

## Symbol Index

This index lists symbols used frequently in the text, followed by the sections in which they first appear.

$ \alpha $	Absolute value, 5.1.
$A^*$	Adjoint of the operator $A$ , 3.7.
$\text{Ad } A, \text{Ad } \alpha$	Adjoint rep operators, 5.1, 5.6.
$B_\alpha$	Algebra of permutation operators, 4.3.
$a, a^*$	Annihilation and creation operators, 9.7.
$A(G)$	Automorphism group of $G$ , 1.3.
$\mathfrak{g}^+, \mathfrak{g}^-, \mathfrak{g}^3$ or $J^+, J^-, J^3$	Basis for a rep of $sl(2)$ , 5.10.
$\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3$	Basis for $so(3)$ , 7.1.
$J_n(z) = [(z/2)^n/n!] \sum_{k=0}^{\infty} [(-z^2/4)^k/(n+1)_k k!]$ , $n = 0, 1, \dots$	Bessel function, 10.2.
$A_\alpha$	Bisymmetric transformations, set of, 4.3.
$\varepsilon_{kl}, \varepsilon_\alpha$	Branch, 9.1.
$\Gamma_n, \Gamma_n^m$	Bravais lattices, 2.8.
$h_m$	Cartan subalgebra of a classical group, 9.1.
$\chi(g), g \in G$	Character of the group $G$ , 3.4.
$C_\infty, U(1), SO(2)$	Circle group, 2.3.
CG	Clebsch–Gordan, 3.5.
$(\mu i, \nu j   \xi sl)$	Clebsch–Gordan (CG) coefficients, 3.5.
$C(u, m; v, n   w, k)$	Clebsch–Gordan coefficients for $SU(2)$ , 7.7.
$\alpha_j$	Clifford algebra, elements of, 9.6.
$C(T)$	Column permutations of a Young tableau, 4.2.
$[A, B], [\alpha, \beta]$	Commutator bracket, 5.1, 5.3.
$\mathbb{C}$	Complex numbers, 1.1.
$\bar{A} = (\bar{A}_{ij})$	Complex conjugate matrix, 3.1.
$\mathcal{C}_m^n$	Conjugacy class in a point group, 3.6.



- $\mathbf{u} \times \mathbf{v}$  Cross product, 3.8.  
 $C_n$  Cyclic group of order  $n$ , 2.4.  
 $C_{\infty v}, C_{\infty h}$  Symmetry groups, 2.9, 7.6.  
 $\Delta(\epsilon_1, \dots, \epsilon_m) = \prod_{j>k} (\epsilon_j - \epsilon_k)$ , 9.2.  
 $f'(x), \dot{f}(t)$  Derivative of a function, 5.1.  
 $\dot{A}(t)$  Derivative of a matrix-valued function, 5.1.  
 $\det A$  Determinant of matrix  $A$ , 2.1.  
 $|\epsilon^{l_1}, \dots, \epsilon^{l_m}| = \sum_{s \in S_m} \delta_s \epsilon_{s(1)}^{l_1} \cdots \epsilon_{s(m)}^{l_m}$ , Determinant, 4.4, 9.2.  
 $D_n$  Dihedral group, 2.4.  
 $\dim V$  Dimension of vector space  $V$ , 3.2.  
 $G \times H$  Direct product of groups, 1.5.  
 $T_1 \oplus T_2$  Direct sum of reps  $T_1$  and  $T_2$ , 3.2, 3.5.  
 $V \oplus W$  Direct sum of vector spaces  $V$  and  $W$ , 3.2, 3.5.  
 $D_{\infty h}$  Symmetry group, 2.9.  
 $T \cong T'$  Equivalence of reps, 3.1.  
 $E(n), (E^+(n))$  Euclidean group in  $n$ -space (proper Euclidean group), 2.2.  
 $\{a, O\}$  Euclidean group element, 2.2.  
 $(\varphi, \theta, \psi)$  Euler angles, 7.1.  
