

## The Signature of Semantic Structures

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Let us have the compound particle  $V_{comp}$  represented by its elementary particle sequence and semantic tree  $stree(V_{comp})$ :

$$stree(V_{comp}) =$$

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      /  \  /  \
     /    \ /    \
    /      \ /      \
   /        \ /        \
  /          \ /          \
 /            \ /            \
V1            V2            V3

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The property tree for each  $V$ -particle  $V_k, k = 1..9$  are given with the algebraic notation discussed in [Semantic Tree Operations](#).

$ptree(V_k) = \sum_{k \in \mathfrak{T}(V_k), i \in \mathbb{P}(V_k)} (k, P_i)$ . Here  $k$  denotes the path  $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$  constructed by branching consecutively along the  $k_{l_1}$ -th branch from the top level, then the  $k_{l_2}$ -th branch from the lower level and finally  $k_{l_h}$ -th branch from the  $h$ -th level. The set  $\mathfrak{T}(V_k)$  denotes the set of all paths from the root to a leaf in the property tree of  $V_k$ . The set  $\mathbb{P}(V_k)$  denotes the indices of the vertices in the property tree of  $V_k$ .

$$\begin{array}{ccc}
 V_1 & V_2 & \dots & V_9 \\
 \begin{array}{c}
 P_1 \ P_2 \ P_3 \\
 \backslash \ | \ / \ \dots \\
 P_k \ \dots \ o \ \dots P_i \\
 \dots \ / \ | \ \backslash \ \dots \\
 P_{j+1} \ P_j \ P_{j-1}
 \end{array} &
 \begin{array}{c}
 P_1 \ \dots \ o \ \dots P_2 \\
 \backslash \ | \ / \\
 P_3 \ P_4 \ P_5 \ \dots \\
 \backslash \ / \ \backslash \ / \ \backslash \\
 P_6 \ P_7 \ P_8 \ P_9 \\
 | \\
 P_{10}
 \end{array} &
 \begin{array}{c}
 P_1 \ \dots \ o \ \dots P_2
 \end{array}
 \end{array}$$

Expressing the property tree of  $V_1$  with the algebraic notation discussed earlier:

$$ptree(V_1) = \sum_{k_j \in \mathfrak{T}(V_1), i \in \mathbb{P}(V_1)} (k_j, P_i)$$

Similarly,  $ptree(V_2)$  is given with

$$\begin{aligned}
 ptree(V_2) = & (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3 k_1, P_6) + (k_3 k_2, P_7) \\
 & + (k_5 k_1, P_8) + (k_5 k_2, P_9) + (k_3 k_1 k_1, P_{10})
 \end{aligned}$$

Here  $P_0$  is  $text(V_2)$ .

Now if we expand the property trees for each  $V$ -particle in the semantic tree for the composite particle  $V_{comp}$  we will have a larger augmented property tree. This augmented property tree represents the semantic structure of  $V_{comp}$  and can be recorded in a matrix form which is the semantic signature of  $V_{comp}$ . The semantic signature matrix of  $V_{comp}$  will have the following structure:

$$ssig(V_{comp}) = [p_0 \ a_{0,1} \ p_1 \ p_0 \ a_{0,2} \ p_2 \ p_0 \ a_{0,3} \ p_3 \ \dots \ p_p \ a_{p,q} \ p_q]$$

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [\mathbf{B}_1 \ \mathbf{B}_2 \ \mathbf{B}_3 \ \dots \ \mathbf{B}_q]$$

$$\mathbf{B}_1 = [\mathbf{p}_0 \ \mathbf{a}_{0,1} \ \mathbf{p}_1], \mathbf{B}_2 = [\mathbf{p}_0 \ \mathbf{a}_{0,2} \ \mathbf{p}_2], \mathbf{B}_3 = [\mathbf{p}_0 \ \mathbf{a}_{0,3} \ \mathbf{p}_3], \dots, \mathbf{B}_q = [\mathbf{p}_p \ \mathbf{a}_{p,q} \ \mathbf{p}_q]$$

Here the block matrix  $\mathbf{B}_1$  fully describes the property  $P_1$  including how it is connected to the property tree  $ptree(V_1)$ . Similarly,  $\mathbf{B}_2$  and  $\mathbf{B}_3$  fully describes the properties  $P_2$  and  $P_3$  and their connectivity to  $ptree(V_1)$ . Finally,  $\mathbf{B}_q$  fully describes the property  $P_q$  and its connectivity to  $ptree(V_9)$ . From now on we will denote the block matrices  $\mathbf{B}_i$  as *semantic elements* of  $V_{comp}$ .

