On the need of Dynamic Simulation when modeling interactions of Semantic Structures

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The machine was all glass retorts and valves and delicately balanced see-saws. Tinted fluids represented different market pressures and financial parameters: interest rates, inflation, trade deficits. The machine sloshed and gurgled, computing ferociously difficult integral equations by the power of applied fluid dynamics. It had enchanted her. She had remade the prototype, adding a few sly refinements of her own. But though the machine had provided some amusement, she had seen only glimpses of emergent behaviour. The machine was too ruthlessly deterministic to throw up any genuine surprises.
From "*Redemption Ark*", Chapter 7, Alastair Reynolds, 2002

Reinforcement learning models for the behavior of Semantic Structures

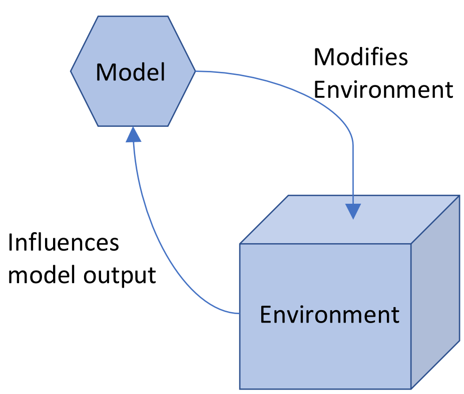
We are considering reinforcement learning models where both are true:

1) The environment influences the model
2) The model influences the environment

The model influences the environment in such a way that the environment gets conditioned to support certain interactions and encourages forming certain types of ensembles. The pertinent question here is what constitutes the environment. We will consider both a static model and dynamic model for semantic structure interactions. In both static and dynamic modeling the environment is represented by some kind of a *field* which will influence the particles entering semantic space either via inference or via parsing. The environment will condition those new particles to form certain kinds of ensembles and to associate them in a certain preferred way with the existing semantic structures.

The Role of the Environment in parsing and inference of semantic structures

The model of semantic structure interactions in conjunction of the Environment determines how the incoming semantic particles will form semantic structures and how the newly formed structures will interact with the existing ones.

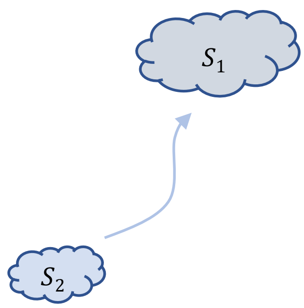


When semantic structures are formed from semantic particles certain semantic paths will be reinforced by the Model while other paths will be discouraged. The reinforcement happens based on certain statistics computed for every newly formed structure. Discouragement happens when the attempt to form a semantic structure fails.

Dynamic vs Static models of the behavior and interactions of Semantic Structures

What is Static Modeling of the Semantic Structures behavior?

Static modeling implies that we do not model *the time* and we do not have *time coordinate* in our models of the interactions of Semantic Structures. We do not have explicit or implicit *time dependence* in any of our relations and the concept of *passed time* is devoid of meaning.
This brings a simplicity to our models as all adjustments in the positions and the interactions between Semantic Structures are instantaneous. For instance, suppose we have an existing semantic structure S_1 which is located on a particular position in Semantic Space. We have just constructed another semantic structure S_2 and we would like to evaluate how much it will displace S_1 and in which direction.



One can use a static model to calculate what will be the new position of S_1 and S_2 given their total mass and given the attractive and repulsive forces acting on substructures of S_1 and S_2. Specifically, let us assume that there are m substructure pairs in S_1 and S_2 which attract each other pairwise. Also, we will assume that there are n substructure pairs in S_1 and S_2 which repel each other pairwise.
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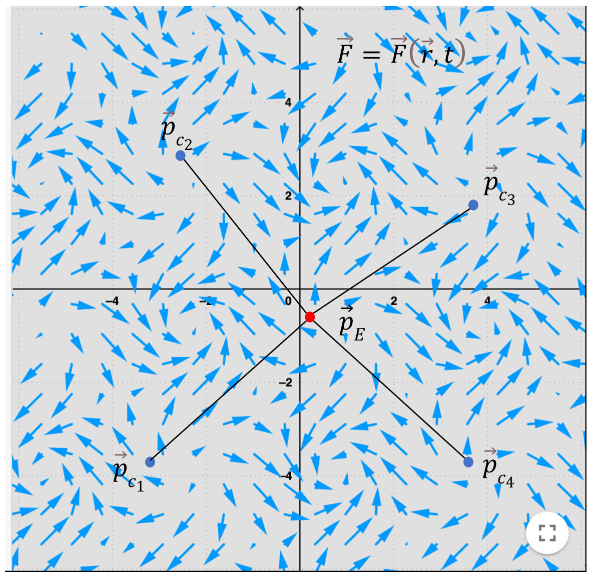
Introducing Dynamic Model of Semantic Structures behavior