 $\exp \alpha: \mathfrak{g} \rightarrow G$  Exponential mapping, 5.5, 5.9.  
 $\exp A = e^A = \sum_{j=0}^{\infty} A^j/j!$  Exponential of a square matrix, 5.1.  
 $G/N$  Factor group, 1.2.  
 $\Gamma(z) = \lim_{n \rightarrow \infty} [n! n^z / (z)_{n+1}]$ ,  $\Gamma(z+1) = z\Gamma(z)$ ,  $\Gamma(n+1) = n!$ ,  $n = 0, 1, 2, \dots$ , Gamma function, 7.5.  
 $GL(n, \mathbb{C}), GL(n, R), GL(n)$  General linear groups, 1.1, 5.4.  
 $G$  Group, 1.1.  
 $R_G$  Group algebra (ring) of the group  $G$ , 3.1, 3.3.  
 $x = \sum_{g \in G} x(g) \cdot g$  Group algebra element, 3.1.  
 $\varphi(g, h)$  Group product in local Lie group, 5.2.  
 $T(g), T(A)$  Group rep matrices, 3.1.  
 $T(g), T(A)$  Group rep operators, 3.1.  
 $H$  Hamiltonian operator, 3.8.  
 $H_n(x) = (-1)^n \exp(x^2) (d^n/dx^n) \exp(-x^2)$ ,  $n = 0, 1, 2, \dots$ , Hermite polynomials, 10.1.  
 $\mathcal{H}$  Hilbert space, 3.8, Appendix.  
 $\mu: G \rightarrow G'$  Homomorphism of groups, 1.3.  
 $\tau: \mathfrak{g} \rightarrow \mathfrak{g}'$  Homomorphism of Lie algebras, 5.3.  
 ${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} [(a)_n (b)_n / (c)_n] (z^n/n!)$ ,  $|z| < 1$ , Hypergeometric series (see Pochhammer symbol), 7.2.  
 $Y$  Icosohedral group, 2.4.  
 $\mathfrak{g}$  Ideal, 4.3.  
 $\mathcal{O}_\alpha, \mathcal{R}_\alpha$  Ideals in the group ring of  $S_\alpha$ , 4.3.  
 $e$  Identity element in a group, 1.1.  
 $E_n$  Identity matrix,  $n \times n$ , 2.1, 3.3.  
 $E$  Identity operator, 2.1.  
 $T^G$  Induced rep, 3.5.  
 $\langle \mathbf{u}, \mathbf{v} \rangle, (\mathbf{u}, \mathbf{v})$  Inner product, 2.1, Appendix.  
 $\int f(x) dx$  Integral of  $f(x)$ , 3.1, 3.8, Appendix.  
 $dA, d_L A, d_r A, \delta A$  Invariant measures on a linear Lie group, 6.1.  
 $I = -E$  Inversion operator, 2.3.  
Irred Irreducible (representation), 3.2.  
 $T^{(\mu)}$  Irred rep of group  $G$ , indexed by the integer  $\mu$ , 3.3.



- $[f_1, \dots, f_m]$  Irred reps of  $GL(m)$ ,  $U(m)$ , and  $SL(m)$ , 9.1.  
 $D^{(u,v)}$  Irred rep of proper homogeneous Lorentz group, 8.3.  
 $D_+^{(u,v)}, D_-^{(u,v)}$  Irred reps of  $L^\dagger$ , 8.3.  
 $D_+^{(l)}, D_-^{(l)}$  Irred reps of  $O(3)$ , 7.6.  
 $D^{(u)}$  Irred reps of  $SU(2)$ ,  $SO(3, R)$ , and  $SL(2, \mathbb{C})$ , 7.2, 7.3.  
 $G \cong H$  Isomorphism of groups, 1.1, 2.4.  
 $G^x$  Isotropy subgroup of  $G$  at  $x$ , 1.4.  
 $\delta_{ij} = 1$  if  $i = j$ ,  $= 0$  if  $i \neq j$ ; Kronecker delta, 1.1.  
