1-bit A/D and D/A Converters

Disclaimer: *This document is not under frequent updating and the author cannot always answer queries regarding specific details presented*

This document is based on the theory introduced in the book:

K.C. Pohlmann, *Principles of Digital Audio*, 3rd ed., McGraw-Hill, 1995

The Pictures and Figures are based on the Matlab simulations I carried out inspired by the discussion about 1-bit converters in news group sfnet.harrastus.audio+video. You should also check out the FAQ of this group for more general Finnish information.

Delta Modulation

Delta modulation (DM) was developed in the 1940s for voice telephony applications. Differential pulse-code modulation is a technique in which the derivative of the signal is quantized. When signal changes between sample periods are small, the quantizer's word length can be reduced. With very high oversampling rates, the changes between sample periods are made very small, thus the quantizer can be reduced to low-bit. A 1-bit DPCM coder is known as a delta modulator (DM). In other words, DM codes the differences in the signal amplitude instead of the signal amplitude itself. Yet another name for DM is pulse width modulation (PWM).

A delta-modulation encoder is shown in Figure 1; it is known as a single integration modulator. The input signal is compared to the integrated output pulses and the delta (difference) signal is applied to the quantizer. The quantizer generates a positive pulse when the difference signal is negative, and a negative pulse when the difference signal is positive. This difference signal moves the integrator step by step closer to the present value input, tracking the derivative of the input signal.

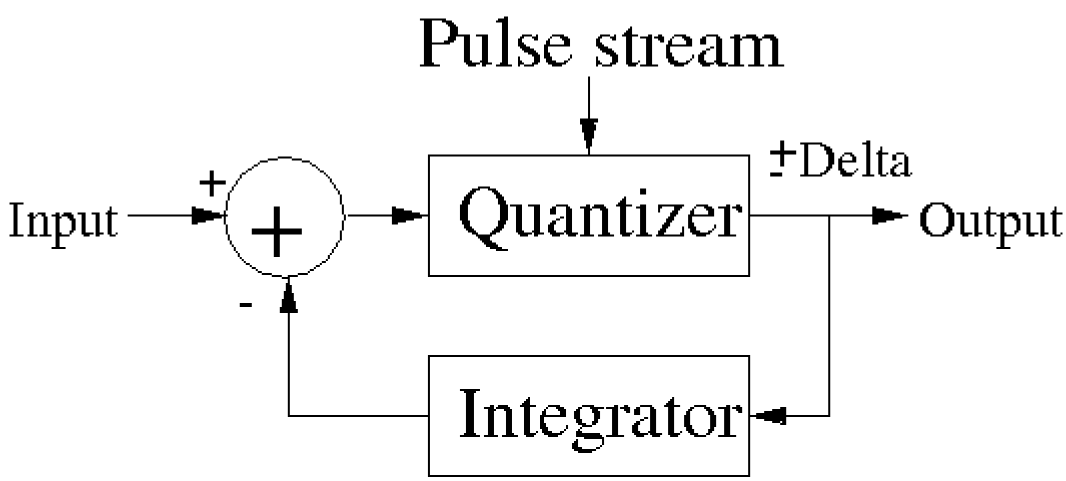

Figure 1. Delta modulation encoder.

For example if we consider 1.5 kHz sinusoidal input signal with maximum amplitude 1 and delta is chosen to be 0.125 which is equivalent to 4 bit quantization i.e. 16 quantization levels. To achieve a resolution equivalent to 4 bit quantization with 4 kHz sampling rate an oversampling ratio of 16 is needed i.e.

(4^2)/(1^2)*4kHz=64 kHz. In Figure 2, 32 times oversampling is used and the output of the integrator tracks nicely the input signal.