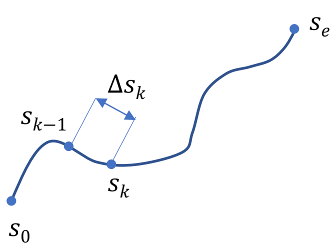
In the Dynamic models for Semantic Structures we define the concept of *time* in semantic space. We also define the vector *velocity* of a semantic particle/structure. We will use both time and velocity to determine the exact time and the amount of interaction between semantic structures which will take place at some future moment.
Let us define a L-dimensional vector field $\mathfrak{F}$ in L-dimensional semantic space. This field will have an effect of the semantic particles in such a way that it will apply an additional momentum to the particle which will be proportional to the travel path of the particle and to the strength and orientation of $\mathfrak{F}$. Let us denote by $\vec{f}$ a vector from the field $\mathfrak{F}$ at position $\vec{r}$ and time t:

$$\vec{f} = \vec{f}(\vec{r}, t) \qquad (1)$$

For instance, let us consider an ensemble of four properties $P_i, i = 1..4$. which are not in their bound state positions but in some general positions.



One could think of the vector field $\vec{f}\,(\vec{r}, t)$ as a Force field distributed along the travel path of a semantic particle. The path of any semantic particle in Semantic Space can be expressed in terms of its natural coordinate s. On the Figure below it is shown the path of a particle with starting point s_0. Let us denote



with t_0 the moment of time in which the particle has been at position s_0. The particle is in position s_{k-1} at a moment t_{k-1}, it is in position s_k at a moment t_k after traveling Δs_k and finally it ends up in position s_e. Let us write the recursive relation between two consecutive positions denoted with $k-1$ and k accordingly. Since we are creating a dynamic model which includes a time dependence in each model parameter we are no longer going to use subscripts to denote the positions but rather will use the functional dependence $(\cdot)$ to indicate implicit time dependence for every model parameter. With that note in mind the recursive relation for two consecutive positions given in their natural coordinates $s(k-1)$ and $s(k)$ become:

$$s(k) = s(k-1) + \Delta s(k) \qquad (2)$$

In case of a property P travelling toward its bound state we will have:

$$E_P(k) = E_P(k-1) + \sum_{j=1}^{N(P)} \vec{f}\left(\vec{p}_j(k-1) + \Delta\vec{p}(k), l_j\right)\Delta\vec{p}(k) \qquad (3)$$

Here $N(P)$ represents the number of semantic aspects in P. The vector in semantic space $\vec{p}_j(k-1)$ represents the position of the aspect $A_j(P)$ in a previous time moment t_{k-1}

Thus, for each aspect $A_j(P)$ we can write the following recurrent relation between $A_j(P)$'s previous position and the current one:

$$\vec{p}_j(k) = \vec{p}_j(k-1) + \Delta\vec{p}(k) \qquad (4)$$

If we denote with $\vec{p}_c(k)$ the position of the centroid of the property P at time t_k we have similar to (4) recurrent relation for the centroid of P:

$$\vec{p}_c(k) = \vec{p}_c(k-1) + \Delta\vec{p}(k) \qquad (5)$$

Let us denote with $\vec{p}_E(k)$ the position of energy weighted center of the ensemble V at time t_k. Then we have:

$$\Delta\vec{p}(k) = \frac{\vec{p}_E(k) - \vec{p}_c(k)}{|\vec{p}_E(k) - \vec{p}_c(k)|}\Delta s(k) \qquad (6)$$