 $L_n^{(\alpha)}(z) = [\Gamma(\alpha + n + 1)/\Gamma(\alpha + 1)n!] \sum_{j=0}^{\infty} [(-n)_j/(\alpha + 1)_j](z^j/j!)$ ,  $n = 0, 1, 2, \dots$ ,  
 Laguerre polynomial, generalized, 7.5, 10.2.  
 $\Delta$  Laplacian, 3.1.  
 $L_2(G)$  Lebesgue square-integrable functions on the group  $G$ , 6.2, Appendix.  
 $L_2(\mathfrak{M})$  Lebesgue square-integrable functions on domain  $\mathfrak{M}$ , Appendix.  
 $L(g)$  Left regular rep, 1.4, 3.1.  
 $P_\mu^m(\cos \theta) = P_\mu^{0,-m}(\cos \theta)$ , Legendre function (associated), 7.2.  
 $P_n(\cos \theta) = {}_2F_1(n + 1, -n; 1; \frac{1}{2}(1 - \cos \theta))$ ,  $n = 0, 1, 2, \dots$ , Legendre polynomial, 7.2.  
 $\mathfrak{g}$  Lie algebra, 5.3.  
 $L(G)$  Lie algebra of local Lie group  $G$ , 5.3.  
 $so(3, 1)$  Lie algebra of homogeneous Lorentz group, 8.1, 9.10.  
 $L_\alpha, D_\alpha$  Lie derivatives, 5.9.  
 $T: V \rightarrow V$  Linear transformation, 2.1, 3.1.  
 $\ln A$  Logarithm of a matrix, 5.1.  
 $L^\dagger$  Lorentz group (complete), 8.3.  
 $L(4)$  Lorentz group (homogeneous), 8.1.  
 $L^{\dagger+}$  Lorentz group (proper), 8.1.  
 $T = (T_{ij})$  Matrix, 2.1.  
 ${}^{2S+1}L_J$  Multiplet, 9.8.  
 $v(\mathbf{x}, g)$  Multiplier, 5.9.  
 $\|\mathbf{u}\|, \|\mathbf{A}\|$  Norm of a vector (operator), 2.1, 5.1, Appendix.  
 $N_A$  Null space of linear operator  $A$ , 3.3.  
 $O$  Octahedral group, 2.4.  
 $n(G)$  Order of group  $G$ , 1.1.  
 $W^\perp$  Orthogonal complement of subspace  $W$ , 3.2.  
 $O(n, \mathbb{C}), O(n, R), O(n)$  Orthogonal groups, 5.4.  
 $\{\lambda_j\} \boxtimes \{\mu_k\}$  Outer product of irred reps of  $S_n$  and  $S_m$ , 9.9.  
 $\delta_s$  Parity of the permutation  $s$ , 4.1, 9.2.  
 $\partial f/\partial x, \partial_x f, f_x$  Partial derivative, 3.1.  
 $\partial_{\pm 1/2}, \pm 1/2$  Partial derivatives, 8.4.  
 $\{f_1, \dots, f_n\}$  Partition or frame, 4.1.  
 $\begin{pmatrix} 1 & 2 & \cdots & n \\ p_1 p_2 \cdots p_n \end{pmatrix}, (p_i \cdots p_j), \sigma(x)$ , Permutations, 1.1, 4.1.  
 $\perp$  Perpendicular, 2.8.  
 $h = 2\pi\hbar \sim 1.054 \times 10^{-27}$  erg sec, Planck's constant, 3.1, 3.8.  
 $(a)_n = a(a + 1) \cdots (a + n - 1) = \Gamma(a + n)/\Gamma(a)$ , Pochhammer's symbol (see Bessel, hypergeometric, and Laguerre functions).  
 $P$  Poincaré group, 8.2.  
 $\mathcal{P}$  Poincaré group (covering group of), 8.4.  
 $C_{nh}, C_{nv}, D_{nh}, D_{nd},$   
 $O_h = O \cup IO, S_{2n}, T_d,$   
 $T_h = T \cup IT, Y_h = Y \cup IY$ , Point groups, 2.5.



