On the signature matrix of semantic property

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Notation

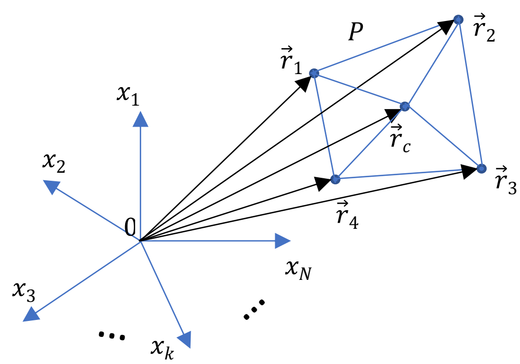
L : the number of semantic dimensions
K : the number of semantic dimensions in a property represented as K-polytope
N : number of semantic aspects in a property
$\mathcal{P}$: set of points forming the K-polytope of a semantic property
A_i : denotes the i-th semantic aspect of a semantic property
P_i : denotes semantic property
V_i : denotes primitive semantic particle
$\vec{r}_c$: in the context of a property: the center of mass of the property
In the context of an ensemble of properties: the center of mass of the ensemble
$\vec{r}_i$: In the context of a property: semantic position of the aspect A_i
In the context of an ensemble of properties: the center of mass of the property
l_i : the type of the aspect A_i
θ_j : angle between the current aspect and semantic axis x_j
$\boldsymbol{\theta}$: a vector with all angular coordinates of the current aspect to the semantic axes

Matrix Representation of Semantic Property

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \mathfrak{m}_i \vec{r}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \qquad (1)$$

If $\mathfrak{m}_l = \mathfrak{m} = const$

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \vec{r}_i}{|\mathcal{P}|} \qquad (2)$$



$$\vec{p}_i = \vec{r}_i - \vec{r}_c \qquad (3)$$

$$\vec{p}_i = \left(1 - \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}\right) \vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \frac{\mathfrak{m}_j}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \vec{r}_j \qquad (4)$$

With $\widehat{\mathfrak{m}}_i = \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}$ we write:

$$\vec{p}_i = (1 - \widehat{\mathfrak{m}}_i)\vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \widehat{\mathfrak{m}}_j \vec{r}_j \qquad (5)$$

In a matrix form:

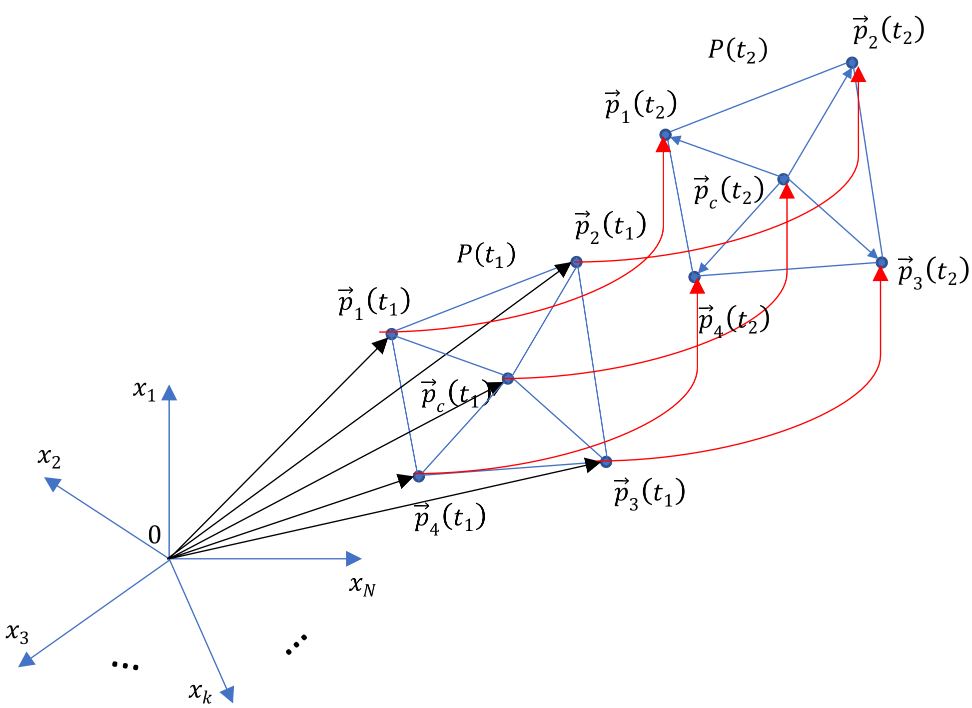
$$P = \begin{bmatrix} 1-\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ -\hat{\mathfrak{m}}_1 & 1-\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & 1-\hat{\mathfrak{m}}_N \end{bmatrix} \begin{Bmatrix} \vec{r}_1 \\ \vec{r}_2 \\ \vdots \\ \vec{r}_N \end{Bmatrix} \quad (6)$$

or succinctly

$$P = MX \quad (7)$$

where $M = \begin{bmatrix} 1-\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ -\hat{\mathfrak{m}}_1 & 1-\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & 1-\hat{\mathfrak{m}}_N \end{bmatrix}$ $X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,L} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ x_{N,1} & x_{N,2} & \cdots & x_{N,L} \end{bmatrix}$ (8)