Note that at every moment t_k we can calculate:

- the energy-weighted center $\vec{p}_E(k)$ of the ensemble
- the increments $\Delta\vec{p}_i(k)$ of the new positions of the centroids $\vec{p}_{c,i}$ of the properties P_i
- the energy $E\left(\vec{p}_{c,i}(k)\right)$ at the new positions of the centroids $\vec{p}_{c,i}$ of the properties P_i
- harmonic energy aggregate $\tilde{E}(k)$ of the ensemble

from the coupled system of equations:

$$\begin{cases} E\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) = E\left(\vec{p}_{c,i}\right) + \sum_{j=1}^{N_i} \vec{f}\left(\vec{p}_{i,j} + \Delta\vec{p}_i, l_{i,j}\right)\Delta\vec{p}_i \\ \tilde{E} = \dfrac{\sum_{i=1}^{M} \mathfrak{m}_i}{\sum_{i=1}^{M} \frac{\mathfrak{m}_i}{E_t\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right)}} \\ \vec{p}_E = \sum_{i=1}^{M} \dfrac{\tilde{E}_t\, \mathfrak{m}_i}{E_t\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right)}\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) \\ \Delta\vec{p}_i = \dfrac{\vec{p}_E - \vec{p}_{c,i}}{|\vec{p}_E - \vec{p}_{c,i}|}\Delta s \end{cases} \qquad (7)$$

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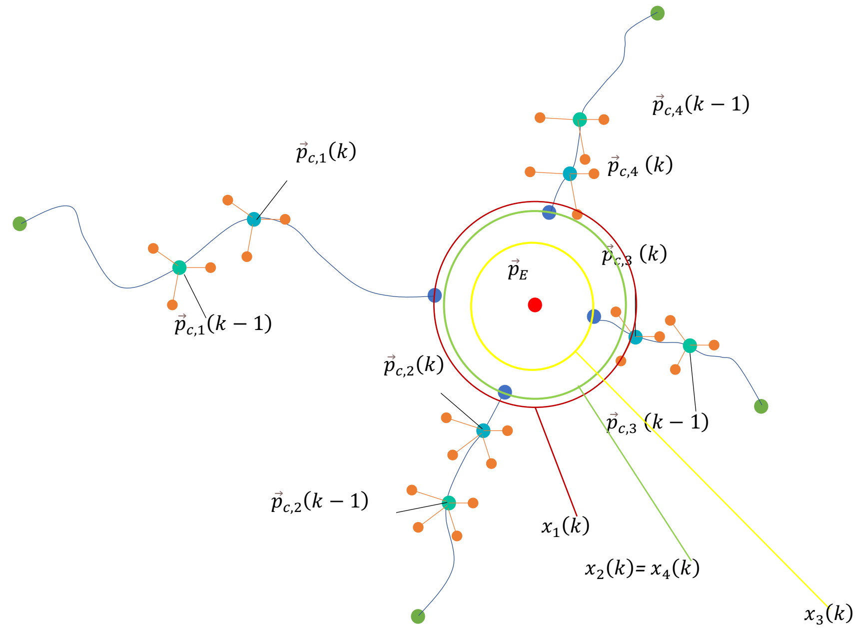
So using the relations (1) - (7) one can determine the movements of the centroids of each of the properties, the energies stored in the properties and the position of the energy weighted centroid of the ensemble for each moment of time. However, we will not be able to determine the positions or the behavior of each property near their bound states. For this we will need to add one extra piece to our dynamic model.