$P$	Projection operator, 3.3, 3.7.
$R_A$	Range of linear operator $A$ , 3.3.
$R_n$	Real Euclidean $n$ -space or group ring of $S_n$ , 2.1, 4.2.
$R$	Real numbers, 1.1.
$\sigma_k$	Reflection in plane perpendicular to $k$ , 2.1.
Rep	Representation, 3.1.
$T W$	Restriction of $T$ to $W$ , 3.2, 3.5.
$R(g)$	Right regular rep, 3.1.
$\alpha(\mathcal{H})$	Root form, 9.1.
$C_k(\theta)$	Rotation through angle $\theta$ about axis $k$ , 2.1.
$S_k(\theta) = \sigma_k C_k(\theta)$ ,	Rotation-inversion, 2.1.
$R(T)$	Row permutations of a Young tableau, 4.2.
$\text{Spin}(m)$	Simply connected covering group of $SO(m, \mathbb{C})$ , 9.6.
$SL(n, \mathbb{C}), SL(n, R), SL(n)$ $[sl(n, \mathbb{C}), sl(n, R), sl(n)]$ ,	Special linear groups (Lie algebras), 1.1, 5.4.
$SO(n, \mathbb{C}), SO(n, R), SO(n)$ $[so(n, \mathbb{C}), so(n, R), so(n)]$ ,	Special orthogonal groups (Lie algebras), 1.4.
$SU(m) [so(m)]$	Special unitary group (Lie algebra), 5.4.
$Y_l^m(\theta, \varphi) = [(2l+1)(l-m)!/4\pi(l+m)!]^{1/2} P_l^m(\cos \theta) e^{im\varphi}$ , $m = -l, -l+1, \dots, l$ , $l = 0, 1, 2, \dots$ ,	Spherical harmonic, 7.4.
$P_u^{-m,n}(\cos \theta) = [(\sin \theta)^{m-n}(1+\cos \theta)^{u+n-m}/2^u \Gamma(m-n+1)] {}_2F_1(-u-n, m-u;$ $m-n+1; [(\cos \theta - 1)/(\cos \theta + 1)])$ ,	Spherical function (generalized), 7.2.
$S_n, S_\alpha$	Symmetric group, 1.1, 4.4.
$S_X$	Symmetric group on set $X$ , 1.4.
$Sp(m)$	Symplectic group, 5.4, 9.4.
$a_{j_1 \dots j_n}$	Tensor components, 3.8.
$T_1 \otimes T_2 (T^{\otimes n})$	Tensor product of group reps ( $n$ -fold), 3.5 (3.8).
$u \otimes v$	Tensor product of vectors, 3.5.
$V \otimes W (V^{\otimes n})$	Tensor product of vector spaces ( $n$ -fold), 3.5 (3.8).
$2S+1L$	Term, 9.8.
$T$	Tetrahedral group, 2.4.
$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$	3- $j$ coefficients, 7.7.
$\text{tr } A = \sum_{i=1}^n A_{ii}$	Trace of $n \times n$ matrix $A$ , 2.1.
$T_a$	Translation, 2.2.
$T(n)$	Translation group in $n$ -space, 2.2.
$A^t$	Transpose of matrix $A$ , 2.1.
$U(m), USp(2m), SU(m)$	Unitary classical groups, 5.4, 9.4.
$u, v, x$	Vectors, 2.1, 3.1.
$V, W$	Vector spaces, 3.1.
$V_G$	Volume of compact group $G$ , 6.2.
$V(Q)$	Volume of parallelepiped $Q$ , 2.6.
$\Lambda(\mathcal{H})$	Weight, 9.1.
$S^\alpha \Lambda = \Lambda - 2(\Lambda_\alpha / \alpha_\alpha) \alpha$	Weyl reflection, 9.1.
$Z$	Zero matrix, 3.7.
$\mathbf{Z}$	Zero operator ( $\mathbf{Z}v = \mathbf{0}$ ), 3.7.
$\mathbf{0}$	Zero vector, 2.2.