Semantic Templates and Semantic Functions

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Semantic Functions

Semantic Functions will be denoted with small cap Greek letters capturing the semantics of the specific function such as the *Action function* δ (from Greek δράση for *Action*).
Real valued Semantic function $\varphi: \mathbf{\Sigma} \to \mathbb{R}$ is function defined on semantic space $\mathbf{\Sigma}$ and having a value on the real axis $\mathbb{R}$. Every object in semantic space is represented by its semantic signature.

Semantic Action Functions

What is a Semantic Template?

Every Semantic Template is represented by an incomplete semantic structure which contains missing substructures (i.e. *compound semantic particles*) and/or missing *primitive semantic particles* and/or missing *semantic property particles*. The place of each missing particle is occupied by a relevant *replacement particle* which contains properties generating the necessary binding force and has an appropriate semantic mass which match the position of the particle in the semantic template. The Semantic Templates will be denoted capital fraktur letters ($\mathfrak{T}$, $\mathfrak{P}$, $\mathfrak{I}$, …) subscripted with an index appropriately. The constructs within the Semantic Templates will be denoted with capital Latin letters $M(\mathfrak{T})$, $I(\mathfrak{T})$ where the fraktur letter inside the parentheses indicates the template those constructs are part of. The various semantic spaces such regular semantic space and template space will be denoted with capital Greek letters $\mathbf{\Sigma}$, $\mathbf{T}$ subscripted with index appropriately.

Depending on what is being matched we divide all Semantic Templates into two categories – *Logical Semantic Templates* and *Physical Semantic Templates*. With Logical Semantic Templates we are matching **only** traits of the semantic signature within the specified Semantic region. With Physical Semantic Templates we are matching **any physical properties** pertaining to the structures and particles within the specified Semantic region.

Every Semantic Template $\mathfrak{T}$ consists of two constructs – *pattern matching construct* $M(\mathfrak{T})$ and *inference construct* $I(\mathfrak{T})$.

Definition: *Centroid of Semantic Template*: represents the mass center of the template structure using the semantic masses of the replacement particles.

Definition: *Regular Semantic Space (or just Semantic Space)*: Semantic space of dimension L which is populated with the semantic structures created by parsing external constructs or by inference. Denoted with $\mathbf{\Sigma}$.

Definition: *Semantic Template Space (or just Template Space)*: Pattern-matching structures exist in a space having the same number of dimensions L as regular semantic space. The template space is parallel to *regular semantic space*. Denoted with $\mathbf{T}$ (tau). Unlike regular semantic space the *template space* is populated with (fuzzy) semantic constructs in which some of the particles (properties, primitive semantic particles, compound semantic particles) are replaced by *template particles*. Each semantic template $\mathfrak{T}$ is associated with a region $\mathfrak{A}(\mathfrak{T})$ (region of *applicability*) of regular semantic space $\mathbf{\Sigma}$ in which the template is valid. To be precise, $\mathfrak{A}(\mathfrak{T}) \subset \mathbf{\Sigma}$ is a region in which its centroid $C(\mathfrak{T})$ is allowed to be positioned without violating the applicability condition of $\mathfrak{T}$. Obviously, this region changes with the elapsed time as the semantic structures of interest (relevant semantic context) move through $\mathbf{\Sigma}$.

Definition: *Template Inference Space* (or just *Inference Space*):
Denoted with $\mathbf{I}$ (*iota* Greek). Has the same number of dimensions L as regular semantic space $\mathbf{\Sigma}$.

Definition: *Intermediate Space Stack*:
A countable set of Semantic Space sheets each of which is parallel to $\mathbf{\Sigma}$ used to facilitate the template matching and inference. In a sense it plays the role of a semantic scratch pad. Each semantic sheet has the same number of dimensions L as regular semantic space $\mathbf{\Sigma}$. Each semantic sheet is denoted with $\mathbf{\Sigma}_{\boldsymbol{\alpha}}$ where $\boldsymbol{\alpha}$ is the sheet index.

