot{r}(t)} - \cancel{k_1 \dot{e}_R(t)} - k_2 \sigma_R(t) + \cancel{\hat{d}_L(t)} - \cancel{\hat{d}_L(t)} + \hat{d}_L(t) - d_L + \cancel{k_1 \dot{e}_R(t)}$$

$$= -k_2 \sigma_R(t) + \hat{d}_L(t) - d_L$$

$$= -k_2 \sigma_R(t) + H\tilde{\beta}$$

$$d_L - \hat{d}_L = H\beta^* - H\beta = H[\beta^* - \hat{\beta}] = H\tilde{\beta}$$

$$-d_L + \hat{d}_L = -H\tilde{\beta}$$

$$\dot{\tilde{\beta}}^T = \dot{\beta}^{*T} - \dot{\hat{\beta}}^T = 0 - \dot{\hat{\beta}}^T = -\eta \sigma_R H$$

* Define Lyapunov function as

$$V = \frac{1}{2} \sigma_R^2(t) + \frac{1}{\eta} \tilde{\beta}^T \tilde{\beta}$$

* ~~Itac~~ Taking derivative

$$\dot{V} = \sigma_R \dot{\sigma}_R + \frac{1}{\eta} \dot{\tilde{\beta}}^T \tilde{\beta}$$

$$= \sigma_R [-k_2 \sigma_R(t) + H\tilde{\beta}] + \frac{1}{\eta} \dot{\tilde{\beta}}^T \tilde{\beta}$$

$$= -k_2 \sigma_R^2(t) + \sigma_R H \tilde{\beta} + \frac{1}{\eta} \dot{\tilde{\beta}}^T \tilde{\beta}$$

$$= -k_2 \sigma_R^2(t) + \sigma_R H \tilde{\beta} + \frac{1}{\cancel{\eta}} (-\cancel{\eta} \sigma_R H) \tilde{\beta}$$

$$= -k_2 \sigma_R^2(t) + \cancel{\sigma_R H \tilde{\beta}} + \cancel{(-\sigma_R H \tilde{\beta})}$$

$$= -k_2 \sigma_R^2(t)$$

(k_2 positive gain)