*Statement:* Every semantic particle, primitive or composite, can be represented as a sequence of *semantic elements*.

*Definition:* Semantic distance between two semantic elements  $B_1$  and  $B_2$

The semantic element  $B_1$  represents two properties -  $P_i$  and  $P_j$  connected through association link  $A_{i,j}$ . The properties  $P_i$  and  $P_j$  are represented by their property signatures  $\mathbf{p}_i$  and  $\mathbf{p}_j$ . The association link  $A_{i,j}$  is represented with its association matrix  $\mathbf{a}_{i,j}$  and semantic significance vector  $\mathbf{w}_{i,j}$ . (Note: Sometimes for clarity all vectors in a block matrix representing semantic element will be denoted with the vector symbol  $\vec{\phantom{x}}$  when clear distinction needs to be made). For details refer to the document [Note On Binding of Association Property to Semantic Properties](#). Similarly the semantic element  $B_2$  represents the properties  $P_k$  and  $P_l$  connected through association link  $A_{k,l}$ . As before the properties  $P_k$  and  $P_l$  are represented by their property signatures  $\mathbf{p}_k$  and  $\mathbf{p}_l$ . The association link  $A_{k,l}$  is represented with its association matrix  $\mathbf{a}_{k,l}$  and semantic significance vector  $\mathbf{w}_{k,l}$ .

Let  $\mathbf{B}_1$  denotes the matrix of the first semantic element  $B_1$  given with  $\mathbf{B}_1 = [\mathbf{p}_i \ \mathbf{a}_{i,j} \ \mathbf{p}_j]$

Let  $\mathbf{B}_2$  denotes the matrix of the second semantic element  $B_2$  given with  $\mathbf{B}_2 = [\mathbf{p}_k \ \mathbf{a}_{k,l} \ \mathbf{p}_l]$

Then the semantic distance between the two is given with:

$$sdist(B_1, B_2) = sdist(P_i, P_k) + sdist(A_{i,j}, A_{k,l}) + sdist(P_j, P_l)$$

where

$$sdist(P_i, P_k) = |\mathbf{p}_i - \mathbf{p}_k|, \quad sdist(P_j, P_l) = |\mathbf{p}_j - \mathbf{p}_l|$$

$$sdist(A_{i,j}, A_{k,l}) = |\mathbf{w}_{i,j} - \mathbf{w}_{k,l}| \times sdist(\mathbf{a}_{i,j}, \mathbf{a}_{k,l})$$

*Definition:* The semantic distance of two semantic matrices  $\mathbf{a}$  and  $\mathbf{b}$  which have the same number of columns is given with:

$$sdist(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^n |\vec{\mathbf{a}}_i - \vec{\mathbf{b}}_i| \text{ where } \mathbf{a} = [\vec{\mathbf{a}}_1 \ \vec{\mathbf{a}}_2 \ \dots \ \vec{\mathbf{a}}_n] \text{ and } \mathbf{b} = [\vec{\mathbf{b}}_1 \ \vec{\mathbf{b}}_2 \ \dots \ \vec{\mathbf{b}}_n].$$

In the block matrix for  $ssig(V_{comp})$   $\mathbf{p}_0$  denotes the signature column vector of the property  $P_0$ ,  $\mathbf{a}_{0,1}$  denotes the association matrix of the arc between property  $P_0$  and property  $P_1$ ,  $\mathbf{a}_{p,q}$  denotes the association matrix of the arc between property  $P_p$  and  $P_q$ . Let us denote the number of rows of  $ssig(V_{comp})$  by  $N$  and the number of columns by  $M$ .