Definition: *Extended Semantic Space*.
Denoted with $\mathbf{E}$. Has one more dimension $L+1$ than regular semantic space $\mathbf{\Sigma}$. Introduced to facilitate semantic computations. All defined so far semantic spaces will be considered part of the extended semantic space. Thus $\mathbf{\Sigma}, \mathbf{\Sigma}_{\boldsymbol{\alpha}}, \mathbf{I}, \mathbf{T} \subset \mathbf{E}$.

Definition: *Semantic Template*: It is a semantic relation which maps a semantic structure from semantic space $\mathbf{\Sigma}$ to new *non-empty* semantic structure from $\mathbf{\Sigma}$ if the pattern matching region $\mathfrak{P}(\mathfrak{T})$ has been matched to some semantic structure S from $\mathbf{\Sigma}$ or it is the empty semantic structure [] if no match is found.

~~**Definition**: *Centroid and Radius of Semantic Template*: *Centroid* is the current point C in Semantic Space where the pattern matching construct $M(\mathfrak{T})$ is centered thereby assuming *Radius (or Range)* R. The centroid of the pattern-matching construct is mapped to a point in regular semantic space indicating the possible location of the root node of the semantic structure S which will be pattern-matched by $M(\mathfrak{T})$. Let us denote with $O(\mathfrak{T}, C, R)$ the semantic output of the pattern matching region centered in C with radius R. The structures of $O(\mathfrak{T}, C, R)$ will be created in one of the semantic sheets $\mathbf{\Sigma}_{\boldsymbol{\alpha}}$.~~

~~**Definition**: *Matching of Semantic Template*: the centroid of the pattern matching construct $M(\mathfrak{T})$ moves within the region of applicability $\mathfrak{A}(\mathfrak{T})$ in Semantic Template Space $\mathbf{T}$. When the semantic latch μ associated with $\mathfrak{T}$ is triggered the centroid of $\mathfrak{T}$ is affixed to the point which has triggered the latch. The radius of the pattern matching region starts expanding until *optimal match* is selected. For definition of optimal match refer to Pattern Matching Structure of Semantic Template.~~

Matching of logical semantic template

Matching of physical semantic template

Pattern Matching Structure of Semantic Template

The pattern matching construct $\mathfrak{P}(\mathfrak{T})$ of any template $\mathfrak{T}$ is represented by a semantic tree T in which every node is one of the three:

a) semantic structure S
b) primitive semantic particle V
c) template particle X

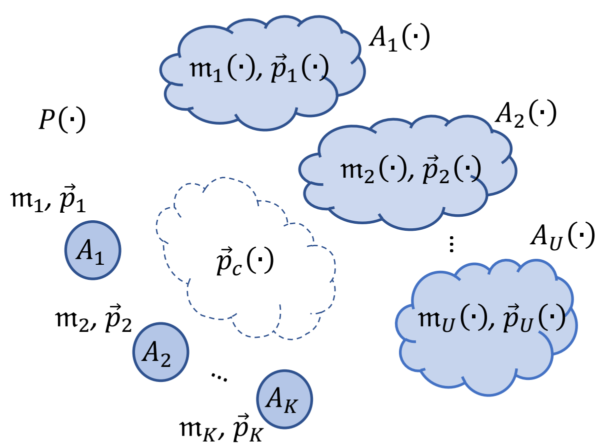
Template Particles

A template particle can be viewed as stochastic generalization of regular semantic particles. Instead of a single semantic particle on a given position in semantic space and semantic time a template particle will define a *cloud* of *stochastic* semantic particles in a region of semantic space-time.

We will consider template property particles and template semantic particles

Definition: *Template property particle*:

