Aristotle University of Thessaloniki
Department of Electrical and Computer Engineering

Advanced Signal Processing

Second Assignment – Summer Semester 2020/2021

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13/4/2021

Exercise 2:

Consider the real discrete process $\boldsymbol{X(k)}$ given by

$$\boldsymbol{X(k)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{6}} \boldsymbol{cos(\omega_i k + \varphi_i)} \ , \boldsymbol{k} = \boldsymbol{0, 1, \ldots, N-1},$$

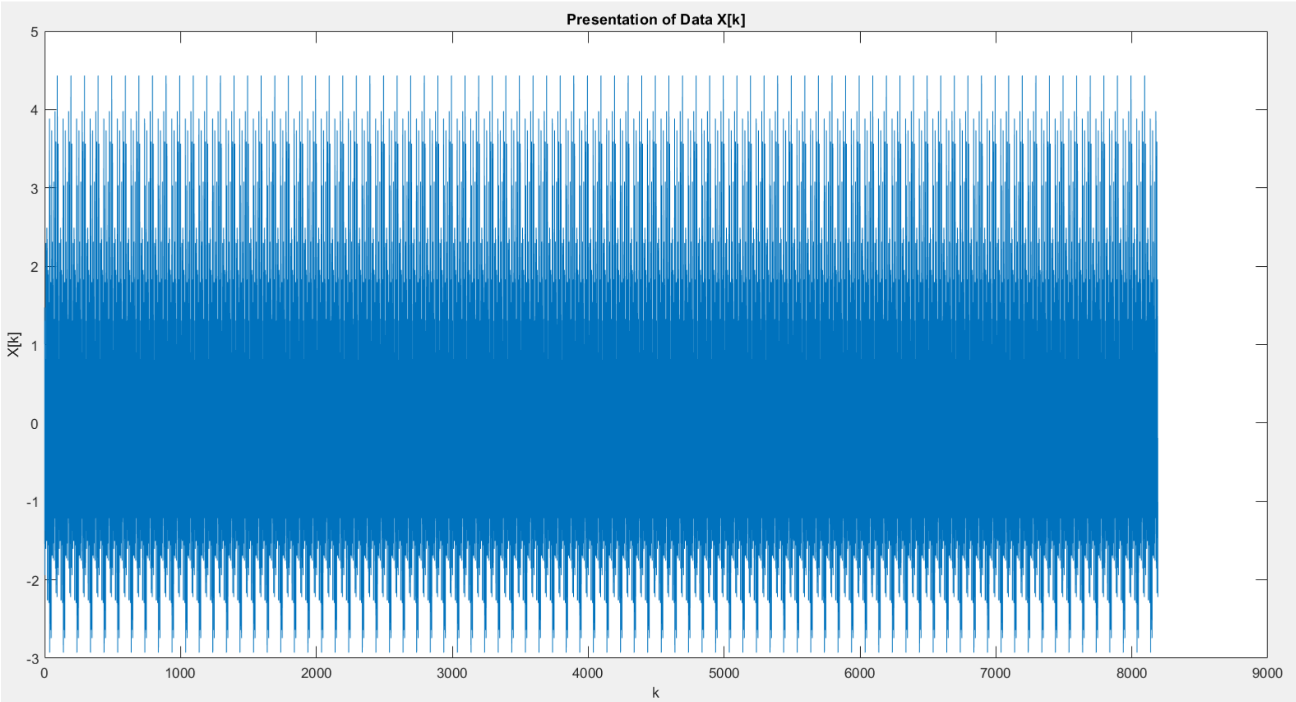
where $\omega_i = 2\pi\lambda_i$, $\lambda_3 = \lambda_1 + \lambda_2$ and $\lambda_6 = \lambda_4 + \lambda_5$, $\varphi_3 = \varphi_1 + \varphi_2$, $\varphi_6 = \varphi_4 + \varphi_5$ and $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6$ are independent and uniformly distributed random variables on $[0,2\pi]$. Consider that $\boldsymbol{\lambda_1 = 0.12Hz}$, $\boldsymbol{\lambda_2 = 0.30Hz}$, $\boldsymbol{\lambda_4 = 0.19Hz}$ and $\boldsymbol{\lambda_5 = 0.17Hz}$ (hence, $\boldsymbol{\lambda_3 = 0.42Hz}$ and $\boldsymbol{\lambda_6 = 0.36Hz}$). Moreover, let N = 8192 as the data length.

1) Construct the $X(k)$.
2) Estimate the power spectrum $C_2^x(f)$. Use $L_2 = 128$ max shiftings for autocorrelation.
3) Estimate the bispectrum (only in the primary area) $C_3^x(f_1, f_2)$ using:
   a) the indirect method with K = 32 and M = 256. Use $L_3 = 64$ max shiftings for the third order cumulants. Use: a_1) rectangular window and a_2) Parzen window.
   b) the direct method with K = 32 and M = 256. Use J = 0.
4) Plot $X(k)$, $C_2^x(f)$, $C_3^x(f_1, f_2)$ (all estimations).
5) Compare the estimations of $C_3^x(f_1, f_2)$ amongst {a_1, a_2, b} settings. Comment on the comparisons.
6) What can you deduce regarding the frequency content from the comparison of $C_2^x(f)$ and $C_3^x(f_1, f_2)$ (all estimations)?
7) How the results will change if you repeat the process from 1 to 5 taking into account:
   a) different segment length: i) K = 16 and M = 512 ii) K = 64 and M = 128?
   b) 50 realizations of the $X(k)$ and comparing the mean values of the estimated $C_2^x(f)$, $C_3^x(f_1, f_2)$?

Analysis:

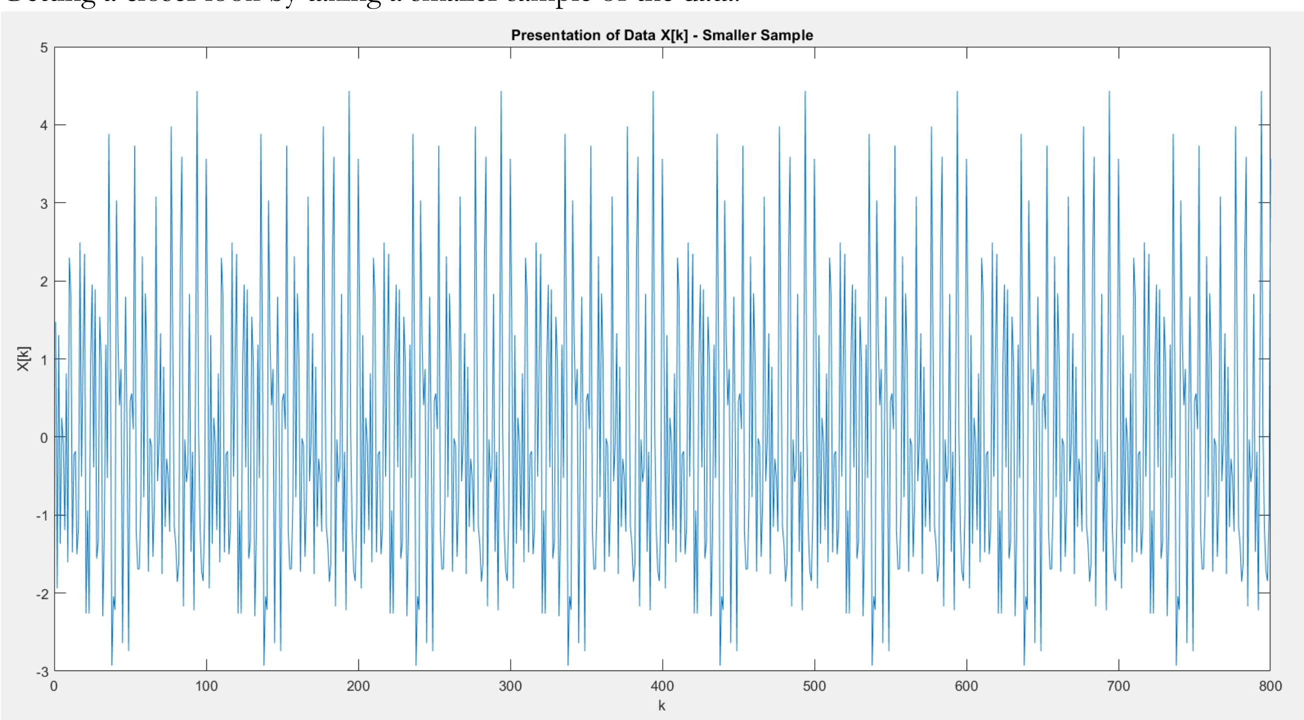
The software we use for the needs of the exercise is MATLAB. Thus, the first three steps are carried out in the script running by the software, producing the following plots:

