# Mixed Integer Programming exercises

**Exercise 1. Modeling and numerical solution.** We give a list of 40 people which must participate to a game. They must be divided in 5 groups of 8 people. For each pair of people $(i, j)$, we have a value of

- 0 if they don't know each other
- 1 if they know each other a bit
- 2 if they know each other well.

We want to create groups so that people know as few people as possible in each group.

1. Formulate this problem by choosing a plausible objective function.
2. Solve it with Pyomo and CBC or Gurobi.

**Exercise 2. Modeling and numerical solution.** A steel factory produce spools. We want to plan the production for the next 12 weeks. We know exactly the demand in spools for the next 12 weeks. The cost for using the furnace can be divided in two parts : a fixed cost to get the furnace turned on and a variable cost proportional to the number of spools produced. In order to store spools from a period to another, a cost proportional to the number of spools must be paid.

The data for each weak are given by :

| Weak | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand (10 spools) | 7 | 5 | 3 | 5 | 5 | 9 | 1 | 8 | 5 | 6 | 2 | 2 |
| Variable cost (1000 €/10 spools) | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Fixed cost (1000 €) | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| Storage (1000 €/10 spools) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

(a) Formulate the problem of determining a production plan while minimizing the costs.

(b) Reformulate the production planning problem if we add the following constraint : If we start the blast-furnace, it must stay active at least 3 and at most 4 consecutive periods.

(c) Solve it with Pyomo and CBC or Gurobi.

**Exercise 3.** Consider the enumeration tree (minimization problem) in Figure 1 :

(a) Give tightest possible lower and upper bounds on the optimal value $z$.

(b) Which nodes can be pruned and which must be explored further?

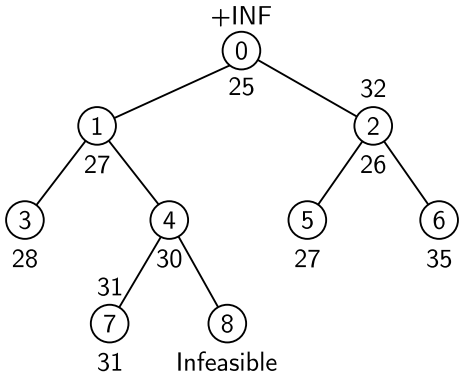


Figure 1: Enumeration tree (min)

**Exercise 4.** Solve geometrically by branch-and-bound

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 && (1)\\ s.t. \quad & -\tfrac{2}{3}x_1 + x_2 \leq \frac{5}{2} && (2)\\ & \tfrac{1}{3}x_1 + x_2 \leq \frac{9}{2} && (3)\\ & 2x_1 + x_2 \leq 14 && (4) \end{aligned}$$

**Exercise 5.** Find a valid inequality for the following sets :

- $X = \{x \in \{0,1\}^2 : 12x_1 - 6x_2 \leq 7\}$
- $X = \{(x,y) \in \mathbb{R}^1_+ \times \{0,1\} : x \leq 20y, x \leq 7\}$
- $X = \{(x,y) \in \mathbb{R}^1_+ \times \mathbb{Z}^3_+ : -x - \frac{10}{3}y_1 + y_2 + \frac{11}{4}y_3 \leq \frac{21}{2}\}$
- $X = \{(x,y) \in \mathbb{R}^1_+ \times \mathbb{Z}^1_+ : x \leq 6y, x \leq 16\}$.