

## Note on binding of match-seeking and match-repelling particles

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### Binding between two primitive particles through match-seeking particle

Let us consider two  $V$ -particles which are not composite – they are given with their semantic signatures respectively:

$$\begin{aligned} \text{ssig}(V') &= [\mathbf{p}'_0 \ \mathbf{a}'_{0,1} \ \mathbf{p}'_1 \ \mathbf{p}'_0 \ \mathbf{a}'_{0,2} \ \mathbf{p}'_2 \ \mathbf{p}'_0 \ \mathbf{a}'_{0,3} \ \mathbf{p}'_3 \ \dots \ \mathbf{p}'_i \ \mathbf{a}'_{i,n} \ \mathbf{p}'_n] \\ \text{ssig}(V'') &= [\mathbf{p}''_0 \ \mathbf{a}''_{0,1} \ \mathbf{p}''_1 \ \mathbf{p}''_0 \ \mathbf{a}''_{0,2} \ \mathbf{p}''_2 \ \mathbf{p}''_0 \ \mathbf{a}''_{0,3} \ \mathbf{p}''_3 \ \dots \ \mathbf{p}''_j \ \mathbf{a}''_{j,m} \ \mathbf{p}''_m] \end{aligned}$$

Here each of the quantities  $\mathbf{p}$  denotes the property signature vector of the corresponding property  $P$  of the  $V$  particle. The matrix  $\mathbf{a}_{r,s}$  represents the property association particle  $A_{r,s}$  which binds to a pair of properties  $P_r$  and  $P_s$  in the property graph  $\mathcal{P}$  of the  $V$  particle. Also there is a semantic significance vector  $\mathbf{w}_{r,s}$  which is associated the property association particle (a.k.a link)  $A_{r,s}$ . For details refer to [The Signature of Semantic Structures](#).

Match-seeking particle  $MA$  binds to a subgraph  $\mathcal{S}$  of the property graph  $\mathcal{P}$  of the  $V$  particle.

There is a closeness condition which needs to be obeyed in order the particle  $MA$  to bind to the particle  $V$ .

### Binding matrix of a match-seeking particle

The match-seeking particle  $MA$  exposes a binding matrix  $\text{mbind}(MA)$ :

$$\text{mbind}(MA) = [\mathbf{B}^1 \ \mathbf{B}^2 \ \mathbf{B}^3 \ \dots \ \mathbf{B}^q]$$

$$\mathbf{B}^1 = [\mathbf{p}^0 \ \mathbf{a}^{0,1} \ \mathbf{p}^1], \mathbf{B}^2 = [\mathbf{p}^0 \ \mathbf{a}^{0,2} \ \mathbf{p}^2], \mathbf{B}^3 = [\mathbf{p}^0 \ \mathbf{a}^{0,3} \ \mathbf{p}^3], \dots, \mathbf{B}^q = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q]$$

Obviously, each of the blocks  $\mathbf{B}^i$  is  $N \times 4$  matrix where  $N$  is the dimension of semantic space. From now on we will denote these blocks of any match-seeking particle as *binding elements*  $B^i$  of the match-seeking particle  $M$ . Each binding element  $B^i$  of a match-seeking particle consists of a couple of property particles  $P^a$  and  $P^b$  connected with association particle  $A^{a,b}$ . Each binding element  $B^i$  is represented by its binding matrix  $\mathbf{B}^i$  and its semantic significance vector  $\mathbf{w}^i$ .

Note that in each of those blocks having the general form  $\mathbf{B}^i = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q]$  it is possible to have  $\mathbf{a}^{p,q} = \mathbf{p}^q = \mathbf{0}$  where  $\mathbf{0}$  represents the null vector in semantic space. However,  $\mathbf{p}^p$  is never close to the null vector i.e.  $|\mathbf{p}^p| > \mathbf{0}$ .