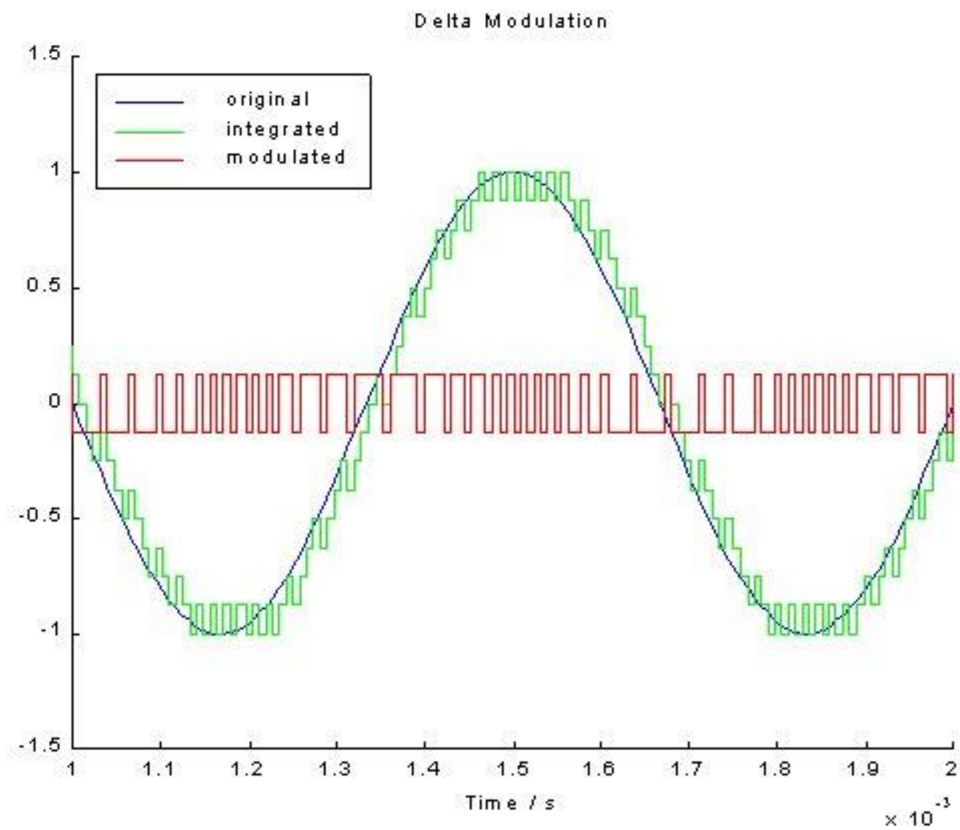

Figure 2. 32 times oversampled DM signal.

We can easily see that a delta modulation decoder has to integrate the modulated signal and lowpass filter the output of the integrator. In Figure 3, 16 times oversampling is used and the output of the integrator hardly tracks the input signal.