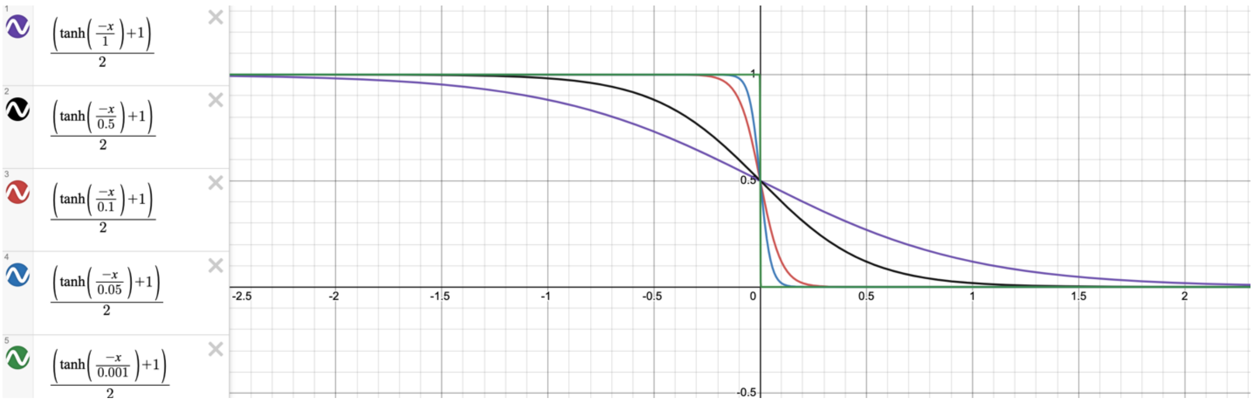
Dynamic modeling of bound state behavior

With static models the bound state of each property is represented by two semantic quantities – the ending position of the travel path and the net energy of the property at that position. Implicitly assumed in static models is that the environment (e.g. scalar energy field) is conditioned in such way that each property will reach the ending position of its bound state.



$$\begin{cases} E(\vec{p}_{c,i}+\Delta\vec{p}_i) = E(\vec{p}_{c,i}) + \sum_{j=1}^{N_i} \vec{f}(\vec{p}_{i,j}+\Delta\vec{p}_i, l_{i,j})\Delta\vec{p}_i \\ x_i = \sqrt{\frac{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}{\mathfrak{m}_i} K_i} \quad ; \quad K_i = 1 + \frac{\lambda_i}{2} + \frac{{\lambda_i}^2}{3} + \mathcal{O}\left({\lambda_i}^3\right) \ ; \ \lambda_i = \frac{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}{\sum_{i=1}^{M} E(\vec{p}_{c,i}+\Delta\vec{p}_i)} \\ \tilde{E} = \frac{\sum_{i=1}^{M} \mathfrak{m}_i}{\sum_{i=1}^{M} \frac{\mathfrak{m}_i}{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}} \\ \vec{p}_E = \sum_{i=1}^{M} \frac{\tilde{E}\,\mathfrak{m}_i}{E(\vec{p}_{c,i}+\Delta\vec{p}_i)} (\vec{p}_{c,i}+\Delta\vec{p}_i) \\ H_i = \frac{\tanh\left(\frac{|\vec{p}_E-\vec{p}_{c,i}|-x_i}{x_u}\right)+1}{2} \\ \Delta\vec{p}_i = \frac{\vec{p}_E-\vec{p}_{c,i}}{|\vec{p}_E-\vec{p}_{c,i}|} H_i\,\Delta s \end{cases} \tag{8}$$

Here x_u controls the slope from 1 to 0 of the factor H_i when the property P_i approaches bound state x_i. Usually, we would want $x_u \ll \min_i x_i$.



Let us understand how the coupled system of equations (8) behaves.
Initially, we have a set of n property particles $P_1, \dots P_n$ in their in-situ positions. Let us consider the trajectory of the i-th property P_i toward its bound state. The trajectory of each property depends on the initial positions and count of the other properties as well as the internal structure of those i.e. the aspect content of each of them. Let us denote the *trajectory* of P_i from moment t_0 to t_k with $T(P_i, t_0, t_k)$. For brevity we will use the concise notation $T_{i,0,k} \equiv T(P_i, t_0, t_k)$. With $E_{i,0,k}$ we denote the *energy signature* of P_i along the trajectory $T_{i,0,k}$ that is the total energy of the set of aspects which belong to P_i when the centroid $\vec{p}_{c,i}$ follows the trajectory $T_{i,0,k}$. So, for every point $t_{i,l}$ (with $0 \leq l < k$) in semantic space which belong to the trajectory $T_{i,0,k}$ we will have a real value $E_{i,l}$ which is the total energy accumulated in P_i for the interval $\left[\vec{p}_{c,i}(l), \vec{p}_{c,i}(l+1)\right)$.
So based on the equations (8) obviously

$$T_{i,0,k} = f\left(T_{1,0,k-1}, \dots, T_{i-1,0,k-1}, T_{i+1,0,k-1}, \dots, T_{n,0,k-1}, E_{1,0,k-1}, \dots, E_{n,0,k-1}\right)$$