### Binding of match-seeking particle against $V$ -particle formulated as optimization problem

Let a primitive particle  $V$  has the following semantic signature:

$$\text{ssig}(V) = [\mathbf{B}_1 \ \mathbf{B}_2 \ \dots \ \mathbf{B}_m]$$

Let us denote by  $f_j^i$  the semantic distance between the binding element  $B^i$  of  $MA$  and the semantic element  $B_j$  of  $V$

$$f_j^i = \text{sdist}(B^i, B_j), \mathbf{B}^i = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q], \mathbf{B}_j = [\mathbf{p}_r \ \mathbf{a}_{r,s} \ \mathbf{p}_s]$$

Then we define the following metric:

$$\text{sdist}(B^i, B_j) = |\mathbf{p}^p| \text{sdist}(P^p, P_r) + |\mathbf{p}^p| |\mathbf{p}^q| \text{sdist}(A^{p,q}, A_{k,l}) + |\mathbf{p}^q| \text{sdist}(P^q, P_s)$$

where

$$sdist(P^p, P_r) = |\mathbf{p}^p - \mathbf{p}_r|, \quad sdist(P^q, P_s) = |\mathbf{p}^q - \mathbf{p}_s|$$

$$sdist(A^{p,q}, A_{r,s}) = |\mathbf{w}^{p,q} - \mathbf{w}_{r,s}| \times sdist(\mathbf{a}^{p,q}, \mathbf{a}_{r,s})$$

The semantic distance of two matrices  $\mathbf{a}$  and  $\mathbf{b}$ , having the same number of columns, is given with:

$$sdist(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^n |\vec{\mathbf{a}}_i - \vec{\mathbf{b}}_i| \text{ where } \mathbf{a} = [\vec{\mathbf{a}}_1 \vec{\mathbf{a}}_2 \dots \vec{\mathbf{a}}_n] \text{ and } \mathbf{b} = [\vec{\mathbf{b}}_1 \vec{\mathbf{b}}_2 \dots \vec{\mathbf{b}}_n].$$

Notice that if  $B^i$  is incomplete that is – contains only a single property not connected to anything then  $sdist(B^i, B_j)$  becomes simply the semantic distance between its sole property particle of the binding element  $B^i$  and the corresponding property of the semantic element  $B_j$ .

//TODO: finish the optimization problem formulation

Closeness condition for a bind between match seeking particle and primitive semantic particle

Let us denote by  $sfil(MA, V)$  the following diagonal matrix which will be named *Filter matrix* of the match seeking particle:

$$sfil(MA, V) = \begin{bmatrix} I_1 & & & & \\ & 0 & & & \\ & & I_2 & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & I_k \end{bmatrix}$$

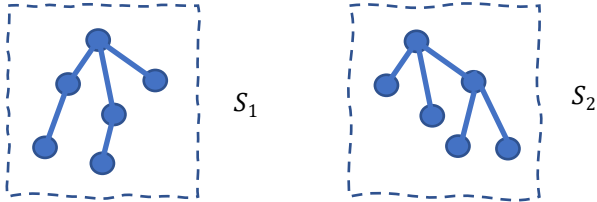
Here  $I_i, i = 1, 2, \dots, k$  are identity matrices which represent the regions of interest in the semantic signature matrix of  $V$  to the match seeking particle  $MA$ .

The regions of interest  $sreg(MA, V)$  in the semantic signature of  $V$  are obtained by multiplying  $sfil(MA, V)$  with  $ssig(V)$ :

$$sreg(MA, V) = sfil(MA, V) \times ssig(V)$$

Between two semantic structures

Let us have two semantic structures  $S_1$  and  $S_2$ .



Let the semantic signature of  $S_1$  is given with:

$$ssig(S_1) = [\mathbf{V}_1 \mathbf{A}_{1,2} \mathbf{V}_2 \mathbf{A}_{1,3} \mathbf{V}_3 \dots \mathbf{A}_{r,p} \mathbf{V}_p]$$

and the semantic signature of  $S_2$  is given with:

Between a primitive  $V$  particle and a semantic structure  $S$