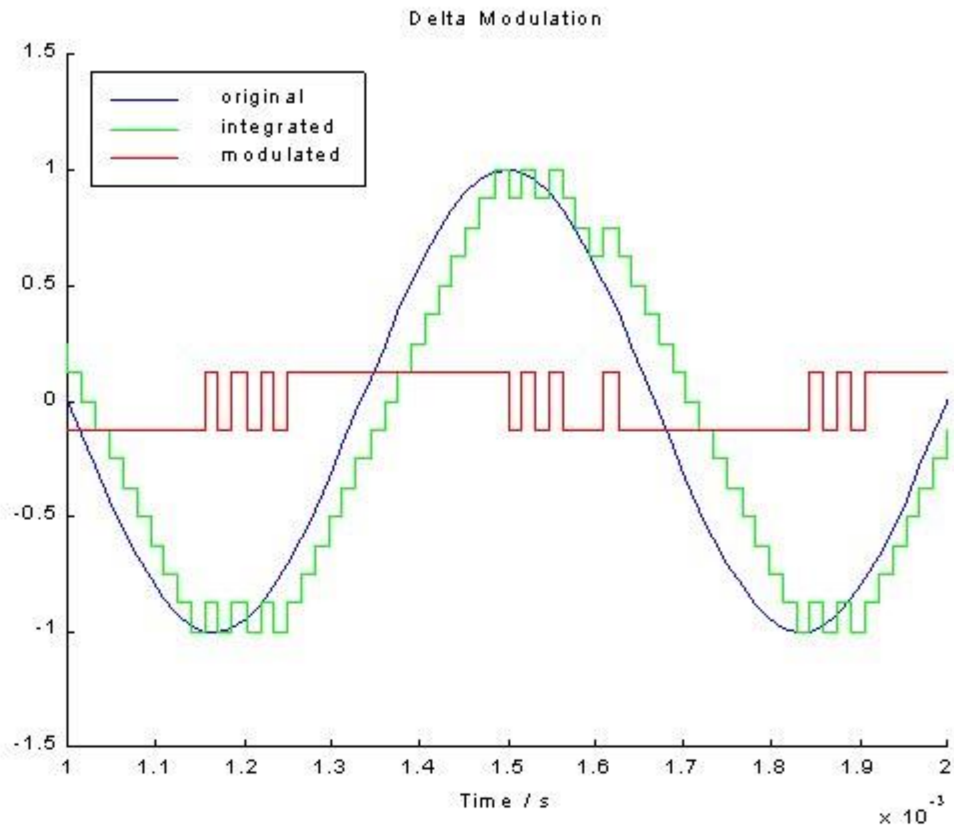

Figure 3. 16 times oversampled DM signal.

If the size of delta or the oversampling ratio is too low, a slope overload occurs. In Figure 4 delta is chosen to be half the previous which is equivalent to 5 bit quantization i.e. 32 levels. Now the oversampling ratio should be 32 but in the Figure 4 it is 16 and slope overload is inevitable.

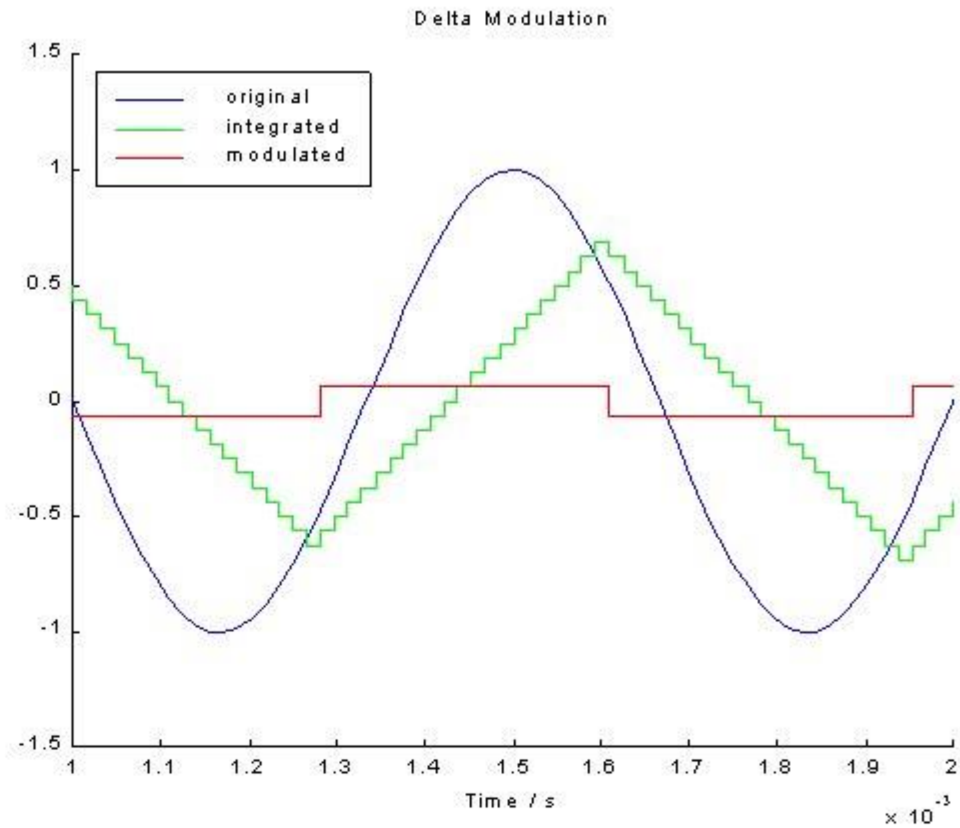

Figure 4. Slope overload.

