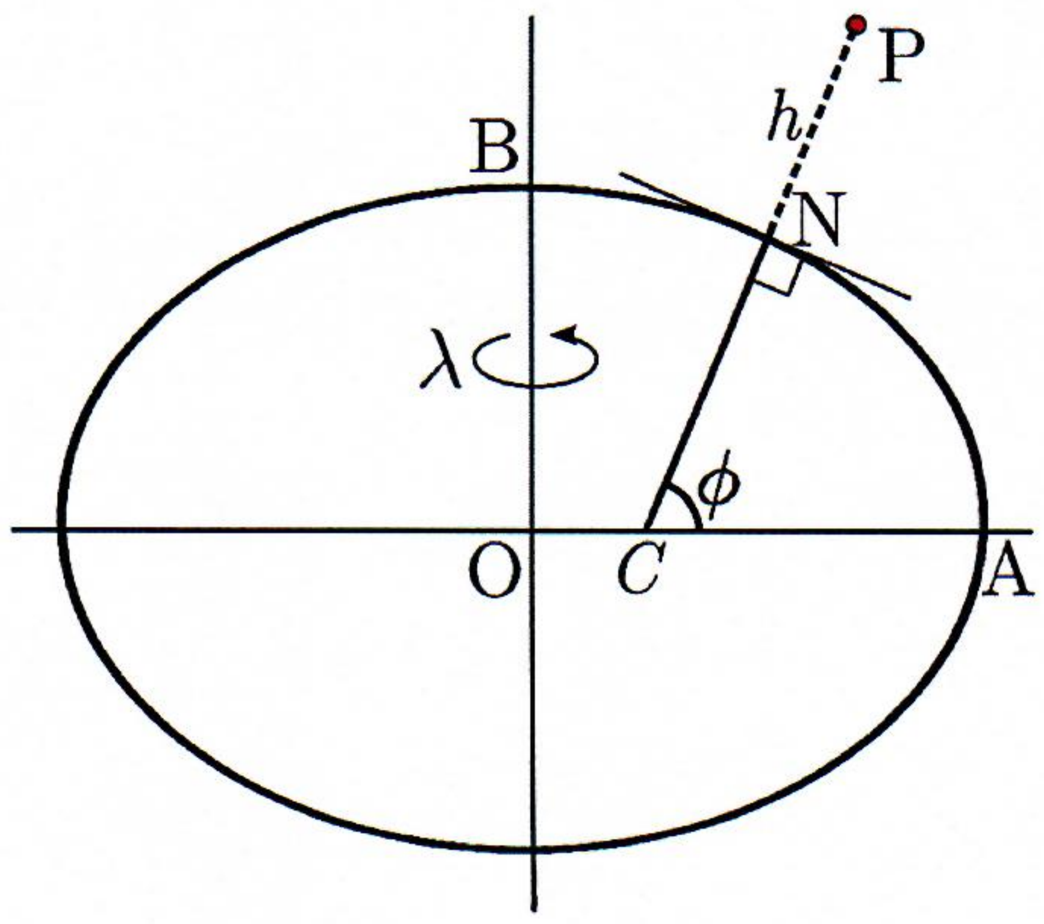

Figure 1.1: Geodetic ellipsoid coordinates

1.1.2 Local-tangential systems

The local tangential plane coordinates, are based on the local *vertical direction* and the earth's axis of rotation.

Two variants exist, the selection of which is a matter of convention but lead to slightly different state vectors in tracking applications.

- East-North-Up (**ENU**)
- North, East, Down (**NED**)

The North-East-Down and East-North-Up coordinate systems, are local geodetic systems fixed to the Earth. They are determined by a tangent plane attached to the geodetic reference ellipsoid at a reference point of interest, which determines the origin of the system. The differentiation between the two systems is the direction that the unit vectors are pointing, which is indicated by their ~~actual~~ names.

Tangential coordinate systems are convenient in the context of tracking nearby ships because the tracking problem can be reduced to tracking a target on a two-dimensional plane.

1.1.3 Earth Centered Earth-fixed (ECEF)

ECEF is a *Cartesian coordinate system* $[X, Y, Z]^T$ with its origin placed at the earth's *center of mass*, hence the naming convention. It is a rotating system as the *z-axis* is pointing through *true north* while the *x-axis* intersects earth's spheroid at the geodetic coordinates $[\phi, \lambda]^T = [0, 0]^T$(fig. 1.2).