4.1. First, the data $X(k)$.



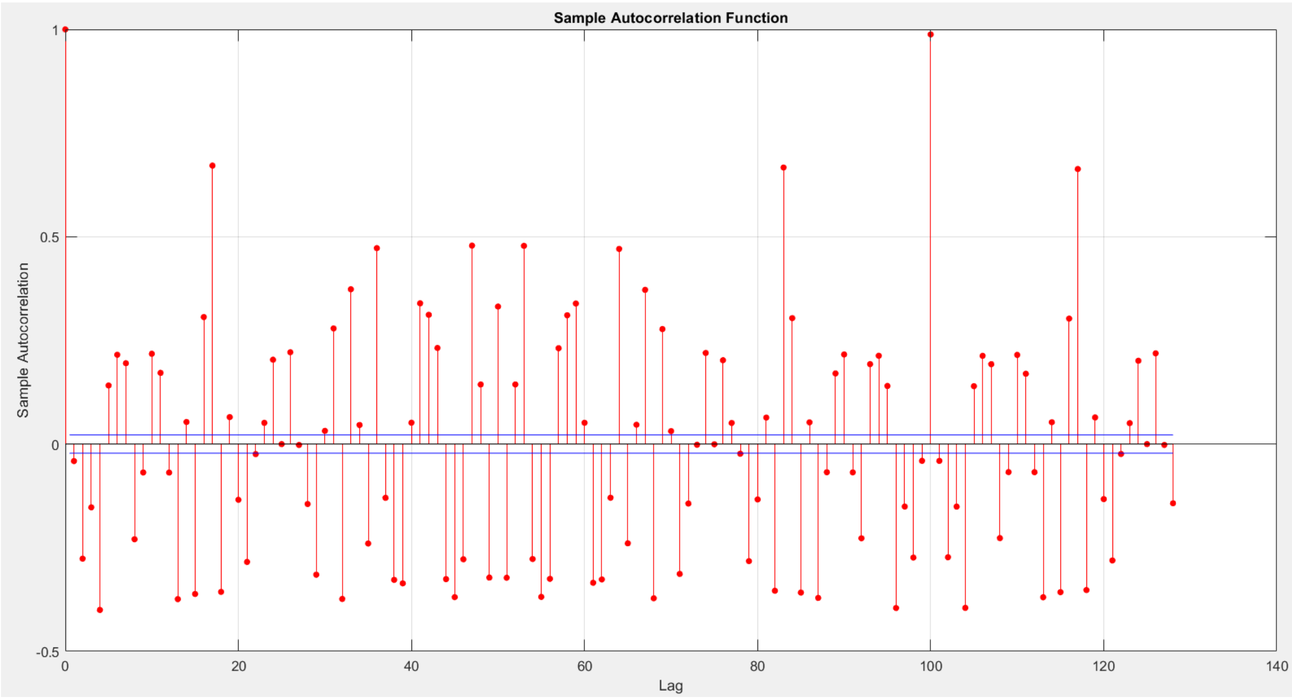
Plot 1 - X(k)

Getting a closer look by taking a smaller sample of the data:



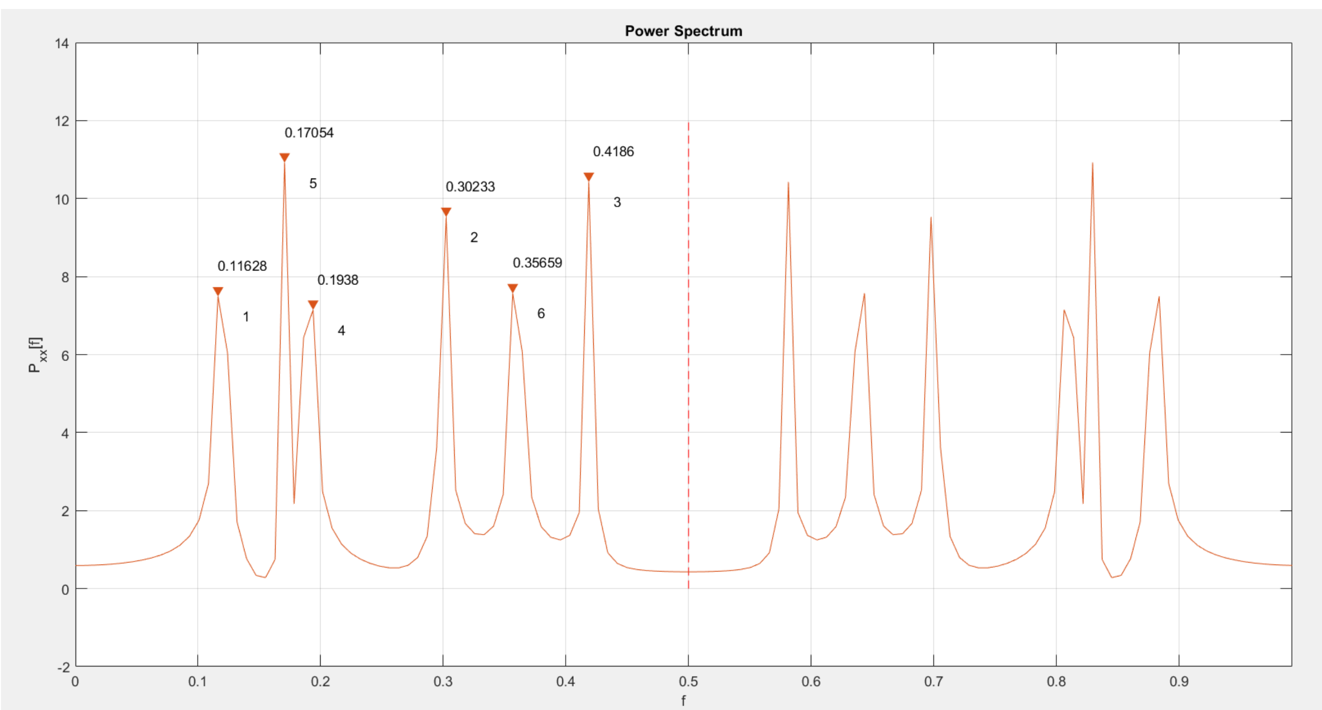
Plot 2 - X(k) / Smaller Sample

4.2. We estimate the power spectrum using the autocorrelation function of $X(k)$.The plot of the autocorrelation function:



Plot 3 – ACF of X(k)

Then, we get the plot of the power spectrum:

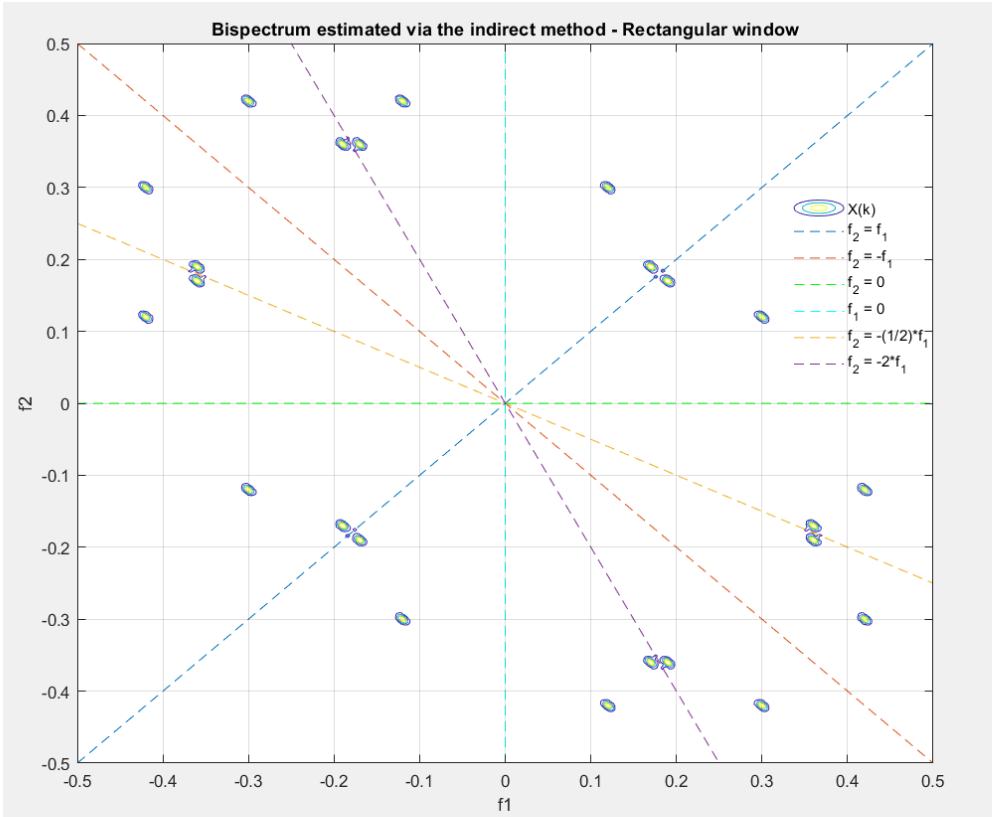


Plot 4 – Estimated Power Spectrum – Numbers describing No of λ and frequency on that peak.

As we expect, the frequencies given by $\boldsymbol{\lambda}$ are close to the ones described by the frequencies on the peaks of the estimated power spectrum.