While P is traversing the semantic space each aspect A_i retains its properties – this means that each $\vec{p}_i$ is invariant when P traverses semantic space. That is - l_i and $\boldsymbol{\theta}_i$ remain invariant.



The last statement, obviously, implies that there does not exist inverse matrix M^{-1} as the set of $N \times N$ matrices X which map to a given matrix P is a continuum.

Thus, we conclude that each semantic property is uniquely defined by the pair of two quantities: the semantic signature matrix P and the mass vector of the property $\mathfrak{m} = \{\mathfrak{m}_1, \mathfrak{m}_2, \cdots, \mathfrak{m}_N\}$.

In Situ Position of Semantic Property

Definition: *In-situ position of semantic property*

This is the initial position in Semantic Space from which each semantic property starts its travel to bound state.

Each semantic property will be mapped to a portion of semantic space which will contain its initial / in-situ position. In order to determine the portion of semantic space which will map to semantic property we will look into the singular value decomposition of the property signature matrix P given with

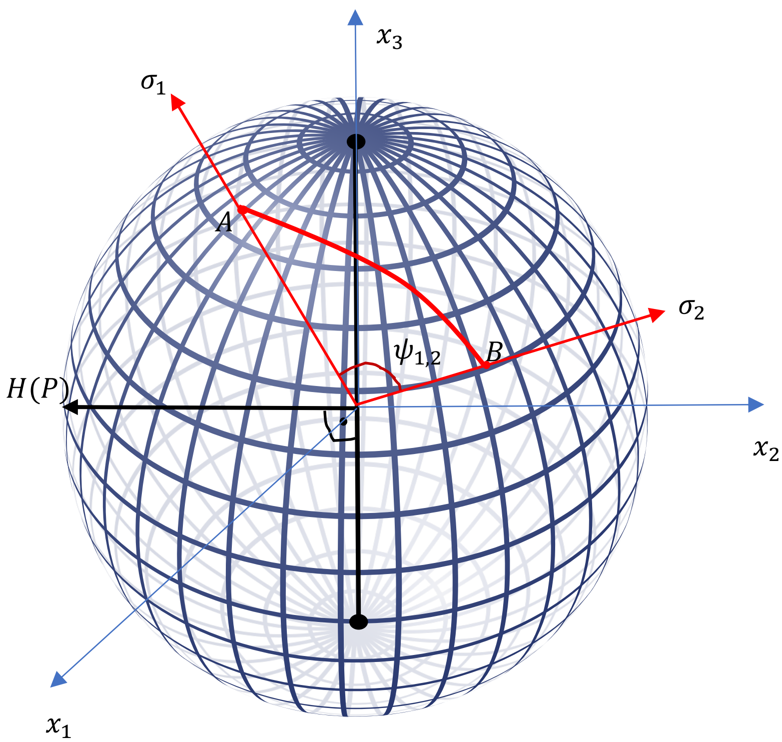
$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,L} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ p_{N,1} & p_{N,2} & \cdots & p_{N,L} \end{bmatrix} \quad (9)$$

Recall, in (9) each row corresponds to an aspect definition encoding the aspect type l_i and its angular coordinates $\boldsymbol{\theta}_i$. Thus, we will be looking for factorization in the form:

$$P = U\Sigma V^T \quad (10)$$

where U is $N \times N$ orthonormal matrix, V is $L \times L$ orthonormal matrix and Σ is $N \times L$ diagonal matrix with at most N non-zero values on the main diagonal. Let us denote those non-zero values on the main diagonal of Σ with $\sigma_1, \sigma_2, \dots, \sigma_N$. We can always normalize P such that $\sigma_1 + \sigma_2 + \cdots + \sigma_N = 1$. Then the entropy of the property P is given with:

$$H(P) = -(\sigma_1 \log \sigma_1 + \sigma_2 \log \sigma_2 + \cdots + \sigma_N \log \sigma_N) \quad (11)$$



x_3
σ_1
A
σ_2
$\psi_{1,2}$
B
$H(P)$
x_2
x_1