In the special case when $k = 1$ we obviously have two discrete points and a single interval at each trajectory T_i. Then

$$T_{i,0,1} = f\left(t_{1,0}, \dots, t_{i-1,0}, t_{i+1,0}, \dots, t_{n,0}, E_{1,0}, \dots, E_{n,0}\right)$$

We can divide the functional arguments of $T_{i,0,1}$ into two sets:

- Those which represent the semantic content of the properties

These are $t_{1,0}, \dots, t_{i-1,0}, t_{i+1,0}, \dots, t_{n,0}$. It is obvious why this is the case.

- Those which represent the influence of the environment

Obviously, these are $E_{1,0}, \dots, E_{n,0}$.
Let us have

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Initial conditions in the dynamic equations of an ensemble

Let us have the following initial conditions: Each property P_i of the ensemble is subjected to initial impulse $L_0^{(i)} = \mathrm{m}_i \vec{v}_0^{(i)}$ applied at the centroid $\vec{p}_{c,i}$. With

$$
\begin{cases}
E\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) = E\left(\vec{p}_{c,i}\right) + \sum_{j=1}^{N_i} \vec{f}\left(\vec{p}_{i,j} + \Delta\vec{p}_i, l_{i,j}\right)\Delta\vec{p}_i \\
\tilde{E} = \frac{\sum_{i=1}^{M} \mathfrak{m}_i}{\sum_{i=1}^{M} \frac{\mathfrak{m}_i}{E_t\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)}} \\
\vec{p}_E = \sum_{i=1}^{M} \frac{\tilde{E}_t\, \mathfrak{m}_i}{E_t\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)} \left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) \\
\Delta\vec{p}_i = \frac{\vec{p}_E - \vec{p}_{c,i}}{\left|\vec{p}_E - \vec{p}_{c,i}\right|} \frac{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)}{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)+E_0^{(i)}} \Delta s + \frac{\vec{p}_0^{(i)} - \vec{p}_{c,i}}{\left|\vec{p}_0^{(i)} - \vec{p}_{c,i}\right|} \frac{E_0^{(i)}}{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)+E_0^{(i)}} \Delta s
\end{cases}
$$

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Accounting for attractive and repulsive forces between aspects of different Properties near bound state

$$
\begin{cases}
E\left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) = E\left(\vec{p}_{c,i}\right) + \sum_{j=1}^{N_i} \vec{f}\left(\vec{p}_{i,j} + \Delta\vec{p}_i, l_{i,j}\right)\Delta\vec{p}_i \\
x_i = \sqrt{\frac{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)}{\mathfrak{m}_i} K_i} \;\; ; \;\; K_i = 1 + \frac{\lambda_i}{2} + \frac{\lambda_i^{\,2}}{3} + \mathcal{O}\left(\lambda_i^{\,3}\right) \; ; \; \lambda_i = \frac{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)}{\sum_{i=1}^{M} E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)} \\
\tilde{E} = \frac{\sum_{i=1}^{M} \mathfrak{m}_i}{\sum_{i=1}^{M} \frac{\mathfrak{m}_i}{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)}} \\
\vec{p}_E = \sum_{i=1}^{M} \frac{\tilde{E}\, \mathfrak{m}_i}{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)} \left(\vec{p}_{c,i} + \Delta\vec{p}_i\right) \\
H_i = \frac{\tanh\left(\frac{\left|\vec{p}_E - \vec{p}_{c,i}\right| - x_i}{x_u}\right) + 1}{2} \\
\Delta\vec{p}_i = \frac{\vec{p}_E - \vec{p}_{c,i}}{\left|\vec{p}_E - \vec{p}_{c,i}\right|} H_i \, \Delta s
\end{cases}
$$

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Initial ensemble velocity at creation time of the ensemble