4.3. The plots of the estimated bispectrum

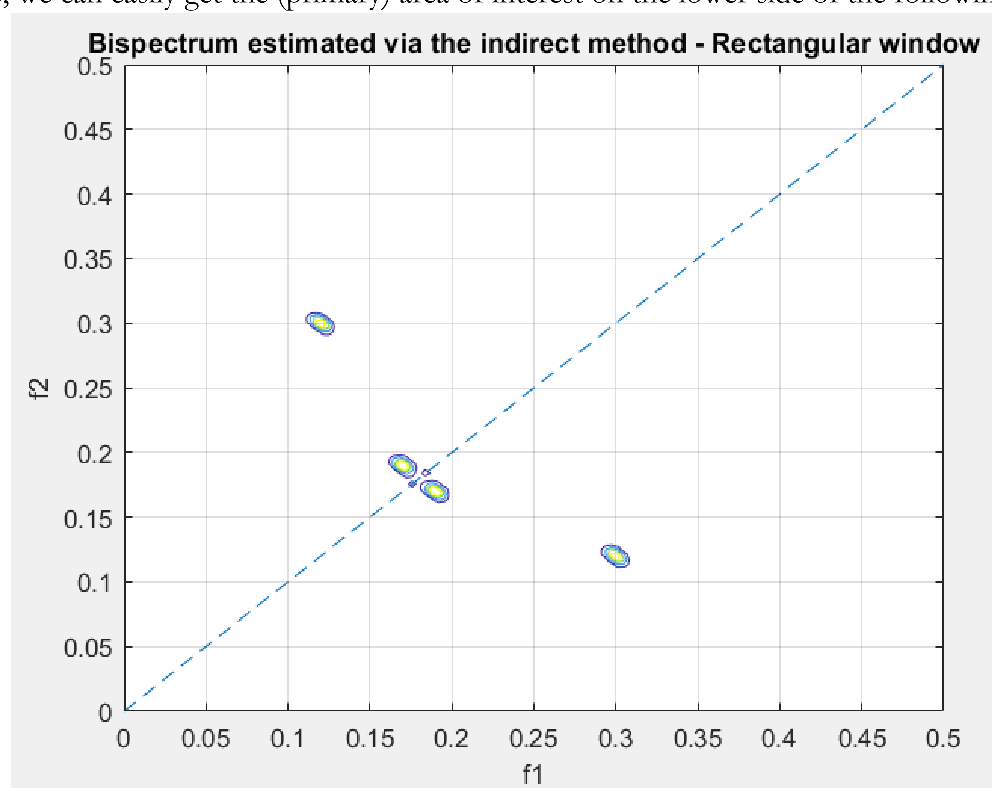
a$_1$) using the indirect method with rectangular window:



Plot 5 – Estimated Bispectrum plot / Indirect method – Rectangular Window – K = 32, M = 256

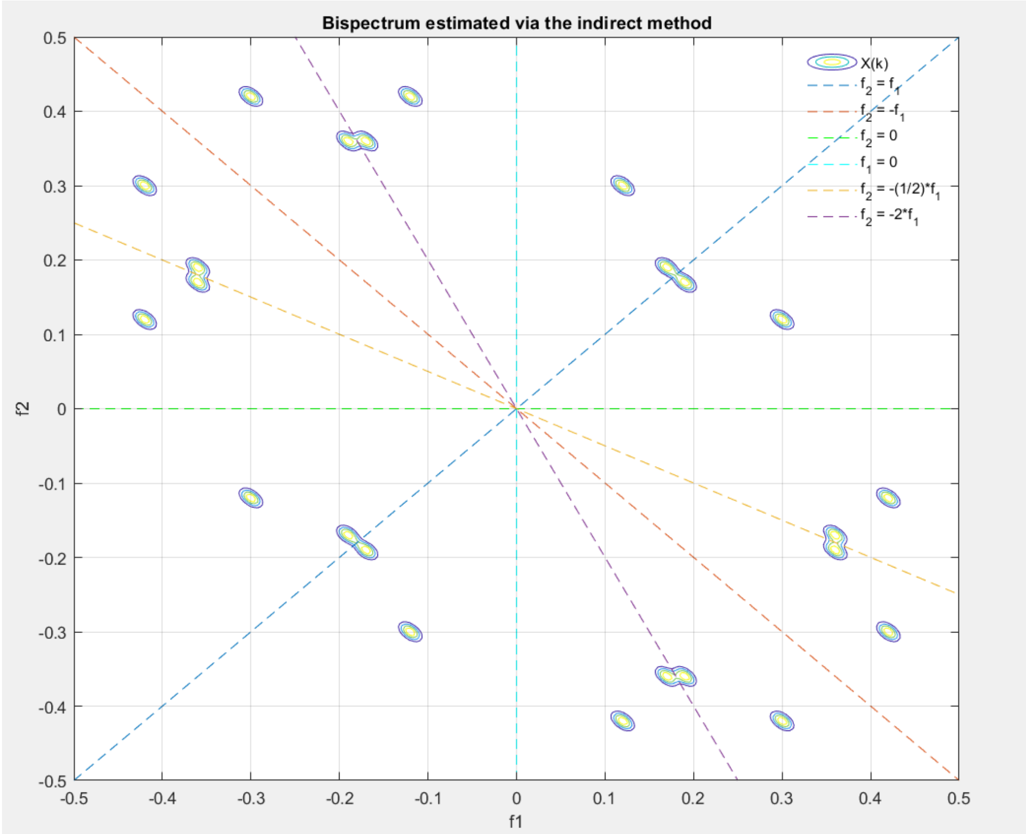
Drown, are the lines of symmetries. What we are interested in though is the primary area.

Thus, we can easily get the (primary) area of interest on the lower side of the following plot:

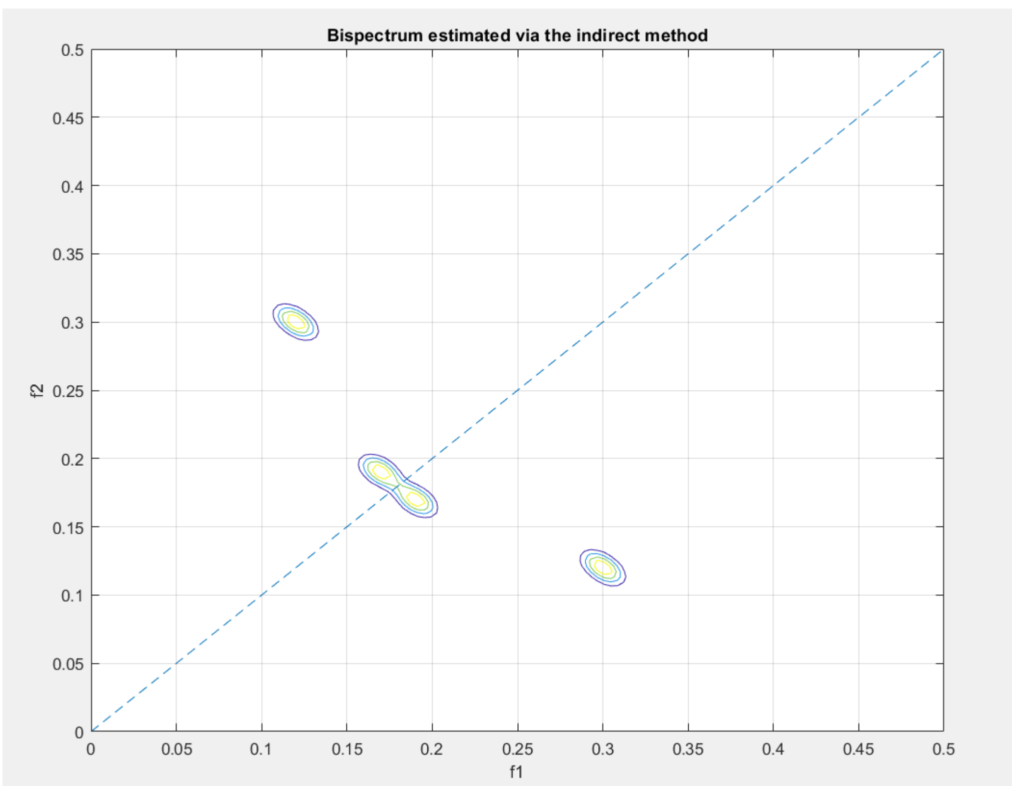


Plot 6 – Estimated Bispectrum plot / Indirect method – Rectangular Window – K = 32, M = 256: Primary Area

a_2) Accordingly, using the indirect method with Parzen window:

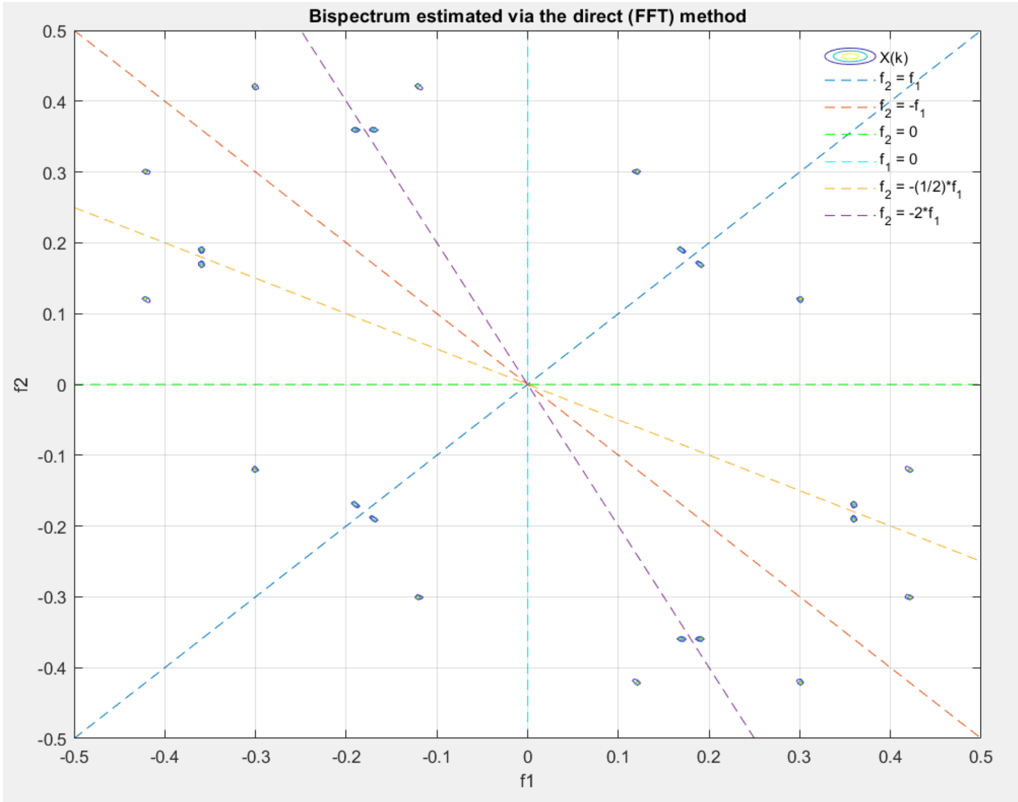


Plot 7 – Estimated Bispectrum plot / Indirect method – Parzen Window – K = 32, M = 256