To convert a maximum amplitude 16-bit word, a 1-bit modulator would have to perform 2^{16} toggles per conversion period; with a sampling frequency of 44.1 kHz, this would demand a toggle rate to approximately 2.9 GHz, an impossibility with today's technology. As the rate is slowed to accomodate hardware limitations, noise levels increase to intolerable level. Looked at in another way, bit reduction at a high sampling frequency is required to output a low-bit signal from a high-bit source; this greatly degrades the signal's dynamic range. The quantization step delta can also be increased in order to lower the sampling rate but the quantization noise will become more apparent due to its uniform distribution in the frequency band from dc to half the sampling rate.

Sigma-Delta Modulation

Sigma-delta modulation (SDM) was developed in 1960s to overcome the limitations of delta modulation. Sigma-delta systems quantize the delta (difference) between the current signal and the sigma (sum) of the previous difference. An integrator is placed at the input to the quantizer; signal amplitude is constant with increasing frequency; thus SDM is also known as pulse density modulation (PDM). Like PCM, SDM quantizes the signal directly, and not its derivative as in DM. Thus the maximum quantizer range is determined by the maximum signal amplitude and is not dependent on signal spectrum. As with other low-bit coders, to achieve high resolution, high sampling rates are required; for example, with an audio band 22,1 kHz and 64 times oversampling, the internal sampling frequency rises to 2.8224 MHz, thus quantization noise is spread from dc to 1.4112 MHz. However, sigma-delta modulation adds noise-shaping benefits.

A first-order (single integration) sigma-delta modulation encoder is shown in Figure 5. The input to the quantizer is the integral of the difference between the input and the quantized output. The difference between the input signal and the output signal approaches zero; the average value of the clocked output tracks the input. There is little dc error in the output signal; the frequency spectrum of the quantizing error rises with increasing frequency (6 dB/octave). The integrator forms a lowpass filter on the difference signal thus providing low frequency feedback around the quantizer. This feedback results in a reduction of quantization noise at low (in-band) frequencies. Unlike PCM and DM, the noise is not white, but shaped by a first-order highpass characteristic seen in pictures below. In practise, the in-band noise floor level is not satisfactory with first-order SDM. Further noise shaping must be achieved with higher-order (multiple integration) sigma-delta modulation coders.

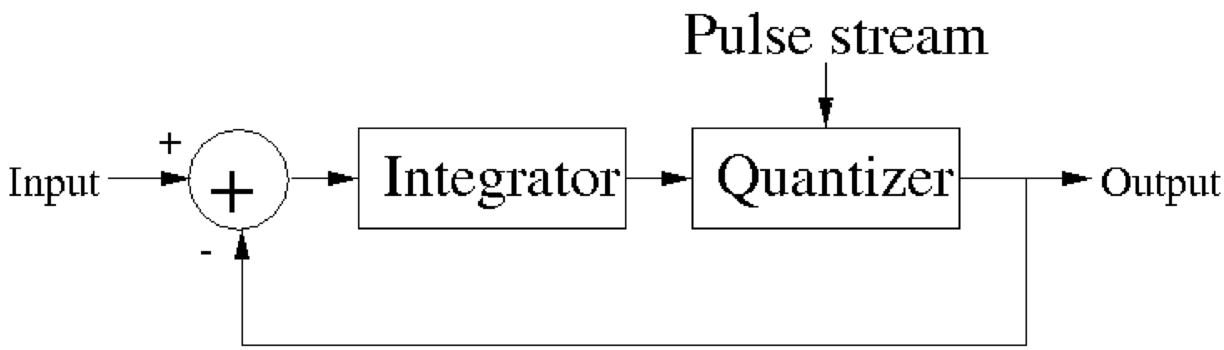

Figure 5. Sigma-delta modulation encoder.

Now we use the same input signal as in DM examples with amplitude 0.9. In Figure 6 is a SDM signal with 16 times oversampling.