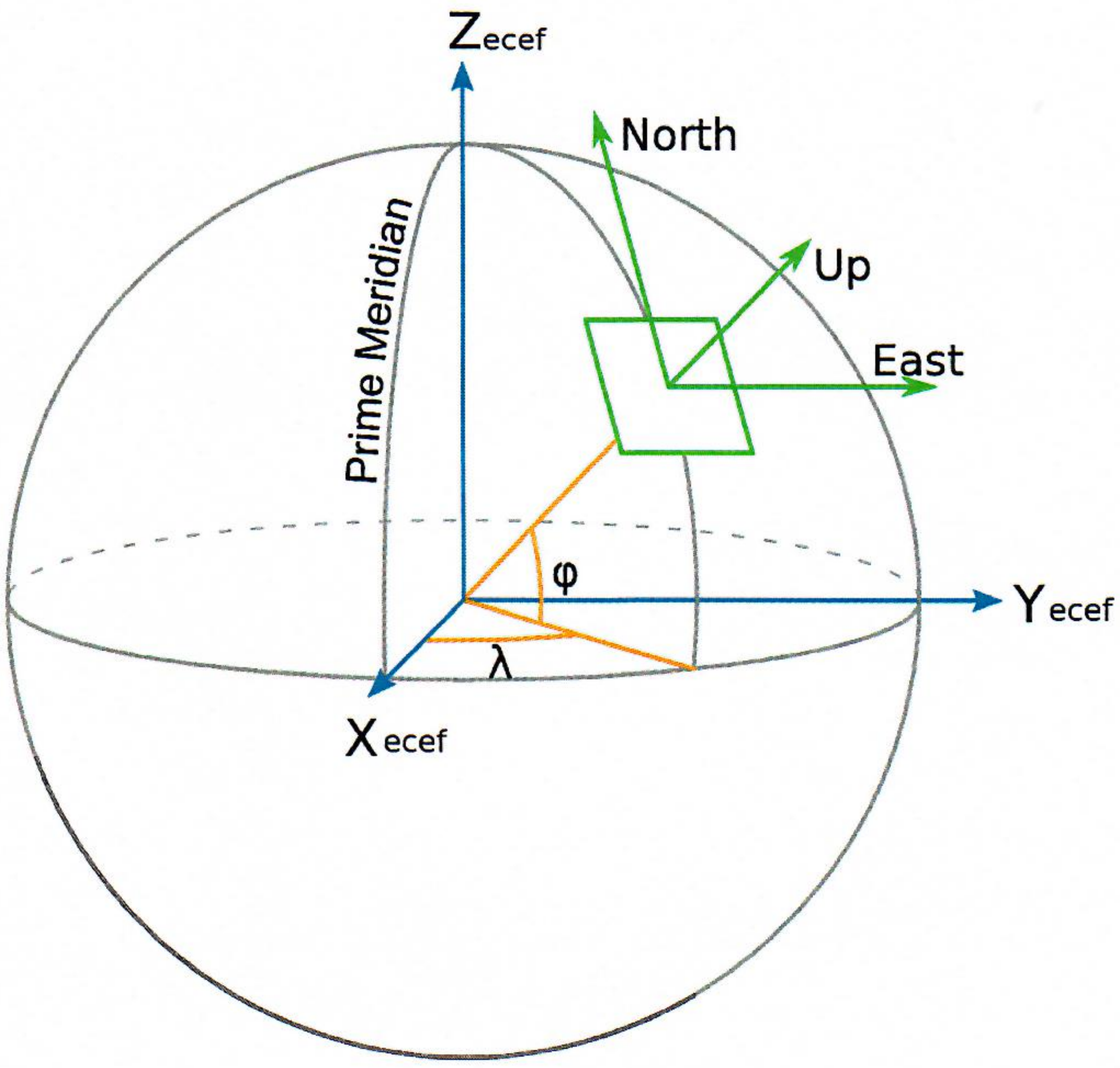

Figure 1.2: ECEF and ENU systems [3]

1.2 Transformations

There is a need to define transformations between different coordinate systems, since tracking is performed in a common configuration space according to the state-space representation of the dynamical targets. Some of the transformations are non-linear, due to the trigonometric functions involved, a factor that signifies the importance of using a tracking framework that is able to handle these nonlinearities.

1.2.1 Geodetic to ECEF

GNSS measurements usually arrive in **geodetic** coordinates, where as tracking at sea is usually performed on a **local tangential plane**.