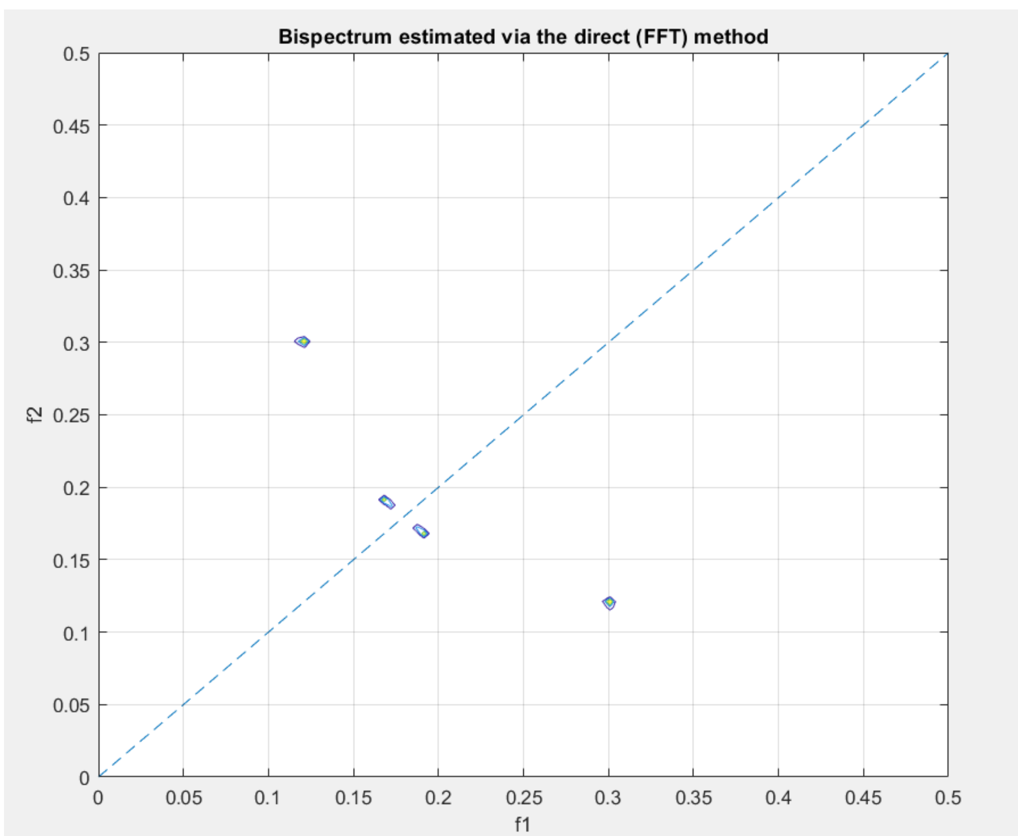


Plot 8 – Estimated Bispectrum plot / Indirect method – Parzen Window – K = 32, M = 256: Primary Area

b) using the direct method:



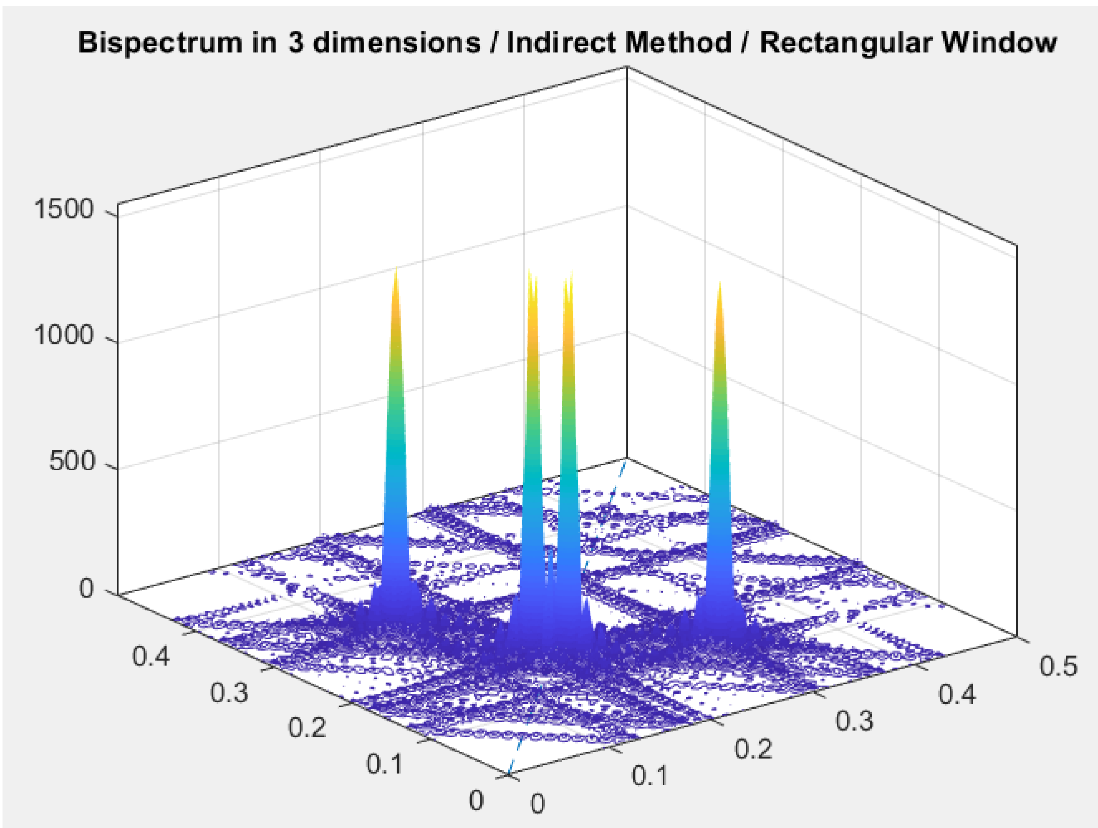
Plot 9 – Estimated Bispectrum plot / Direct Method – K = 32, M = 256



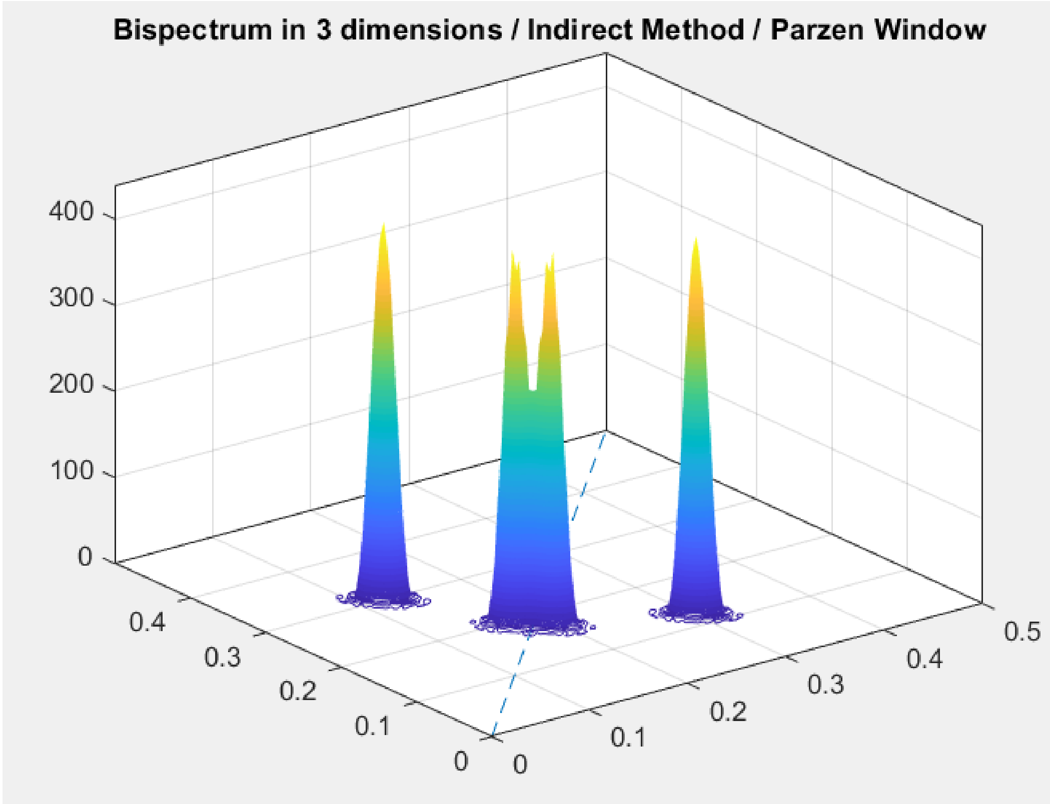
Plot 10 – Estimated Bispectrum plot / Direct Method – K = 32, M = 256: Primary Area