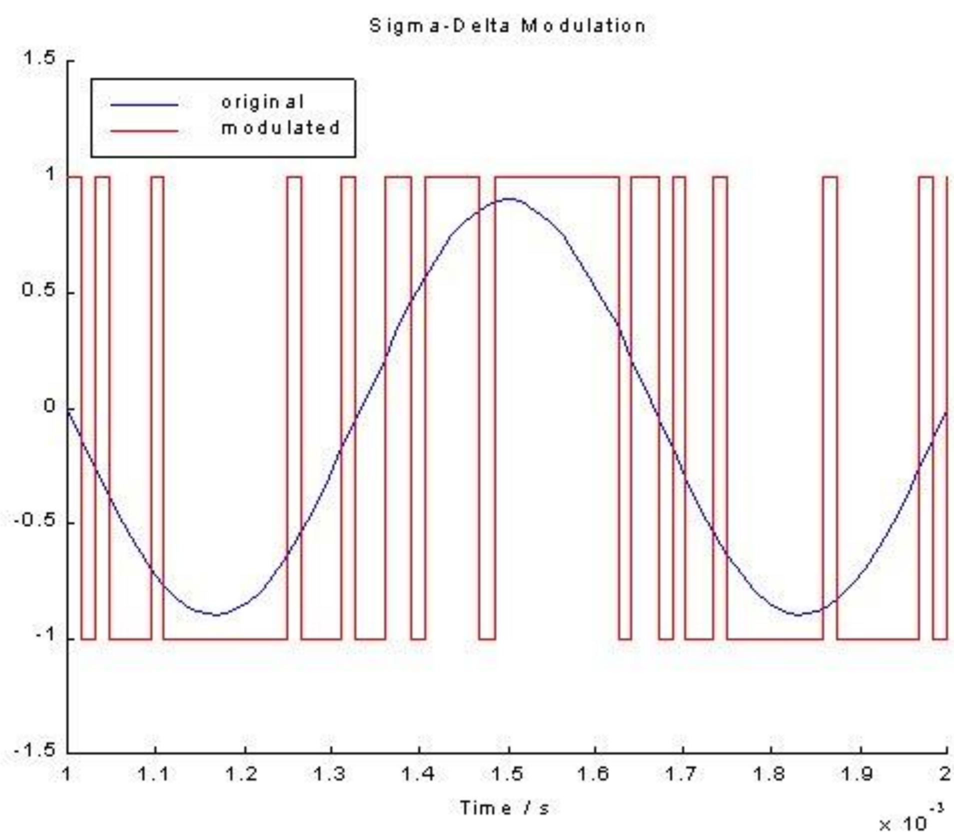

Figure 6. 16 times oversampled SDM signal.

In Figure 7 is the spectrum of the previous modulated signal. It can be seen that in the band of interest i.e. dc to 2 kHz the noise floor is lower than in higher frequencies.