We would like to model the velocity of the ensemble at the time of its creation. We know that the aspect centroids and the mass center of the ensemble slow down to a standstill when approaching bound state. This model does not account for the residual energy stored in the ensemble which we want it to be transferred into initial velocity of the ensemble. The questions which need to be answered are: what is the initial direction of motion of the newly created ensemble and what is the magnitude of the initial velocity. The question about the initial direction of the ensemble can be answered after constructing the semantic signature matrix of the ensemble V:

//TODO: use the signature separators concept from the Semantic Signature of Semantic Properties.

$$\vec{p}_E = \sum_{i=1}^{M} \frac{\tilde{E}\, \mathfrak{m}_i}{E\left(\vec{p}_{c,i}+\Delta\vec{p}_i\right)} \left(\vec{p}_{c,i} + \Delta\vec{p}_i\right)$$

$$\Delta\vec{p}_i = \frac{\vec{p}_E - \vec{p}_{c,i}}{\left|\vec{p}_E - \vec{p}_{c,i}\right|} H_i \, \Delta s$$

$$
\begin{cases}
E(\vec{p}_{c,i}+\Delta\vec{p}_i) = E(\vec{p}_{c,i}) + \sum_{j=1}^{N_i} \vec{f}(\vec{p}_{i,j}+\Delta\vec{p}_i, l_{i,j})\Delta\vec{p}_i \\
x_i = \sqrt{\frac{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}{\mathfrak{m}_i}K_i} \;\; ; \;\; K_i = 1 + \frac{\lambda_i}{2} + \frac{{\lambda_i}^2}{3} + \mathcal{O}\left({\lambda_i}^3\right) \; ; \; \lambda_i = \frac{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}{\sum_{i=1}^{M} E(\vec{p}_{c,i}+\Delta\vec{p}_i)} \\
\tilde{E} = \frac{\sum_{i=1}^{M} \mathfrak{m}_i}{\sum_{i=1}^{M} \frac{\mathfrak{m}_i}{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}} \\
\vec{p}_E = \sum_{i=1}^{M} \frac{\tilde{E}\,\mathfrak{m}_i}{E(\vec{p}_{c,i}+\Delta\vec{p}_i)}(\vec{p}_{c,i}+\Delta\vec{p}_i) \\
H_i = \frac{\tanh\left(\frac{|\vec{p}_E-\vec{p}_{c,i}|-x_i}{x_u}\right)+1}{2} \\
\Delta\vec{p}_i = \frac{\vec{p}_E-\vec{p}_{c,i}}{|\vec{p}_E-\vec{p}_{c,i}|} H_i\, \Delta s
\end{cases}
$$

Dynamic Modeling of Semantic Structure Aggregates

Let us now consider a set of primitive semantic particles $V_i, i = 1..n$.

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Appendices

Appendix A: Work-Energy Theorem

The net work done on semantic particle is equal to the change of the particle internal energy.

$W_{net} = E_b - E_a$

Derivation:

$d W_{net} = \vec{f}_{net}\Delta\vec{p}_c$

The Newton third law is still applicable in semantic structures:

$\vec{f}_{net} = \mathfrak{m}\frac{d\vec{v}}{dt}$. Thus, we have $d W_{net} = \mathfrak{m}\frac{d\vec{v}}{dt}\Delta\vec{p}_c = \mathfrak{m}\, d\vec{v}\, \frac{d\vec{p}_c}{dt} = \mathfrak{m}\,\vec{v}\, d\vec{v}$

Integrating the last differential equality from a to b yields
$W_{net}(a,b) = \int_a^b \mathfrak{m} \sum_{l=1}^{L} v_l d v_l$

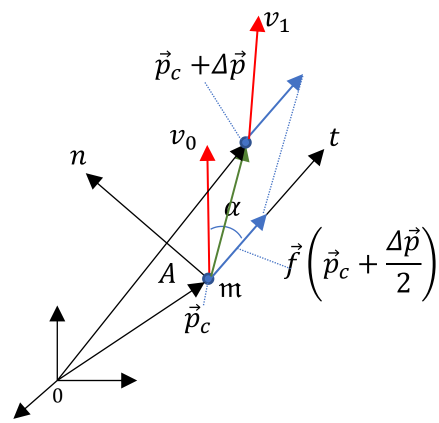
Here $\vec{v} = \sum_{l=1}^{L} v_l \vec{e}_l$ where $\vec{e}_l$ is the unit vector of the l-th semantic dimension.
Thus $W_{net}(a,b) = \frac{1}{2}\mathfrak{m}\sum_{l=1}^{L} {v_l}^2\Big|_a^b = \frac{1}{2}\mathfrak{m}v^2|_a^b = E_b - E_a$.