5. **<u>Comparison of the estimations</u>**

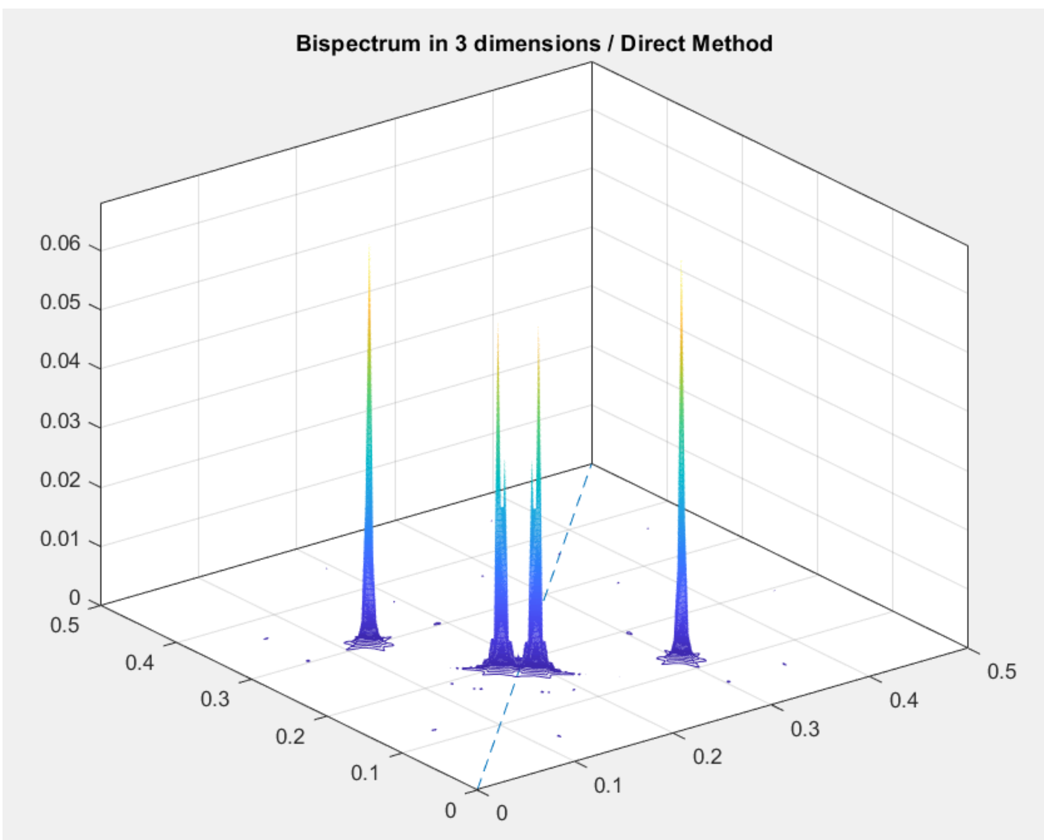
When using FT on cosine functions for infinite time, we get delta functions (estimated by a sinc function as its width gets smaller and smaller). Since time is not infinite in our case, we have a sinc function. Thus, the smaller the width of the sinc is, the better the estimation of delta is in the bispectrum, giving a more accurate estimation of the amplitude on these peaks (frequencies). By using different methods on the estimation of the bispectrum we see the changes on those peaks of the sinc function (that expresses the amplitude on the points of interest where the **quadratic phase coupling (QPC)** occurs), as well as changes in the variance. Thus, on plot 8 (Parzen window) we see that the area on which QPC occurs is wider than on plot 6 (Rectangular window), and that area is wider on plot 6 than on plot 10 (Direct method). We conclude that the results with the best bispectral resolutions are with the direct method. Even though Parzen window offers the largest main lobes among the methods, it also has the best variance of the estimator (among other windows). The following diagrams show the bispectrum in 3 dimensions:



Plot 11 – Bispectrum plot in 3 dimensions / Indirect Method / Rectangular Window / Primary Area



Plot 12 – Bispectrum plot in 3 dimensions / Indirect Method / Parzen Window / Primary Area



Plot 13 - Bispectrum plot in 3 dimensions / Direct Method

We can see that with the Parzen window we get zero values on the surface where we do not have peaks, while at the same time we get an estimation with wider peak (according to what was analyzed above regarding sinc function and delta estimation). Parzen window process "removes" some of the frequency information. On the other hand, bispectral resolution is better with the rectangular window, something we can see from how thin the peaks are (more accurate), but we see a "rougher" surface (spectral leakage). Also, while comparing the magnitude values of direct and indirect method, they seem to be significantly lower in the case of the direct method. Eventually, our choice of window depends on some quality measures of the estimator like variance, bispectral resolution and leakage effect.

6. The power spectrum only allows for a view of the amplitude of the frequencies, without giving information about nonlinear interactions among them. As we mentioned above, we can see on the bispectrum plots, that there are 2 points (in the primary area) where QPC occurs. Generally, three harmonics with frequencies f_k and phases φ_k, k = 1,2,3, are said to be quadratically phase coupled if $\lambda_3 = \lambda_1 + \lambda_2$ and $\varphi_3 = \varphi_1 + \varphi_2$. As we expected, those points are (approximately):

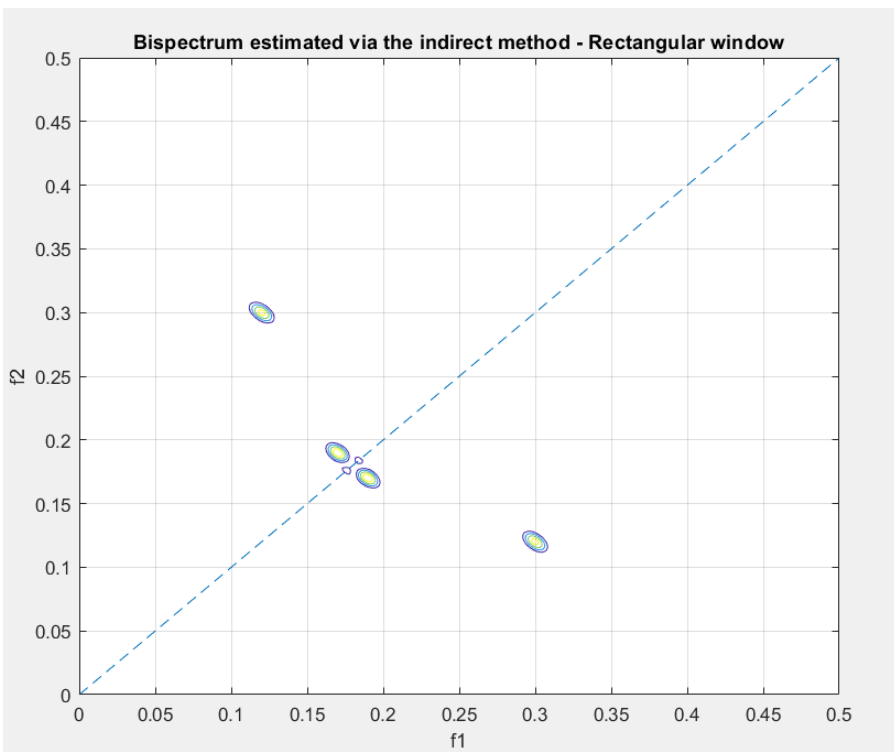
→ ~ (0.30,0.12) indicating the phase coupling on the components with these 2 frequencies.

→ ~ (0.19,0.17) indicating the phase coupling on the components with these 2 frequencies.

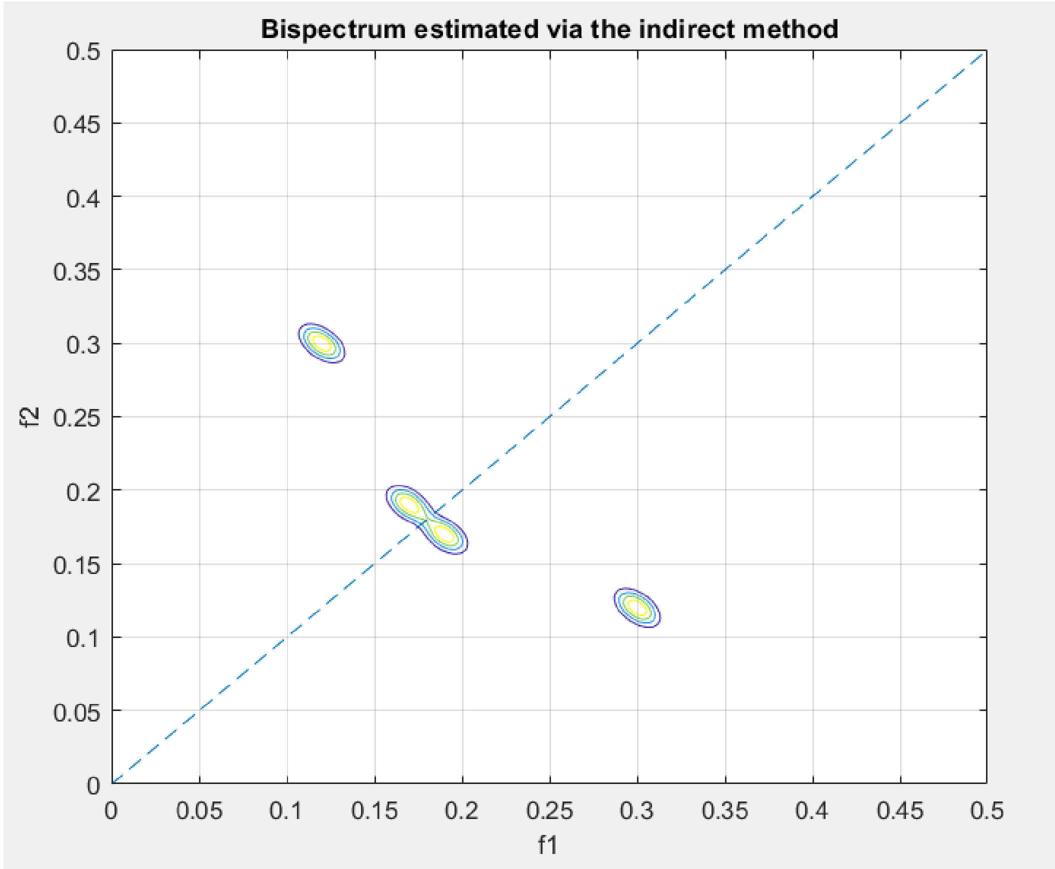
Thus, we see that the nonlinear interaction between harmonic components is the reason that QPC occurs, and the bispectrum is a great tool for the detection of this coupling.

