

## Note on binding of match-seeking and match-repelling particles

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### Binding between two primitive particles through match-seeking particle

Let us consider two  $V$ -particles which are not composite – they are given with their semantic signatures respectively:

$$\text{ssig}(V') = [p'_0 \ a'_{0,1} \ p'_1 \ p'_0 \ a'_{0,2} \ p'_2 \ p'_0 \ a'_{0,3} \ p'_3 \ \dots \ p'_i \ a'_{i,n} \ p'_n]$$
$$\text{ssig}(V'') = [p''_0 \ a''_{0,1} \ p''_1 \ p''_0 \ a''_{0,2} \ p''_2 \ p''_0 \ a''_{0,3} \ p''_3 \ \dots \ p''_j \ a''_{j,m} \ p''_m]$$

Here each of the quantities  $p$  denotes the property signature vector of the corresponding property  $P$  of the  $V$  particle. The vector  $a_{r,s}$  denotes the signature of the property association particle  $A_{r,s}$  which binds to a pair of properties  $P_r$  and  $P_s$  in the property graph  $\mathcal{P}$  of the  $V$  particle.

Match-seeking particle  $MA$  binds to a subgraph  $\mathcal{S}$  of the property graph  $\mathcal{P}$  of the  $V$  particle.

There is a closeness condition which needs to be obeyed in order the particle  $MA$  to bind to the particle  $V$ .

### Binding matrix of a match-seeking particle

The match-seeking particle  $MA$  exposes a binding matrix  $mbind(MA)$ :

$$mbind(MA) = [B^1 \ B^2 \ B^3 \ \dots \ B^q]$$

$$B^1 = [p^0 \ a^{0,1} \ p^1], B^2 = [p^0 \ a^{0,2} \ p^2], B^3 = [p^0 \ a^{0,3} \ p^3], \dots, B^q = [p^p \ a^{p,q} \ p^q]$$

Obviously, each of the blocks  $B^i$  is  $N \times 3$  matrix where  $N$  is the dimension of semantic space. From now on we will denote the block matrices  $B^i$  as *binding elements* of the match-seeking particle  $M$ .

Note that in each of those blocks having the general form  $B^i = [p^p \ a^{p,q} \ p^q]$  it is possible to have  $a^{p,q} = p^q = \mathbf{0}$  where  $\mathbf{0}$  represents the null vector in semantic space. However,  $p^p$  is never close to the null vector i.e.  $|p^p| > 0$ .

### Binding of match-seeking particle against $V$ -particle formulated as optimization problem

Let a primitive particle  $V$  has the following semantic signature:

$$\text{ssig}(V) = [B_1 B_2 \dots B_m]$$

Let us denote by  $f_j^i$  the semantic distance between the binding element  $B^i$  of  $MA$  and the semantic element  $B_j$  of  $V$

$$f_j^i = \|B^i \ominus B_j\|, B^i = [p^p \ a^{p,q} \ p^q], B_j = [p_r \ a_{r,s} \ p_s]$$

Here the operation  $\|\ominus\|$  denotes the following metric:

$$\|B^i \ominus B_j\| = |p^p| |p^p - p_r| + |a^{p,q}| |a^{p,q} - a_{r,s}| + |p^q| |p^q - p_s|$$

We also use the operation  $\odot$  to denote columnar dot product defined as:

$$B^i \odot B_j = p^p \cdot p_r + a^{p,q} \cdot a_{r,s} + p^q \cdot p_s$$

TODO: finish this

Recursive definition of the semantic distance?

$$f_j^i(B) = \|B^i \ominus B_j\|, f_q^p(a) = \|a^p \ominus a_q\|, f_s^r(w) = \|w^r \ominus w_s\|$$

Here  $w \subset a \subset B$ .

$$f_j^i(B) = |\mathbf{p}^p| |\mathbf{p}^p - \mathbf{p}_r| + |\mathbf{a}^{p,q}| f_{r,s}^{p,q}(a) + f_{r,s}^{p,q}(w) |\mathbf{p}^q| |\mathbf{p}^q - \mathbf{p}_s|$$

In abbreviated form:

$$f(B) = |\mathbf{p}^1| |\mathbf{p}^1 - \mathbf{p}_1| + |\mathbf{a}^{1,2}| f(a) + f(w) |\mathbf{p}^2| |\mathbf{p}^2 - \mathbf{p}_2|$$

Closeness condition for a bind between match seeking particle and primitive semantic particle  
Let us denote by  $sfil(MA, V)$  the following diagonal matrix which will be named *Filter matrix* of the match seeking particle:

$$sfil(MA, V) = \begin{bmatrix} I_1 & & & & \\ & 0 & & & \\ & & I_2 & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & I_k \end{bmatrix}$$

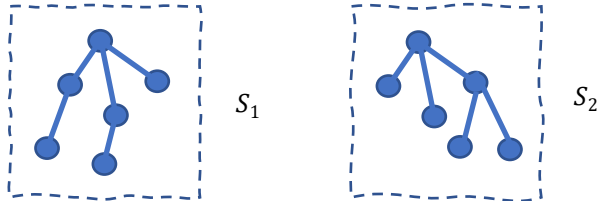
Here  $I_i, i = 1, 2, \dots, k$  are identity matrices which represent the regions of interest in the semantic signature matrix of  $V$  to the match seeking particle  $MA$ .

The regions of interest  $sreg(MA, V)$  in the semantic signature of  $V$  are obtained by multiplying  $sfil(MA, V)$  with  $ssig(V)$ :

$$sreg(MA, V) = sfil(MA, V) \times ssig(V)$$

Between two semantic structures

Let us have two semantic structures  $S_1$  and  $S_2$ .



Let the semantic signature of  $S_1$  is given with:

$$ssig(S_1) = [\mathbf{V}_1 \mathbf{A}_{1,2} \mathbf{V}_2 \mathbf{A}_{1,3} \mathbf{V}_3 \dots \mathbf{A}_{r,p} \mathbf{V}_p]$$

and the semantic signature of  $S_2$  is given with:

Between a primitive  $V$  particle and a semantic structure  $S$