The transformation between these two systems requires an intermediate transformation from **geodetic** (ϕ, λ, h) to **ECEF** (X, Y, Z) coordinates 1.1.

we use kapital letters for forces

$$
\begin{aligned}
X &= (N+h)cos\lambda cos\phi \\
Y &= (N+h)cos\lambda sin\phi \\
Z &= [N(1-e^2)+h]sin\lambda
\end{aligned}
\tag{1.1}
$$

X_e
y_e
z_e

\mathbf{X} for matrices

1.2.2 ECEF to ENU

Given a reference point in Geodetic $[\phi_0, \lambda_0]$ and the corresponding ECEF $[X_0, Y_0, Z_0]$ coordinates, then any other point's ECEF coordinates $[X, Y, Z]$ can be converted to local tangential plane coordinates $[x, y, z]_t$ using the transformation in 1.2

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix}_t = \mathcal{L}\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}\right) \tag{1.2}
$$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_e - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_0$ or X_e, y_e, z_e — X_0, y_0, z_0

where $\mathcal{L}$ is the tangential plane projection matrix of the geodetic ellipsoid with the origin being the reference point $[\phi_0, \lambda_0, h_0]$ 1.3

$$\mathcal{L} = \begin{bmatrix} -sin\phi_0 & cos\phi_0 & 0 \\ -sin\lambda_0 cos\phi_0 & -sin\lambda_0 sin\phi_0 & cos\lambda_0 \\ cos\lambda_0 cos\phi_0 & cos\lambda_0 sin\phi_0 & sin\lambda_0 \end{bmatrix} \tag{1.3}$$

1.3 Own-ship coordinate systems

To describe the position and orientation of a ship, one can use the following orthogonal coordinate systems fig. 1.3 [4] .

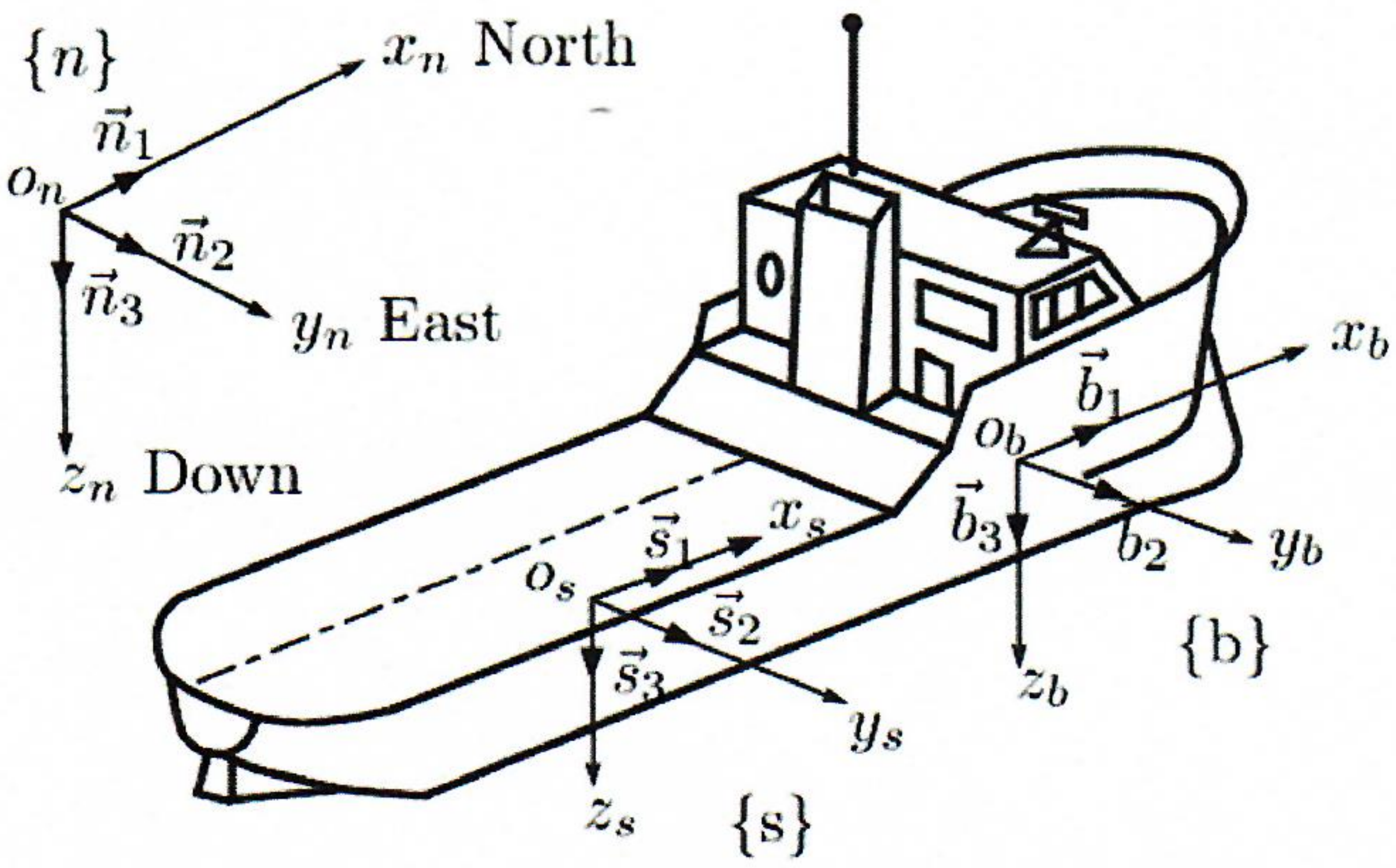

Figure 1.3: Body fixed, sea-keeping and NED systems [4]