7. The plots when changing the segment length:

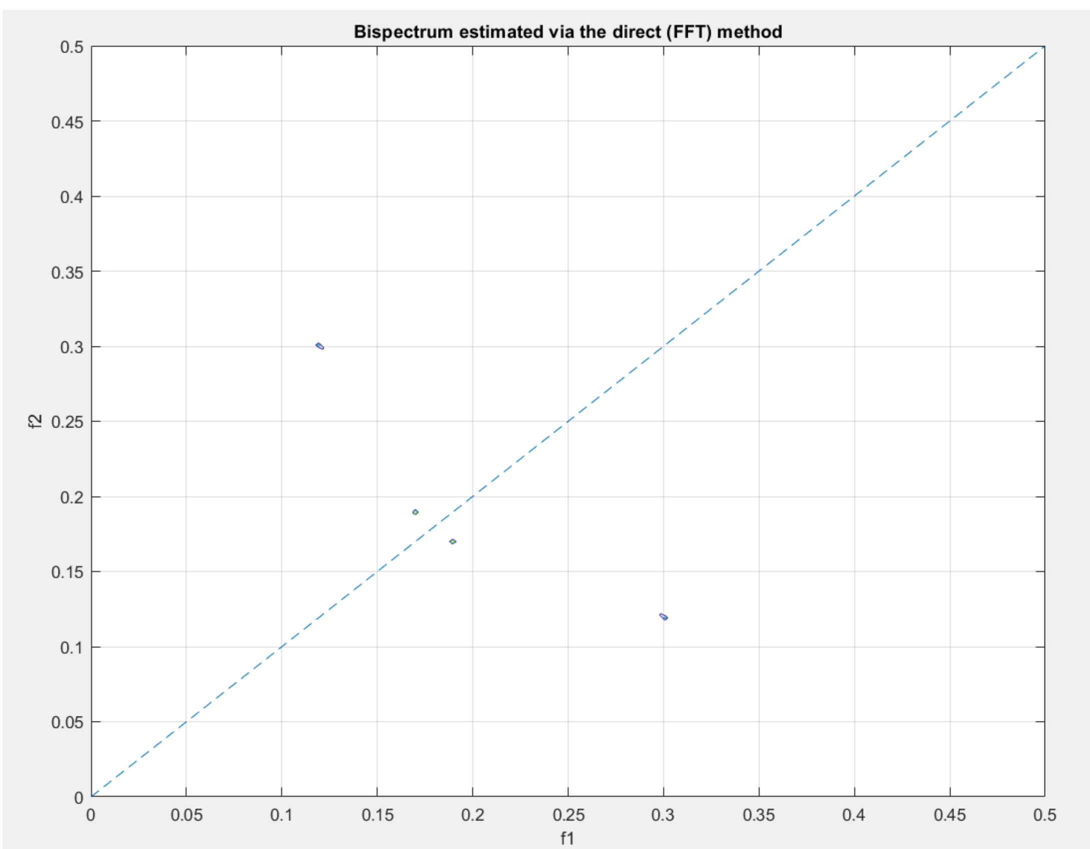
a_1)



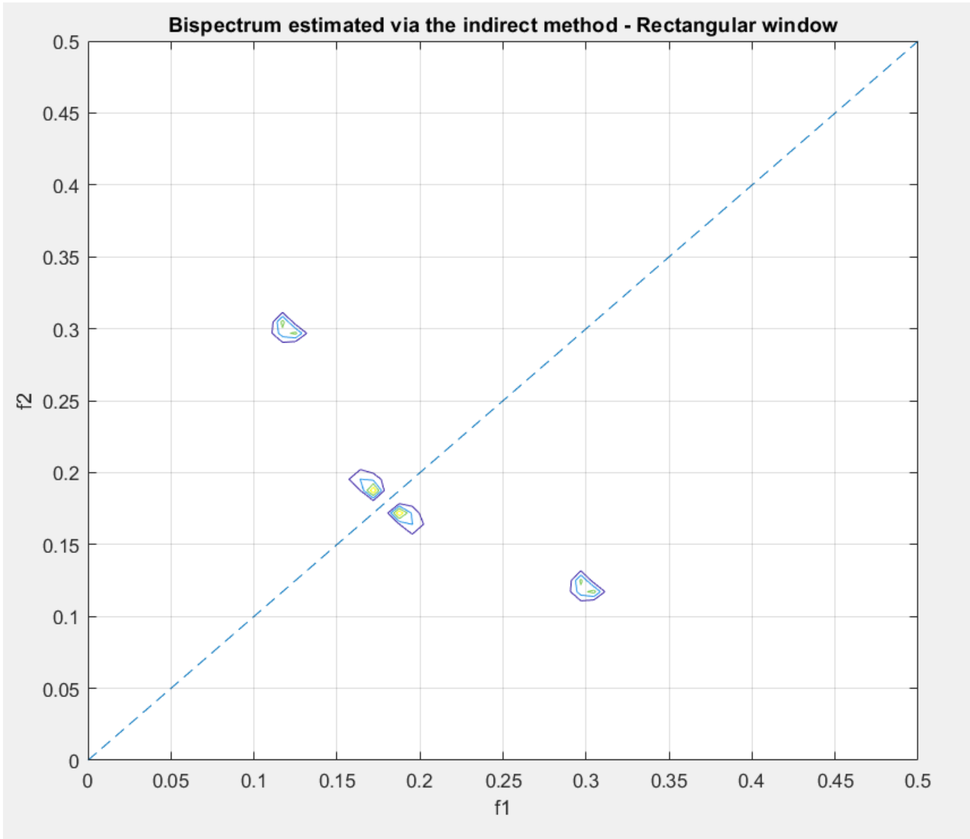
Plot 14– Estimated Bispectrum plot / Indirect method – Rectangular Window – K = 16, M = 512: Primary Area



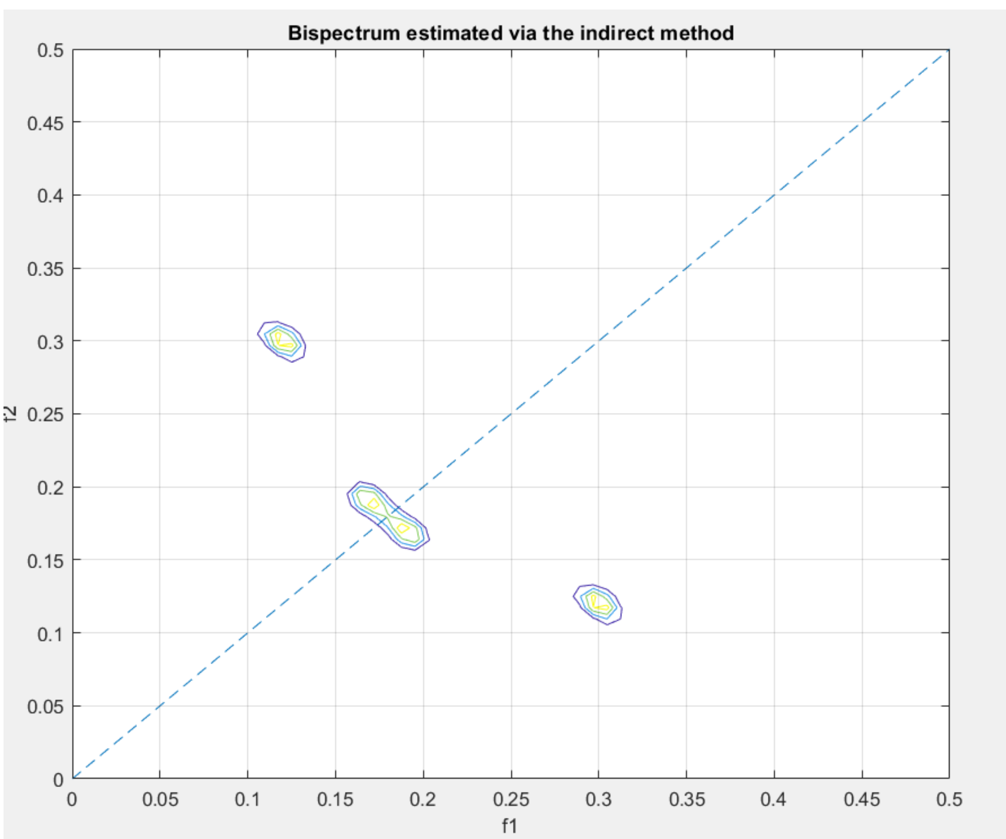
Plot 15 – Estimated Bispectrum plot / Indirect method – Parzen Window – K = 16, M = 512: Primary Area