Every template property particle P at a moment of semantic time t is defined in terms of two sets of aspects – the set of *deterministic* aspects $\mathcal{D}$ and set of *stochastic* aspects $\mathcal{T}$.
Each aspect A_i in the set of *deterministic* aspects $\mathcal{D} = \{A_1, \dots, A_K\}$ is characterized by its deterministic mass $\mathfrak{m}_i$ and deterministic position in semantic space $\vec{p}_i$, $i = 1..K$. The semantic aspects in the set of stochastic aspects $\mathcal{T}$ form a *stochastic cloud*. Each stochastic aspect $A_j(\cdot)$ in the *cloud* $\{A_1(\cdot), \dots, A_U(\cdot)\}$ is characterized by its stochastic mass $\mathfrak{m}_j(\cdot)$ and stochastic location $\vec{p}_j(\cdot)$, $j = 1..U$. Here we have introduced the postscript notation $(\cdot)$ to denote that we are dealing with stochastic quantity which is characterized by a probability distribution $\mathfrak{f}(\cdot)$. When the stochastic quantity is scalar the corresponding probability distribution $\mathfrak{f}(\cdot)$ is a function of a single variable; in case the stochastic quantity is a vector then the probability distribution $\mathfrak{f}(\cdot)$ is a function of the same number of variables as the quantity dimension. Using the introduced postscript notation $(\cdot)$ we will denote the cloud of stochastic aspects with $\mathcal{T}(\cdot)$ and the template particle with $P(\cdot)$ as shown on the Figure below:



For the centroid of $P(\cdot)$ we write:

$$\vec{p}_c(\cdot) = \sum_{i=1}^{K} \frac{\mathfrak{m}_i}{\mathfrak{m}_{tot}(\cdot)} \vec{p}_i + \sum_{j=1}^{U} \frac{\mathfrak{m}_j(\cdot)}{\mathfrak{m}_{tot}(\cdot)} \vec{p}_j(\cdot) \text{ where } \mathfrak{m}_{tot}(\cdot) = \sum_{i=1}^{K} \mathfrak{m}_i + \sum_{j=1}^{U} \mathfrak{m}_j(\cdot) \qquad (1)$$

The moment in time t_c in which the centroid of the template property assumes position $\vec{p}_c(\cdot)$ can also be stochastic quantity which will be denoted with $t_c(\cdot)$. We will denote with $\vec{P}_c$ the $L+1$-vector corresponding to the space-time position of the template property centroid. Obviously,

$$\vec{P}_c = \begin{Bmatrix} \vec{p}_c \\ t_c \end{Bmatrix} \quad (2)$$

In general, the $L+1$-vector of the template centroid will be a stochastic quantity: $\vec{P}_c(\cdot)$.
Let us denote the $L+1$-vectors of the aspects of the template property by $\vec{P}_i(\cdot)$.
Let us denote the distance between the i-th aspect and the centroid with $\vec{d}_i(\cdot)$ and the $L+1$-vector corresponding to that distance with $\vec{D}_i(\cdot)$. For $\vec{d}_i(\cdot)$ we have:

$$\vec{d}_i(\cdot) = \vec{p}_j(\cdot) - \vec{p}_c(\cdot) \quad (3)$$

~~In the template property we can have a stochastic cloud $\mathcal{A}(\cdot)$ which matches any subset of aspects $\{A_1, \dots, A_U\}$ such that certain stochastic quantity is preserved. For instance, such quantity could be the total semantic mass of the subset of aspects:~~

$$\cancel{\mathfrak{m}_{\mathcal{A}}(\cdot) = \sum_{j=1}^{U} \mathfrak{m}_j(\cdot) \quad (3)}$$

For simplicity of the representation and without loss of generality we assume that the whole template property is represented by semantic cloud of aspects.

The aggregate characteristics of template property $P(\cdot)$ are given with the stochastic quantities *center of the matched semantic property* $\vec{p}_c(\cdot)$ and *mass of the matched semantic property* $\mathfrak{m}_{tot}(\cdot)$.

$$\vec{p}_c(\cdot) = \sum_{j=1}^{N} \frac{\mathfrak{m}_j(\cdot)}{\mathfrak{m}_{tot}(\cdot)} \vec{p}_j(\cdot) \text{ where } \mathfrak{m}_{tot}(\cdot) = \sum_{j=1}^{N} \mathfrak{m}_j(\cdot) \quad (4)$$

Note that we have defined $N+1$ spatial coordinates but we have only N independent spatial coordinates because:

$$\sum_{j=1}^{N} \frac{\mathfrak{m}_j(\cdot)}{\mathfrak{m}_{tot}(\cdot)} \vec{d}_j(\cdot) = 0 \quad (5)$$

For the whole template property we will assume we have only one independent temporal coordinate t which characterize the position in time of the matched property:

$$\vec{p}_c(\cdot) = f(t) \quad (6)$$

Additionally, we will assume that the matched aspects are *rigid* (not *elastic*), that is $\vec{d}_j(\cdot)$ does not depend on t.

