On the signature matrix of semantic property

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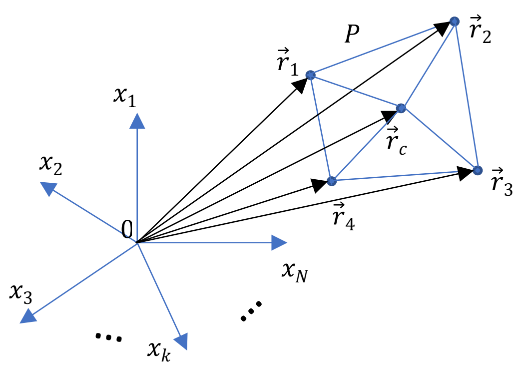
Notation

L : the number of semantic dimensions
K : the number of semantic dimensions in a property represented as K-polytope
N : number of semantic aspects in a property
$\mathcal{P}$: set of points forming the K-polytope of a semantic property
A_i : denotes the i-th semantic aspect of a semantic property
P_i : denotes semantic property
V_i : denotes primitive semantic particle
$\vec{p}_c$: in the context of a property: the center of mass of the property
In the context of an ensemble of properties: the center of mass of the ensemble
$\vec{p}_i$: In the context of a property: semantic position of the aspect A_i
In the context of an ensemble of properties: the center of mass of the property
l_i : the type of the aspect A_i
θ_j : angle between the current aspect and semantic axis x_j
$\boldsymbol{\theta}$: a vector with all angular coordinates of the current aspect to the semantic axes

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \mathfrak{m}_i \vec{r}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \qquad (1)$$



If $\mathfrak{m}_l = \mathfrak{m} = const$

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \vec{r}_i}{|\mathcal{P}|} \qquad (2)$$

$$\vec{p}_i = \vec{r}_i - \vec{r}_c \qquad (3)$$

$$\vec{p}_i = \left(1 - \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}\right)\vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \frac{\mathfrak{m}_j}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \vec{r}_j \qquad (4)$$

With $\hat{\mathfrak{m}}_i = \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}$ we write:

$$\vec{p}_i = (1 - \hat{\mathfrak{m}}_i)\vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \hat{\mathfrak{m}}_j \vec{r}_j \qquad (5)$$

In a matrix form:

$$P = \begin{bmatrix} 1-\widehat{\mathfrak{m}}_1 & -\widehat{\mathfrak{m}}_2 & \cdots & -\widehat{\mathfrak{m}}_N \\ -\widehat{\mathfrak{m}}_1 & 1-\widehat{\mathfrak{m}}_2 & \cdots & -\widehat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\widehat{\mathfrak{m}}_1 & -\widehat{\mathfrak{m}}_2 & \cdots & 1-\widehat{\mathfrak{m}}_N \end{bmatrix} \begin{Bmatrix} \vec{r}_1 \\ \vec{r}_2 \\ \vdots \\ \vec{r}_N \end{Bmatrix} \quad (6)$$

or succinctly

$$P = MX \quad (7)$$

where $M = \begin{bmatrix} 1-\widehat{\mathfrak{m}}_1 & -\widehat{\mathfrak{m}}_2 & \cdots & -\widehat{\mathfrak{m}}_N \\ -\widehat{\mathfrak{m}}_1 & 1-\widehat{\mathfrak{m}}_2 & \cdots & -\widehat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\widehat{\mathfrak{m}}_1 & -\widehat{\mathfrak{m}}_2 & \cdots & 1-\widehat{\mathfrak{m}}_N \end{bmatrix}$ $X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,L} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ x_{N,1} & x_{N,2} & \cdots & x_{N,L} \end{bmatrix}$ (8)

Obviously, there does not exist inverse matrix M^{-1} as the set of matrices X which map to a given matrix P is a continuum.