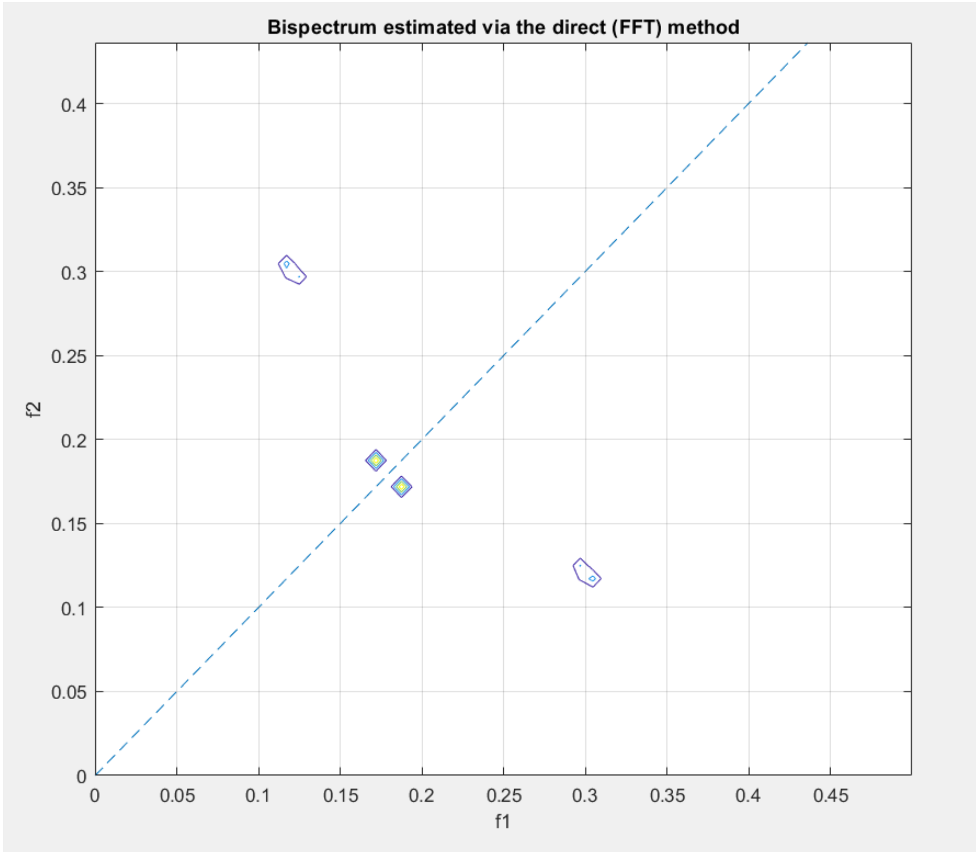
Plot 16 – Estimated Bispectrum plot / Direct method –K = 16, M = 512: Primary Area a$_2$)



Plot 17 – Estimated Bispectrum plot / Indirect method – Rectangular Window – K = 64, M = 128: Primary Area



Plot 18 – Estimated Bispectrum plot / Indirect method – Parzen Window – K = 64, M = 128: Primary Area



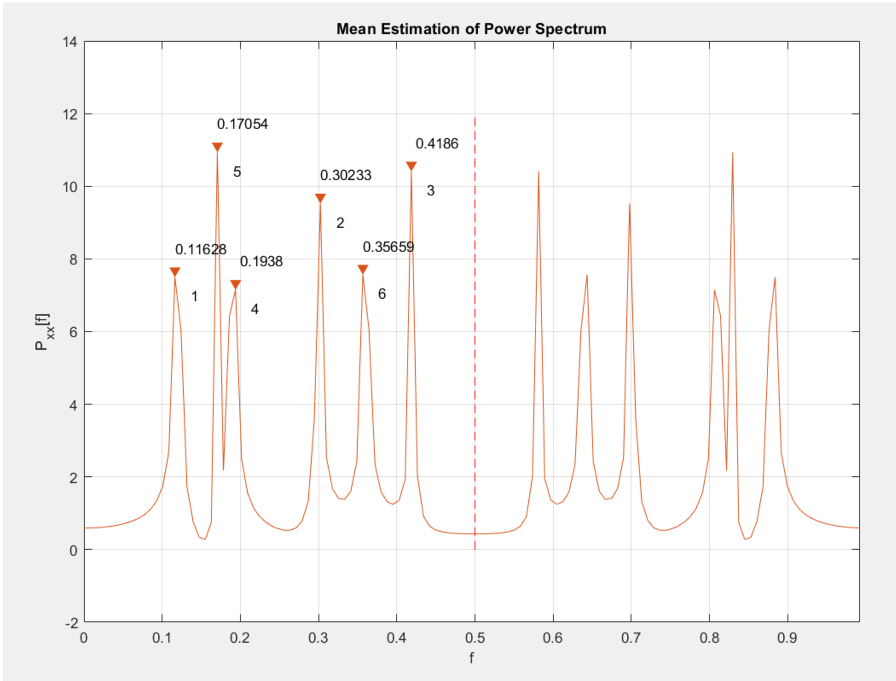
Plot 19 – Estimated Bispectrum plot / Direct method – K = 64, M = 128: Primary Area

As we can see from the plots of a_1, a_2 the change on the segment length changes the quality of the estimation. With larger segment length (a_1) we have better estimation, thus the frequencies are more concentrated on the expected points (where the QPC occurs).
Accordingly, with smaller segment length (a_2), we have more scattered values on the expected points.

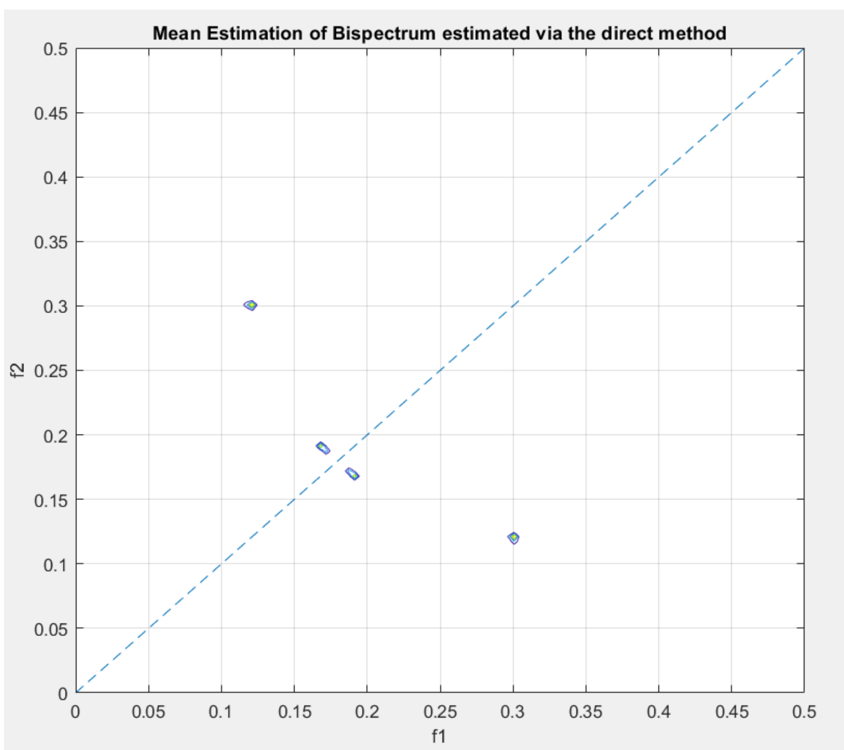
Clearly, the effect of the segment length is bigger when using the direct method than when using the indirect method.

[1] Conventional estimators are generally of high variance and therefore a large number of records (K) is required to obtain smooth bispectral estimates.

b)



Plot 20– Mean Estimation of Power Spectrum plot coming from 50 realization of X[k]– K = 32, M = 256



Plot 21 – Mean Estimation of Bispectrum plot coming from 50 realization of X[k] / Direct method – K = 32, M = 256: Primary Area

In this case, there is no significant difference between the results of the mean estimation and the results of our initial realization. The small difference that occurs comes from the stochastic (random) property of $\boldsymbol{X(k)}$. The fact that the randomness is not on the amplitude of the cosine but inside of it, restricts the variance to a small value resulting to realizations close to the expected.

<u>MATLAB code:</u>

<u>Note 1</u>: For the needs of the exercise, HOSA - Higher Order Spectral Analysis Toolbox for MATLAB was used.