Therefore, the template property $P(\cdot)$ is characterized completely by the following stochastic quantities:

$$\vec{p}_c(\cdot), t_c(\cdot), \mathfrak{m}_{tot}(\cdot), \vec{D}_j(\cdot), \mathfrak{m}_j(\cdot), j = 1..N-1 \quad (7)$$

Accounting for the vector quantities in L dimensional semantic space our template property $P(\cdot)$ made of N aspects will be represented uniquely by $(L+1)N + 1$ scalar random quantities.

Each of the quantities in (7) is defined with its probability distribution which will be generally denoted with $\mathfrak{f}(\cdot)$ using appropriate subscript which will refer to the stochastic quantity of interest. For instance, $\mathfrak{g}_{P_c}(\vec{\omega})$ will refer to the probability distribution that the centroid of P is within some $L+1$-dimensional region of semantic space-time. Here the $L+1$ vector $\vec{\omega}$ will identify uniquely a point in semantic space and time.

$$\vec{\omega} = p_1\vec{e}_1 + p_2\vec{e}_2 + \cdots + p_L\vec{e}_L + t\vec{e}_{L+1} \quad (8)$$

Here $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_L$ represents a basis in the L dimensional semantic space and $t\vec{e}_{L+1}$ represents the time component always advancing in positive direction.

Let us consider the centroid of $P(\cdot)$. Here $\mathfrak{g}_{P_c}(\cdot)$ represents the joint distribution of spatial and temporal position of the centroid of $P(\cdot)$. Then, the probability that the centroid of $P(\cdot)$ can be found in the region of space ∂S_P and within the time interval Δt is given with:

$$\Pr\left(\vec{P}_c \in \partial S_{P_c} \times \Delta t\right) = \int_{\partial S_{P_c} \times \Delta t} \mathfrak{g}_{P_c}(\vec{\omega})\, d\Omega \quad (9)$$

Here $\vec{\omega}$ represents the $L+1$-vector of a point within the region $\partial S_P \times \Delta t$ in Semantic space-time and is given with (8) with respect to some basis. $d\Omega$ denotes elementary volume in $L+1$ dimensional space-time and is given with:

$$d\Omega = dp_1 dp_2 \ldots dp_L dt \quad (10)$$

In the future discussions we will assume that the spatial and the temporal dependences in the joint distributions $\mathfrak{f}(\cdot)$ of all stochastic quantities in the template property are independent of each other. That is

$$\mathfrak{g}(\vec{\omega}) = \mathfrak{f}(\vec{s})\mathfrak{h}(t) \quad (11)$$

In (11) $\mathfrak{f}(\vec{s})$ denotes the spatial probability density function of some stochastic quantity. Obviously

$$\Pr\left(\vec{P}_c \in \partial S_{P_c}\right) = \int_{\partial S_{P_c}} \mathfrak{f}_{P_c}(\vec{s})\, dV \quad (12a)$$

Here the event $\vec{P}_c \in \partial S_{P_c}$ denotes the occurrence of the property centroid in some region of semantic space ∂S_{P_c}.

$$\Pr\left(\vec{P}_c \in \Delta t\right) = \int_{\Delta t} \mathfrak{h}(t)\, dt \quad (12b)$$

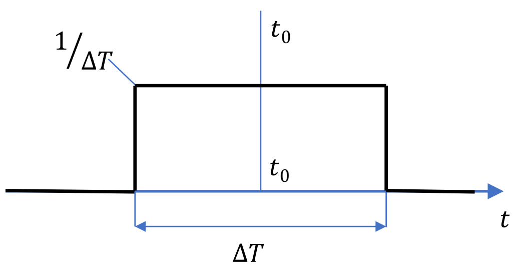
Here the event $\vec{P}_c \in \Delta t$ denotes the occurrence of the property centroid in some interval of time Δt.