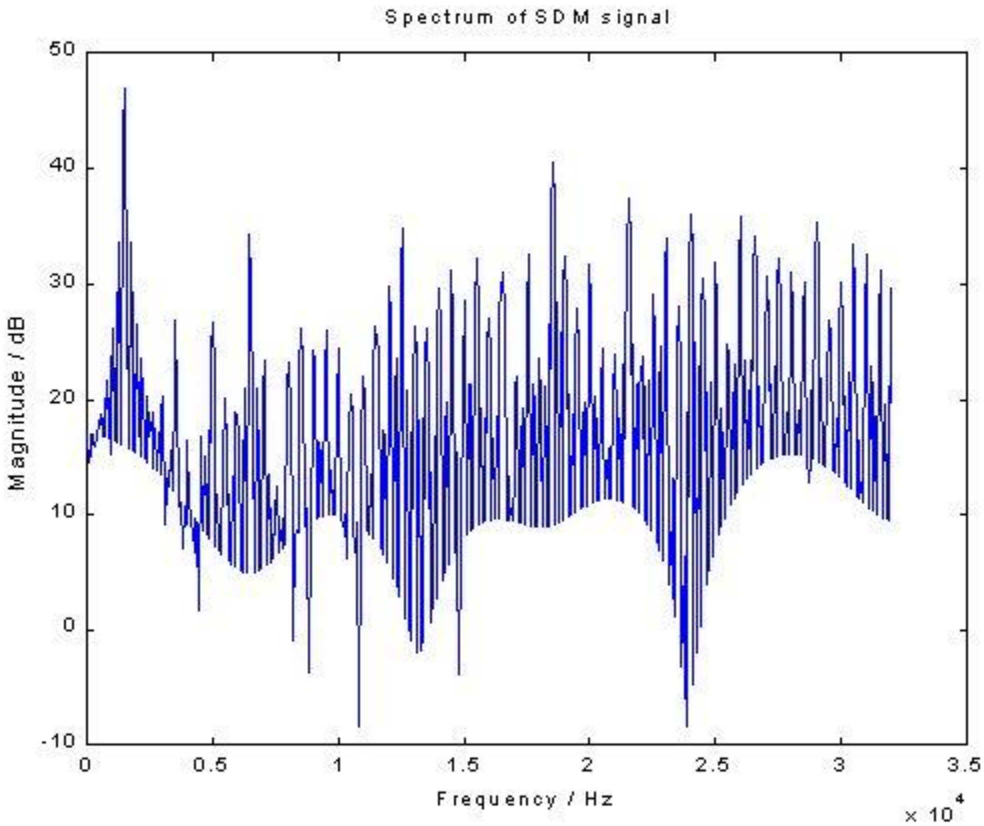

Figure 7. Spectrum of the previous SDM signal.

The roughness of Figure 7 is due to computational limits when dealing with short input signals. In Figure 8 is a SDM signal with 64 times oversampling.

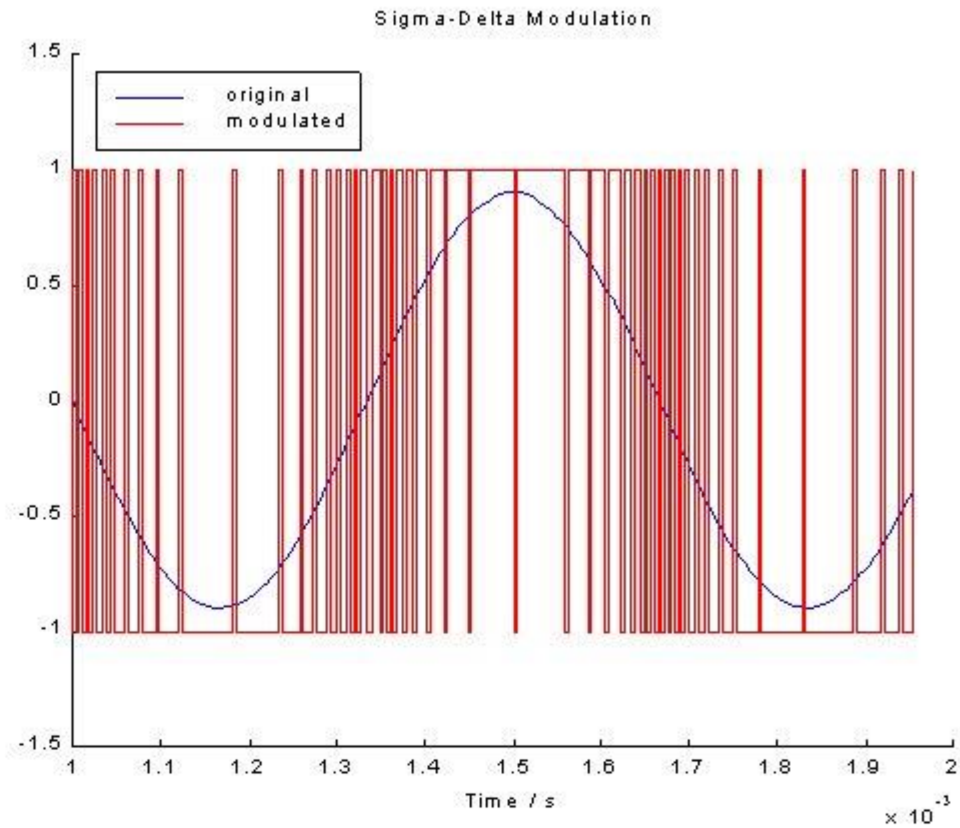

Figure 8. 64 times oversampled SDM signal.