Appendix B: Initial conditions in dynamic equations of semantic ensemble

Let us consider a semantic aspect A with mass $\mathfrak{m}$ moving through semantic space with initial velocity v_0. Let us consider a small incremental step $\Delta\vec{p}$ along the trajectory of A. The position of A when the initial velocity v_0 is applied is given with the vector $\vec{p}_c$. Since the aspect A is immersed in the reinforcement field $\mathfrak{F}$ we will approximate the semantic force acting on A during the small interval $\Delta\vec{p}$ with

$\vec{f}\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)$. Let us select a coordinate system composed of normal n and tangent t axes with origin at A. The direction of the tangent t coincides with the direction of the semantic force from the reinforcement field $\mathfrak{F}$ at $\vec{p}_c$. The projections of the initial velocity v_0 to the normal and tangent are:

$v_{0,t} = v_0 \cos\alpha$, $v_{0,n} = v_0 \sin\alpha$ and $\vec{v}(\vec{p}_c) = v_{0,n}\vec{n} + v_{0,t}\vec{t}$



In the absence of initial velocity at position $\vec{p}_c$ of A the change of the semantic energy ΔE of A during the interval $\Delta\vec{p}$ is given with:

$$\Delta E = \vec{f}\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)\Delta\vec{p} = \frac{1}{2}\mathfrak{m}v_t^2\Big|_{v_{t,0}}^{v_{t,1}}$$

Therefore, the change of the aspect velocity solely due to the influence of the reinforcement field is given with

$$\Delta v_t = \sqrt{\frac{2\vec{f}\left(\vec{p}_c+\frac{\Delta\vec{p}}{2}\right)\Delta\vec{p}}{\mathfrak{m}}} \quad \text{(B.1)}$$

For the new position of A obviously we have:

$$\vec{v}\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right) = \vec{v}_1 = v_n\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)\vec{n} + v_t\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)\vec{t} = v_{1,n}\vec{n} + v_t\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)\vec{t} \qquad \text{(B.2)}$$

For the change in semantic energy due to the change in velocity along t we can write:

$$\frac{1}{2}\mathfrak{m}v_{1,t}^2 = \frac{1}{2}\mathfrak{m}v_{0,t}^2 + \vec{f}\left(\vec{p}_c + \frac{\Delta\vec{p}}{2}\right)\Delta\vec{p} \qquad \text{(B.3)}$$

Thus, we can write:

$$v_{1,t} = v_{0,t}\sqrt{1 + \frac{2\vec{f}\left(\vec{p}_c+\frac{\Delta\vec{p}}{2}\right)\Delta\vec{p}}{\mathfrak{m}v_{0,t}^2}} \qquad \text{(B.4)}$$

Now let us consider the function $f(x)$ which we want to approximate for $|x| \ll 1$

$$f(x) = \sqrt{1+x}\ ,\ |x| \ll 1 \qquad \text{(B.5)}$$

Using Taylor expansion for $|x| \ll 1$ one writes:

$$f(x) \sim 1 + \frac{1}{2}x + \mathcal{O}(x^2) \qquad \text{(B.6)}$$

Substituting (B.6) in (B.4) gives us:

$$v_{1,t} = v_{0,t}\left(\frac{\mathfrak{m}v_{0,t}^2 + \vec{f}\left(\vec{p}_c+\frac{\Delta\vec{p}}{2}\right)\Delta\vec{p}}{\mathfrak{m}v_{0,t}^2}\right) \quad \text{(B.7)}$$

the normal projection of the aspect velocity will not change:

$$v_{1,n} = v_{0,n} \quad \text{(B.8)}$$