Definition: *Binding Region* of semantic template property
The binding region of semantic template property P given with the quantities described in (7) is defined as the region S_P of semantic space defined as:
$S_P = \{\vec{s}: \quad \mathfrak{f}(\vec{s}) > 0\}$ for at least one of the stochastic quantities listed in (7)

Let us consider an aspect $A_j(\cdot)$ from $P(\cdot)$. Obviously, $A_j(\cdot)$ is described completely by the tuple $\vec{D}_j(\cdot)$, $\mathfrak{m}_j(\cdot)$. Here $\mathfrak{f}_{D_j}(\cdot)$ represents the joint distribution of spatial and temporal position of $A_j(\cdot)$ with respect to the centroid of the matched property. That is, the probability that aspect $A_j(\cdot)$ can be found in the region of space ∂S_{D_j} and within the time interval Δt is given with:

$$\Pr\left(\vec{D}_j \in \partial S_{D_j} \times \Delta t\right) = \int_{\partial S_{D_j} \times \Delta t} \mathfrak{g}_{D_j}(\vec{\omega})\, d\vec{\omega} \qquad (13)$$

Let us consider the special case when the temporal position of the matching property is a uniform random variable centered around some value t_0 :



We will assume independence of the spatial and temporal distributions (which in our first order dynamic semantic model will be a reasonable assumption). Then (9) can be rewritten as:

$$\Pr(\vec{p}_c \in \partial S_P \cap t \in \Delta T) = \int_{\partial S_P} \mathfrak{f}_{P_c}(\vec{s})\, d\vec{s} \int_{t_0-\frac{\Delta T}{2}}^{t_0+\frac{\Delta T}{2}} \mathfrak{h}_{P_c}(t)\, dt = \int_{\partial S_P} \mathfrak{f}_{P_c}(\vec{s})\, d\vec{s} \quad \text{(14a)}$$

$$\Pr(\vec{p}_c \notin \partial S_P \cup t \notin \Delta T) = 0 \quad \text{(14b)}$$

Similarly (11) can be rewritten as:

$$\Pr\left(\vec{d}_j \in \partial D_j \cap t \in \Delta T\right) = \int_{\partial S_{D_j}} \mathfrak{f}_{D_j}(\vec{s})\, d\vec{s} \int_{t_0-\frac{\Delta T}{2}}^{t_0+\frac{\Delta T}{2}} \mathfrak{h}_{D_j}(t)\, dt = \int_{\partial S_{D_j}} \mathfrak{f}_{D_j}(\vec{s})\, d\vec{s} \quad \text{(15a)}$$

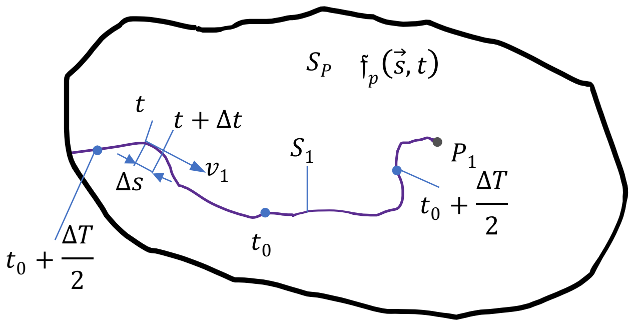
$$\Pr\left(\vec{d}_j \notin \partial D_j \cup t \notin \Delta T\right) = 0 \quad \text{(15b)}$$

Let us consider a semantic property P_1 entering the region ∂S_P of Semantic Space at time $t_0 - \frac{\Delta T}{2}$ with velocity $\vec{v}_1\left(\vec{s}, t_0 - \frac{\Delta T}{2}\right)$. Let us denote the trajectory of P_1 inside ∂S_P from time $t_0 - \frac{\Delta T}{2}$ to $t_0 - \frac{\Delta T}{2} + t_1$ with $S_1\left(t_0 - \frac{\Delta T}{2}, t_0 - \frac{\Delta T}{2} + t_1\right)$.

