Semantic tree operations

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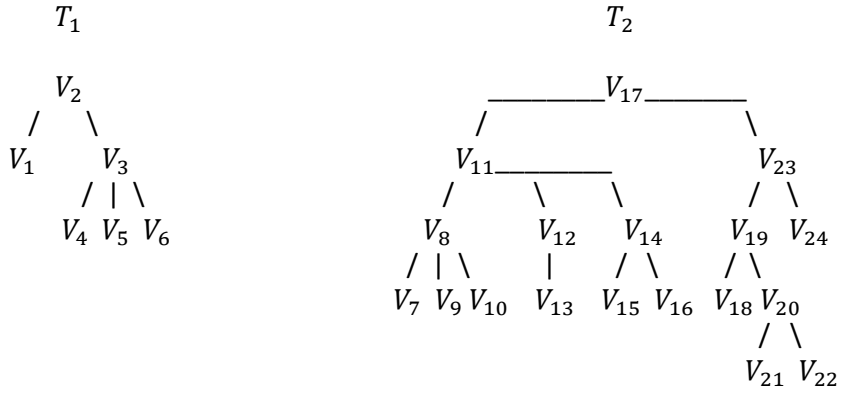
Let us introduce the *tree tuple* (k, v) where v represents certain node - semantic particle or a subtree. Here k is a tree tuple factor which encodes uniquely the position of the node v in the tree.
The following operations are defined for tree tuple factors:
We have a base of m tree tuple factors $k_1, k_2, \dots\ k_m$ which are defined as the digits greater than 0 of $(m+1)$-nary number system such that $0 = k_0 < k_1 < k_2 < \dots < k_m$. We define an operation `*` denoting digit concatenation $k_i * k_j = (m+1)k_i + k_j$. Obviously,
$k_i * k_j > k_i$ for any pair $i, j = 1..m$
Note that the latter implies that
$k_{i_1} * k_{i_2} * \dots * k_{i_n} > k_{j_1} * k_{j_2} * \dots * k_{j_{n-1}}$ for any tuple where $i_p, j_q = 1..m, p = 1..n, q = 1..n-1$

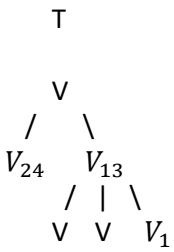
$$\left(k_i, (k_j, v_j)\right) = (k_i * k_j, v_j)$$

Encoding a complete m-ary tree T of height h with the algebraic notation above:
$T = (k_0, v_0^0)$. Further we will assume that $k_0 = 0$.
$v_0^0 = (k_0, v_0^1) + (k_1, v_1^1) + (k_2, v_2^1) + \dots + (k_m, v_m^1)$
In general we have:
$v_q^p = \left(k_0, v_q^{p+1}\right) + \left(k_1, v_{(q-1)m+1}^{p+1}\right) + \left(k_2, v_{(q-1)m+2}^{p+1}\right) + \dots + \left(k_m, v_{qm}^{p+1}\right)$
where $q = 1..m^h, p = 1..h$
Obviously, we have at most $\frac{(m^{h+1}-1)}{m-1}$ distinct terms v_q^p which represent nodes i.e. semantic values.
The expression for the tree also can be written as:
$T = \sum_{i=0}^{N}(k_i^*, v_i)$ where $N \leq \frac{(m^{h+1}-1)}{m-1}$ and k_i^* are the *node factors* given with $k_i^* = k_{i_1} * k_{i_2} * \dots * k_{i_n}; n \leq h$. The node values v_i are the values v_q^p ordered in increasing order of k_i^*. This order corresponds to *level order traversal* of the m-ary tree. Note that with appropriately defined comparison operation `<` we can model different ways of traversing the m-ary tree. For instance, if we define `<` as the comparison for the values of $\left(m+1-k_{i_1}\right)m^{n-1} + \left(m+1-k_{i_2}\right)m^{n-2} + \dots + \left(m+1-k_{i_n}\right)$ we will have ordering which corresponds to the *preorder traversal* of the tree.

Example
Peter is Dimitar's son.
Dimitar's son has a friend in the neighborhood and his friend's name is James.
⇨ James is Peter's friend

Peter is the son of Dimitar.
V_1 V_2 V_3 V_4 V_5 V_6
The son of Dimitar has a friend in the neighborhood and the name of his friend is James.
V_7 V_8 V_9 V_{10} V_{11} V_{12} V_{13} V_{14} V_{15} V_{16} V_{17} V_{18} V_{19} V_{20} V_{21} V_{22} V_{23} V_{24}





Expressing T_1 with the algebraic notation discussed earlier:

$$T_1 = (k_0, v_2) + (k_1, v_1) + \big(k_2, \big((k_0, v_3) + (k_1, v_4) + (k_2, v_5) + (k_3, v_6)\big)\big)$$

which is expanded to:

$$T_1 = (k_0, v_2) + (k_1, v_1) + (k_2 k_0, v_3) + (k_2 k_1, v_4) + (k_2 k_2, v_5) + (k_2 k_3, v_6)$$