1. NED/ENU, $\{n\}, \{e\}$
2. Body-fixed, $\{b\}$
3. Sea-keeping, $\{s\}$
4. Tracking coordinates, $\{t\}$

cannot follow both heading and velocity vector, choose one you have $x_{\not b}$ in heading direction in Fig 1.4

1.3.1 Body fixed coordinate system

The *body fixed*, as the name suggests, is fixed to the hull of a vessel. The x-axis is pointing towards the bow, the y-axis points starboard and the z-axis points downwards completing the orthogonal system. The origin of the system in marine applications is usually the origin of the principal axes of inertia, which simplifies the solution of the dynamical equations of motion for the body. This coordinate system is used to express on-board velocity and acceleration measurements and the equations of motion of the vessel as a rigid body are formulated about the origin o_b.

ship-following

and follows

1.3.2 ~~Sea-keeping~~ coordinate system

The ~~*sea-keeping*~~ frame follows the average speed of the vessel. ~~As such, the system is fixed to the ship's *equilibrium* state, which is defined by~~ the ship's average speed and heading, The positive x-axis is pointing ~~towards the forward velocity vector~~, the y-axis is pointing starboard and the z-axis is pointing down. The origin of the system is selected so that the z-axis ~~is~~ pointing through its mean center of ~~mass,~~ and the x-y plane coincides with the ~~mean free~~ water surface.

calm

~~As a~~ ~~bo~~ has x and y axes in the tangent plane ~~on but is rotated about the vertical axis to follow.~~ ~~in the~~ ~~to follow~~

ahead

bouyancy

2.2 Motion models

As discussed in a previous section, when choosing a coordinate system for the target motion model, it is very convenient to assume the target's state on a the own-ship's two dimensional tracking plane.

In order to improve the quality the stability and the accuracy of the estimations, the ships are assumed to comply with dynamic motion models. The tracking problem therefore is that of estimating the model's parameters for a target, taking into consideration all available observations from various sensors.

The physical constraints and dynamics of the problem of tracking floating targets at sea, justify the use of a 2-D kinematic model. Such a model can capture with sufficient accuracy the configuration space of a remote vessel's maneuvering behavior. In the context of tracking, targets are assumed to be point objects with negligible dimensions corresponding to the mean vessel position. Maneuvering ships exhibit very characteristic trajectories and as such their motion can be sufficiently expressed for the purpose of tracking by second-order dynamical models.

While the motion models proposed in literature are numerous, most of them are maintaining a relatively low level of complexity. *Linear motion* models assume a constant linear velocity(CV) or constant linear acceleration (CA) [5]. These models have the advantage that the state transition equations are linear, and thus Gaussian probability densities can be efficiently and optimally propagated as matrix multiplications. Unfortunately linear models are restricted to straight-line motions and are thus unable to model rotations, especially the yaw rate, into account. In general, a vessel follows slow parabolic-type maneuvers. If one introduces the angular speed of the target around its z-axis as well, the resulting models are referred to as *curvilinear* models. These models can be further distinguished by the selected state variables which are assumed to be constant.

2.2.1 Continuous constant turn rate and velocity (CTRV) model

Starting with a time continuous model:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + w(t) \quad , \mathbf{x} = [x, y, v, \psi, \omega]^T \tag{2.2}$$

then assuming that the linear velocity v and the turn rate ω remain constant — which is a fair assumption for a moving vessel — then the non-linear differential equations describing the motion are according to [5]

u is $\dot{x}_b$

~~Aa~~ v is $\dot{y}_b$

$U = V$

U is ship speed through water

$$f = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v} \\ \dot{\psi} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \overset{u}{\cancel{v}}cos(\psi) \\ vsin(\psi) \\ 0 \\ \omega \\ 0 \end{bmatrix} \tag{2.3}$$

V or U_g is speed over ground (SOG)

which is describing *constant turn rate and velocity* (**CTRV**) model since the linear and angular velocities accelerations are zero.

~~V_g~~

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\{t\}} = R_z(\frac{\pi}{2} - \psi) \begin{bmatrix} E \\ N \\ U \end{bmatrix}_{\{e\}} \tag{1.4}$$

where

$$R_z(\theta) = \begin{bmatrix} cos\cancel{\theta} & -sin\cancel{\theta} & 0 \\ sin\cancel{\theta} & cos\cancel{\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta \rightarrow \psi$