In Figure 9 we see how the noise floor decreases as a result of higher sampling rate and the noise-shaping features together.

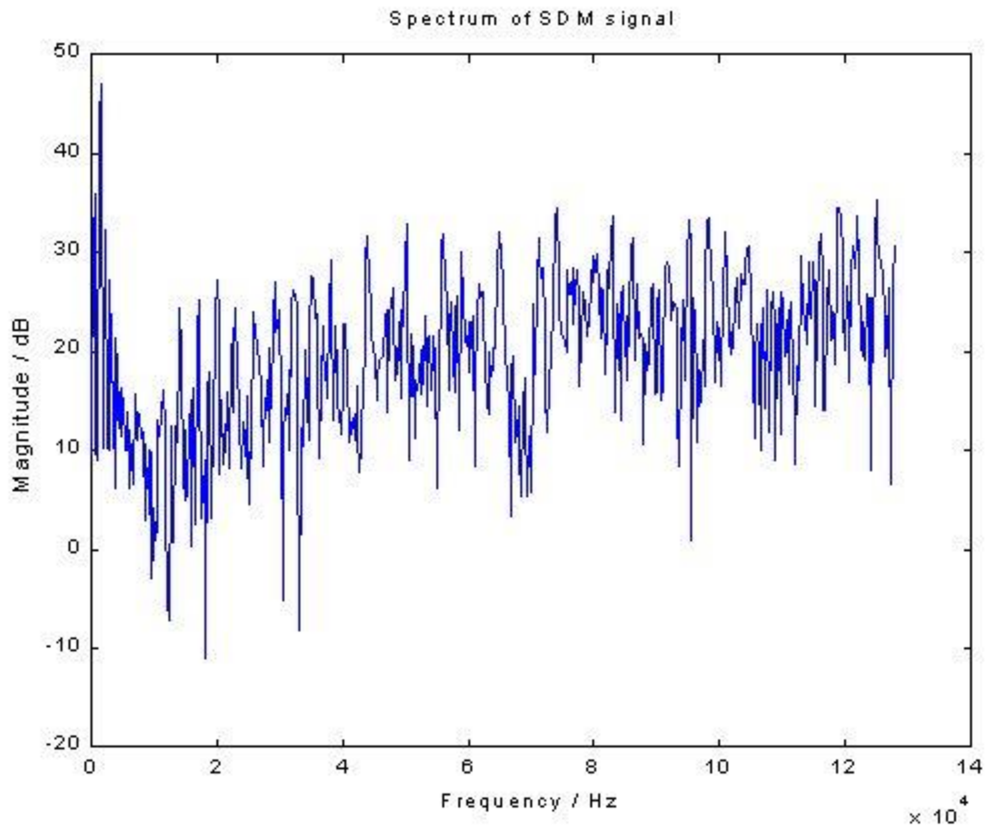

Figure 9. Spectrum of the previous SDM signal.

The previous spectra imply that the SDM decoder is only a lowpass filter. In Figure 9 the noise floor minimum is at frequencies near 10 kHz, which is much higher than the band of interest (dc to 2 kHz). This suggests that with higher-order noise-shaping and adequate oversampling the requirements of the filters transition band get looser than the brick-wall characteristics needed with Nyquist-rate sampled signals.

It might sound unintuitive that the same audio quality can be achieved with lower bit rate. Although it can be noted that the quantization only adds noise to the quantized signal and the quality depends on the distribution of this noise in the audio frequency band. With sampling frequencies large enough the in-band noise can be shaped out of the audio frequency band and the same or even better noise floor can be achieved than with strict Nyquist-rate sampling. Nowadays SDM is used in the DA converters of many CD players. The 16-bit 44.1 kHz data is requantized and oversampled with SDM encoder and converted to analog line level signal with SDM decoder. 1-bit converter is in some cases easier and cheaper to implement and the most important benefit is its better tolerance for small variances in component values compared to the high demands of the conventional weighted-resistor and R-2R ladder high-bit DACs. However neither of these conversion methods has proven superior compared to another.