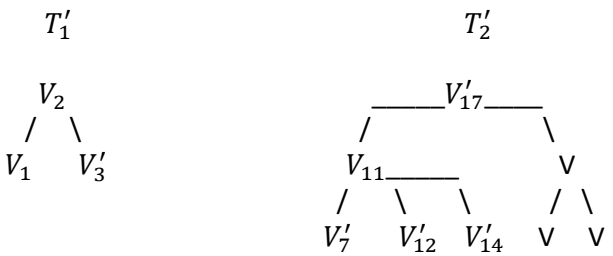
Expressing T_2 with the algebraic notation yields:

$$T_2 = (k_0, v_{17}) + \Big(k_1, \Big((k_0, v_{11}) + \Big(k_1, \big((k_0, v_8) + (k_1, v_7) + (k_2, v_9) + (k_3, v_{10})\big)\Big) + \Big(k_2, \big((k_0, v_{12}) + (k_1, v_{13})\big)\Big) + \Big(k_3, \big((k_0, v_{14}) + (k_1, v_{15}) + (k_2, v_{16})\big)\Big)\Big)\Big) + \Big(k_2, \Big((k_0, v_{23}) + \Big(k_1, \Big((k_0, v_{19}) + (k_1, v_{18}) + \Big(k_2, \big((k_0, v_{20}) + (k_1, v_{21}) + (k_2, v_{22})\big)\Big)\Big)\Big) + (k_2, v_{24})\Big)\Big)$$

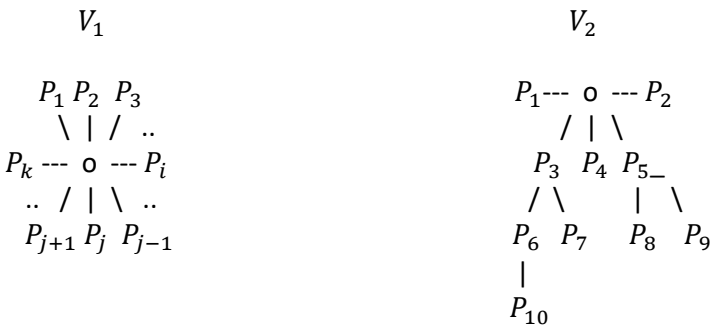
which is expanded to:

$$\begin{aligned} T_2 = {} & (k_0, v_{17}) + (k_1 k_0, v_{11}) + (k_1 k_1 k_0, v_8) + (k_1 k_1 k_1, v_7) + (k_1 k_1 k_2, v_9) + (k_1 k_1 k_3, v_{10}) \\ & + (k_1 k_2 k_0, v_{12}) + (k_1 k_2 k_1, v_{13}) + (k_1 k_3 k_0, v_{14}) + (k_1 k_3 k_1, v_{15}) + (k_1 k_3 k_2, v_{16}) \\ & + (k_2 k_0, v_{23}) + (k_2 k_1 k_0, v_{19}) + (k_2 k_1 k_1, v_{18}) + (k_2 k_1 k_2 k_0, v_{20}) + (k_2 k_1 k_2 k_1, v_{21}) \\ & + (k_2 k_1 k_2 k_2, v_{22}) + (k_2 k_2, v_{24}) \end{aligned}$$

Semantic Aggregation:

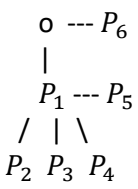


Properties and Dependent Properties:



A V particle is defined as a tree-like structure of attached properties as depicted on the figures above.

Let us construct the V particle of the verb "*is*":
P_1: *key*: 'particle_type', *value*: 'verb'
P_2: *key*: 'is_transitive', *value*: 'true'
P_3: *key*: 'plurality', *value:* 'singular'
P_4 : *key*: 'grammatical_person', *value:* 'third_person'
P_5 : *key*: 'tense', *value:* 'present'
P_6 : *key*: 'text', *value:* 'is'



Each property (P-particle) is constructed from a key (key-particle) and a value (val-particle).
Each key and value particles are registered in a particle registry $\Re^P$. The purpose of the particle registry $\Re^P$ is to facilitate learning of new keys and values and use them for parsing new semantic constructs.
Between any two keys k_1 and k_2, such that it is obeyed exactly one of the following 3 relations:
$k_1 \prec\!\!\succ k_2 \quad \vee \quad k_1 \succ k_2 \quad \vee \quad k_1 \prec k_2$
Here the operator $\prec\!\!\succ$ indicates that there is no parent-child relationship between $k_1$and k_2, the operator $\succ$ indicates that k_2 is a subkey (or a child) of k_1 and $\prec$ indicates that k_2 is a superkey (or a parent) of k_1.
For instance, in the example above $key(P_1) \prec\!\!\succ key(P_6)$

Semantic properties of a V particle

The semantic properties of a particle are under the key 'semantic_properties_available'. The semantic properties of a particle can be frequently updated depending on the context and upon refining concepts as a result of parsing and analysis of new semantic structures.
In our example we can deduce the following semantic properties:

P_7 : *key*: 'semantic_properties_available', *value:* 'true'
P_8 : *key*: 'represents_an_entity', *value:* 'true'

SN.key_relation(k1, k2): KeyRelation
Enum KeyRelation: {'keys_unrelated', 'key_left_is_parent_of_key_right', 'key_left_is_ancestor_of_key_right', 'key_left_is_child_of_key_right', 'key_left_is_descendant_of_key_right'}
SN.key_children(k): set()
SN.key_descendents(k): set()

How do we add properties to a particle? Either via semantic programming or via inference.
Simple scenario to add property via inference:
Dimitar is a son of John.
V_1 V_2 V_3 V_4 V_5 V_6

The red Ford is a car of Bank of America.
V_7 V_8 V9 V10 V11 V12 V13 V14 V15 V16

Maria is a daughter of Dimitar.
V17 V18 V19 V20 V21 V22