The semantic signature matrix  $ssig(V_{comp})$  can be decomposed as a sum of two intrinsic structural matrices – property signature matrix  $psig(V_{comp})$  and connectivity signature matrix  $csig(V_{comp})$ :

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [\mathbf{p}_0 \ 0 \ \mathbf{p}_1 \ \mathbf{p}_0 \ 0 \ \mathbf{p}_2 \ \mathbf{p}_0 \ 0 \ \mathbf{p}_3 \ \dots \ \mathbf{p}_p \ 0 \ \mathbf{p}_q]$$

$$csig(V_{comp}) = [0 \ \mathbf{a}_{0,1} \ 0 \ 0 \ \mathbf{a}_{0,2} \ 0 \ 0 \ \mathbf{a}_{0,3} \ 0 \ \dots \ 0 \ \mathbf{a}_{p,q} \ 0]$$

Let us denote by  $psig(P_1, V_{comp})$  the augmented semantic property signature of property  $P_1$  with respect to  $V_{comp}$ . It is given with:

$$psig(P_0, V_{comp}) = [\mathbf{p}_0 \ 0 \ 0 \ \mathbf{p}_0 \ 0 \ 0 \ \mathbf{p}_0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0 \ 0 \ \mathbf{p}_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

$$psig(P_q, V_{comp}) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ \mathbf{p}_q]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$$

Here  $\mathbb{S}(V_{comp})$  denotes the set of the indices of all semantic particles which the composite  $V_{comp}$  is composed from.

Another way to partition the signature matrix into block matrices is:

$$ssig(V_{comp}) = [\mathbf{V}_1 \ \mathbf{A}_{1,2} \ \mathbf{V}_2 \ \mathbf{A}_{1,3} \ \mathbf{V}_3 \ \dots \ \mathbf{A}_{6,8} \ \mathbf{V}_8 \ \mathbf{A}_{6,9} \ \mathbf{V}_9]$$

The block matrix  $\mathbf{V}_1$  represents the property tree of the particle  $V_1$  and it is given by:

$$\mathbf{V}_1 = [\mathbf{p}_0 \ \mathbf{a}_{0,1} \ \mathbf{p}_1 \ \mathbf{p}_0 \ \mathbf{a}_{0,2} \ \mathbf{p}_2 \ \mathbf{p}_0 \ \mathbf{a}_{0,3} \ \mathbf{p}_3 \ \dots \ \mathbf{p}_0 \ \mathbf{a}_{0,k} \ \mathbf{p}_k]$$

The block matrix  $\mathbf{A}_{1,2}$  describes the connection between the particles  $V_1$  and  $V_2$  connecting the root property  $\mathbf{p}_0$  of  $V_1$  and the root property  $\mathbf{p}_{k+1}$  of  $V_2$ . It is given with:

$$\mathbf{A}_{1,2} = [\mathbf{p}_0 \ \mathbf{a}_{0,k+1} \ \mathbf{p}_{k+1}] \text{ //TODO: expand it – the matrix structure is more complicated!}$$

### Properties of the signature matrix

Here are some interesting properties of  $ssig(V_{comp})$ :

The number of rows  $N$  in  $ssig(V_{comp})$  is  $3 \times$  the number of arcs in the augmented property tree of  $V_{comp}$ .

The rank of

TO DO: finish the property section

### Asymptotic closeness of semantic structures

Let us have two semantic structures S1 and S2.

$$\begin{aligned} \text{ssig}(S_1) &= [\mathbf{V}_{k_1} \mathbf{A}_{k_1, k_2} \mathbf{V}_{k_2} \mathbf{A}_{k_1, k_3} \mathbf{V}_{k_3} \dots \mathbf{A}_{k_p, k_q} \mathbf{V}_{k_q}] \\ \text{ssig}(S_2) &= [\mathbf{V}_{l_1} \mathbf{A}_{l_1, l_2} \mathbf{V}_{l_2} \mathbf{A}_{l_1, l_3} \mathbf{V}_{l_3} \dots \mathbf{A}_{l_r, l_s} \mathbf{V}_{l_s}] \end{aligned}$$

Uniform asymptotic closeness

$K$ -level uniform asymptotic closeness