<u>Note 2</u>: bispeciV2.m is a version of bispeci.m with deactivated window (rectangular window)

<u>Note 3:</u> We can also change the levels in our contour plots if we want see clearer effects on spectral leakage.

```
%% ADVANCED DIGITAL SIGNAL PROCESSING METHODS
% Assignment 2 - Summer Semester 2020/2021
% Kavelidis Frantzis Dimitrios - AEM 9351 - kavelids@ece.auth.gr - ECE AUTH

%% Clearing
clear all
close all
clc

%% Assigning values
% Lamda and Wmega
lamda1 = 0.12;
lamda2 = 0.3;
lamda3 = lamda1+lamda2;
lamda4 = 0.19;
lamda5 = 0.17;
lamda6 = lamda4+lamda5;
lamda = [lamda1 lamda2 lamda3 lamda4 lamda5 lamda6]';
wmega = NaN(6,1);
for i = 1:6
    wmega(i) = 2*pi*lamda(i);
end
% Data Length N:
N = 8192;                % = 2^13
% Universaly distributed random variables phi
phi1 = 2*pi*rand;
phi2 = 2*pi*rand;
phi3 = phi1+phi2;
phi4 = 2*pi*rand;
phi5 = 2*pi*rand;
phi6 = phi4+phi5;
phi = [phi1 phi2 phi3 phi4 phi5 phi6]';

%% 1. Constract the X[k].
X = zeros(N,1);
for k = 1:N              % It is 0:N-1 so we assume our index k on MATLAB is k+1
    for j = 1:6
        X(k) = X(k)+cos(wmega(j)*k+phi(j));
    end
end
% Plot our data
figure()
plot(X)
title("Presentation of Data X[k]")
ylabel("X[k]")
xlabel("k")

% Plot a smaller sample of our data for a clearer review.
figure()
plot(X(1:800))
title("Presentation of Data X[k] - Smaller Sample")
ylabel("X[k]")
xlabel("k")

%% 2. Estimate the power spectrum C2(f) using L2 = 128 max shiftings for autocorrelation
% Estimation of power spectrum using dsp toolbox and welch method
% SE = dsp.SpectrumEstimator;             % Creating SE object
% Pxx = SE(X);
% fvals = (0:length(Pxx)-1)/length(Pxx);
% plotter = dsp.ArrayPlot('XDataMode','Custom','CustomXData',fvals,...
%     'PlotType','Line','YLimits',[0 0.3], ...
%     'YLabel','Power Spectrum ','XLabel','Frequency (Hz)');
% plotter(Pxx)
% [pks1,locs1] = findpeaks(Pxx);
% lamdaFreqEst1 = locs1(1:length(locs1)/2)/length(Pxx)

% Estimation of power spectrum using autocorrelation and 128 shiftings
L2 = 128;                                          % max shiftings
acfX = autocorr(X,L2);                              % Autocorrelation
% Plot Autocorrelation
autocorr(X,L2)
% Power spectrum
Pxx = abs(fft(acfX));                              % Power Spectrum
```

```
% Getting the frequency axis
fvals = (0:length(Pxx)-1)/length(Pxx);
% Plot power spectrum
figure()
plot(fvals,Pxx)
title("Power Spectrum")
ylabel("P_x_x[f]")
xlabel("f")
line([0.5 0.5], [0 12],'Color','red','LineStyle','--')
hold on
[pks2,locs2] = findpeaks(Pxx,fvals);                      % Finding locations of peaks
findpeaks(Pxx,fvals,'NPeaks',6);
text(locs2(1:6)+.02,pks2(1:6)-.5,num2str([1;5;4;2;6;3]))            % Display number of peak
text(locs2(1:6),pks2(1:6)+.8,num2str(locs2(1:6)'))           % Display the frequencies of interest


%% 3. Estimate the bispectrum (only in the primary area) using:
%% a. The indirect method with K = 32 , M = 256 , L3 = 64 and
%% a1) Rectangular window a2) Parzen window
%% b. The direct method with K = 32 , M = 256 , J = 0
syms m
XeqY = m;
K = 32;
M = 256;
L3 = 64;


% 3.a.a1. --- Bispectrum / Indirect Method / Rectangular Window
% Biscpetrum estimation using the indirect method:

%%% For the whole plot, uncomment/turn on bspecPlotInfo2 and comment/turn
%%% off bspecPlotInfo
%%% And for the primary area, comment/turn off bspecPlotInfo and
%%% uncomment/turn on bspecPlotInfo2


figure()
[BspecIn1, waxisIn1] = bispeciV2(X,L3,M,0,'unbiased');      % Rectangular Window
% bspecPlotInfo
bspecPlotInfo2
figure()
contour3(waxisIn1,waxisIn1,abs(BspecIn1),250), grid on
title('Bispectrum in 3 dimensions / Indirect Method / Rectangular Window')
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% 3.a.a2. --- Bispectrum / Indirect Method / Parzen Window
figure()
[BspecIn2, waxisIn2] = bispeci(X,L3,M,0,'unbiased');        % Parzen window
% bspecPlotInfo
bspecPlotInfo2
figure()
contour3(waxisIn2,waxisIn2,abs(BspecIn2),250), grid on
title('Bispectrum in 3 dimensions / Indirect Method / Parzen Window')
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% 3.b --- Bispectrum / Direct Method
figure()
[BspecD, waxisD] = bispecd(X,M,1,M,0);
% bspecPlotInfo
bspecPlotInfo2
figure()
contour3(waxisD,waxisD,abs(BspecD),250), grid on
title('Bispectrum in 3 dimensions / Direct Method')
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum using direct method (different function)
figure()
[bisp,freq,cum,lag] = bisp3cum(X,M,L3,'none','u');



%% 7. How the results change if you repeat the process taking into account
%% a) i) K = 16, M = 512 ii)K = 64, M = 128?
%% b) 50 realizations of the X[k] and comparing mean values of the estimated C2,C3
% 7.a.i)
Ka1 = 16;
Ma1 = 512;

% Bispectrum estimation using indirect method & rectangular window
figure()
[BspecIn1_7a1, waxisIn1_7a1] = bispeciV2(X,L3,Ma1,0,'unbiased');      % Rectangular Window
% bspecPlotInfo
bspecPlotInfo2
```

```
% ------------------------------------------------------------

% Bispectrum estimation using indirect method & Parzen window
figure()
[BspecIn2_7a1, waxisIn2_7a1] = bispeci(X,L3,Ma1,0,'unbiased');        % Parzen window
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum using direct method
figure()
[BspecD_7a1, waxisD_7a1] = bispecd(X,Ma1,1,Ma1,0);
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum using direct method (different function)
figure()
[bisp_7a1,freq,cum,lag] = bisp3cum(X,Ma1,L3,'none','u');

% ------------------------------------------------------------
% ------------------------------------------------------------


% 7.a.ii)
Ka2 = 64;
Ma2 = 128;

% Bispectrum estimation using inverse method & rectangular window
figure()
[BspecIn1_7a2, waxisIn1_7a2] = bispeciV2(X,L3,Ma2,0,'unbiased');      % Rectangular Window
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum estimation using inverse method & Parzen window
figure()
[BspecIn2_7a2, waxisIn2_7a2] = bispeci(X,L3,Ma2,0,'unbiased');        % Parzen window
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum using direct method
figure()
[BspecD_7a2, waxisD_7a2] = bispecd(X,Ma2,1,Ma2,0);
% bspecPlotInfo
bspecPlotInfo2

% ------------------------------------------------------------

% Bispectrum using direct method (different function)
figure()
[bisp_7a2,freq,cum,lag] = bisp3cum(X,Ma2,L3,'none','u');

% ------------------------------------------------------------
% ------------------------------------------------------------

% 7.b)
meanC2_X = zeros(length(Pxx),1);
meanC3_X = zeros(length(BspecIn2),length(BspecIn2));
Xvecs = zeros(N,50);

% Taking 50 realization of the X[k]
for i = 1:50
    X7 = zeros(N,1);
    phi1 = 2*pi*rand;
    phi2 = 2*pi*rand;
    phi3 = phi1+phi2;
    phi4 = 2*pi*rand;
    phi5 = 2*pi*rand;
    phi6 = phi4+phi5;
    phi = [phi1 phi2 phi3 phi4 phi5 phi6]';
    for k = 1:N             % It is 0:N-1 so we assume our index k on MATLAB is k+1
        for j = 1:6
            X7(k) = X7(k)+cos(wmega(j)*k+phi(j));
        end
    end
    Xvecs(:,i) = X7;
end

% Estimating C2 ,C3
for i = 1:50
    acfX7 = autocorr(Xvecs(:,i),L2);                                % Autocorrelation
    Pxx7 = abs(fft(acfX7));                                 % Power Spectrum
    meanC2_X = meanC2_X + Pxx7;
    [BspecIn2_7, waxisIn2_7] = bispecd(Xvecs(:,i),L3,1,M,0);      % Rectangular Window
    meanC3_X = meanC3_X + BspecIn2_7;
```

```matlab
end
meanC2_X = meanC2_X./50;
meanC3_X = meanC3_X./50;

nfft = 256;
if (rem(nfft,2) == 0)
        waxis = [-nfft/2:(nfft/2-1)]/nfft;
    else
        waxis = [-(nfft-1)/2:(nfft-1)/2]/nfft;
end

% Plot Mean Estimation of Power Spectrum
% Getting the frequency axis
fvals_X = (0:length(meanC2_X)-1)/length(meanC2_X);
% Plot power spectrum
figure()
plot(fvals_X,meanC2_X)
title("Mean Estimation of Power Spectrum")
ylabel("P_x_x[f]")
xlabel("f")
line([0.5 0.5], [0 12],'Color','red','LineStyle','--')
hold on
[pks2_X,locs2_X] = findpeaks(meanC2_X,fvals_X);                      % Finding locations of peaks
findpeaks(meanC2_X,fvals_X,'NPeaks',6);
text(locs2_X(1:6)+.02,pks2_X(1:6)-.5,num2str([1;5;4;2;6;3]))            % Display number of peak
text(locs2_X(1:6),pks2_X(1:6)+.8,num2str(locs2_X(1:6)'))           % Display the frequencies of interest

% Plot Mean Estimation of Bispectrum
figure()
contour(waxis,waxis,abs(meanC3_X),4), grid on
title('Mean Estimation of Bispectrum estimated via the direct method')
xlabel('f1'), ylabel('f2')
set(gcf,'Name','Hosa BISPECI')

% bspecPlotInfo
bspecPlotInfo2
%% ------------------------ End of Assignment 2 ------------------------
```

References:

[1] Bispectrum estimation: A digital signal processing framework, C.L. Nikias, vol. 75, no. 7, pp. 880 ,1987