Then the total length of the trajectory of P_1 from the moment $t_0 - \frac{\Delta T}{2}$ to moment $t_0 + \frac{\Delta T}{2}$ can be obtained via the integral:

$$S_1\left(t_0 - \frac{\Delta T}{2}, t_0 + \frac{\Delta T}{2}\right) = \int_{t_0-\frac{\Delta T}{2}}^{t_0+\frac{\Delta T}{2}} \vec{v}_1(\vec{s}, t)\vec{s}_t dt = \int_{t_0-\frac{\Delta T}{2}}^{t_0+\frac{\Delta T}{2}} v_{1,t}(\vec{s}, t) dt \qquad (16)$$

Here $v_{1,t}$ is the tangential projection of the velocity and $\vec{s}_t$ is the tangent unit vector.



Definition: *Template-Weighted Trajectory Length*

Let us consider again the trajectory of a semantic property P_1

$$W_1\left(t_0 - \frac{\Delta T}{2}, t_0 - \frac{\Delta T}{2}\right) = \int_{t_0 - \frac{\Delta T}{2}}^{t_0 + \frac{\Delta T}{2}} \mathfrak{f}_p(\vec{s}, t) v_{1,t}(\vec{s}, t) dt \qquad (17)$$

Definition: *Average Trajectory Weight of semantic Property*

Matching of Template Property Particles

In order to understand how match between template property and semantic property occurs we need to define one more quantity which will be included in the template property signature:

Definition: *Matching Threshold* Θ

//TODO: finish the Matching Threshold definition

// This paragraph does not belong to *Matching of Template Property*
Each template aspect is paired with an association particle AP. In turn AP can be paired with a semantic particle in semantic sheet $\mathbf{\Sigma}_\alpha$ for some sheet index α. Also, the template property particle P can be paired with semantic particle in $\mathbf{\Sigma}_\alpha$ via another association particle attached to the semantic position of the centroid $\vec{p}_c(\cdot)$.

Definition: *Representation of Template Property*
The representation of a Template Property exists in *Template Space* $\mathbf{T}$. We use the following notation:

$$\mathbf{P}(\cdot) = [\mathbf{p}_c\, \mathbf{p}_1\, \mathbf{p}_2 \ldots \mathbf{p}_N](\cdot)$$
$$\mathfrak{m}_P(\cdot) = [\mathfrak{m}_c\, \mathfrak{m}_1\, \mathfrak{m}_2 \ldots \mathfrak{m}_N](\cdot)$$

Here $(\cdot)$ as before denotes that the quantity on the left is a stochastic quantity. This means that the quantity on the left is not a scalar, but it is a quantity represented with some probability distribution function $f(\,\cdot\,,\mathfrak{P})$. Here the dot $\cdot$ denotes the domain of the function which is the same semantic space on which the corresponding stochastic quantity is defined. The second argument $\mathfrak{P}$ (*fraktur P*) denotes the set of parameters of the specific distribution. The set $\mathfrak{P}$ in general will be considered a subset of the n-dimensional real space $\mathbb{R}^n$ for some large enough n. Each specific distribution function which is a realization of the representation function $f(\,\cdot\,,\mathfrak{P})$ will be written as $f(\,\cdot\,,\mathscr{p})$, where the parameter vector $\mathscr{p} \in \mathfrak{P}$.
With this notation

//TODO: elaborate on this item

Definition: Repulsion between template properties
//TODO: elaborate on this item

Definition: Primitive Template Particle
//TODO: elaborate on this item

Definition: Semantic Template Structure
//TODO: elaborate on this item

Definition: *Representation of Template Particle*
The representation of a Template Particle exists in *Template Space* $\mathbf{T}$. We use the following notation:

//TODO: elaborate on this item

Definition: Optimal Match of The Pattern Matching Region

Inference Structure of Semantic Template

Example of Semantic Template: Calculation of attractive force between semantic structures

Let us consider a newly formed semantic structure S_1. The closest semantic structure will be denoted with S_0. On aggregation level l the nearby semantic structure S_0 can be represented as a graph of n_l substructures.