Note on binding of an association particle to semantic particles

D. Gueorguiev 11/25/21

Primitive semantic particles

Let us consider two primitive semantic particles - V_i and V_j connected through association particle (link) $A_{i,j}$.

V_i----$A_{i,j}$----V_j

The particles V_i and V_j are represented by their semantic signatures $\mathbf{V}_i$ and $\mathbf{V}_j$. The association link $A_{i,j}$ is represented with its association matrix $\mathbf{A}_{i,j}$ and semantic significance vector $\mathbf{W}_{i,j}$.
The association matrix $\mathbf{A}_{i,j}$ captures the affinity force $F(V_i, V_j, t)$ between the particles V_i and V_j at the time t of constructing the compound structure involving those particles. Note that the magnitude of affinity force between the particles may change as their semantic positions and signatures are altered in the future. A change in the affinity force $F(V_i, V_j, t + \Delta t)$ at a future moment $t + \Delta t$ may change the matrix $\mathbf{A}_{i,j}$ of the association link between the altered particles. Altering the semantic position of a particle will require reevaluating the semantic links of this particle with the relevant enclosing contexts.

The association matrix has the following structure:
$\mathbf{A}_{i,j} = [\mathbf{a}_{p_1,q_1} \dots \ \mathbf{a}_{p_m,q_n}]$ where the pairs p, q denote all relevant property pairs where the left property belongs to V_i and the right property belongs to V_j. Let us denote with $\mathcal{P}$ the set of property indices which belong to V_i and with $\mathcal{Q}$ the set of indices which belong to V_j. Then $p \in \mathcal{P}$ and $q \in \mathcal{Q}$. Note that the map $\mathcal{P} \to \mathcal{Q}$ is many-to-many. That is, the same index p may appear multiple times with different $q \in \mathcal{Q}$ and the same index q may appear multiple times with different $p \in \mathcal{P}$. The property association matrices $\mathbf{a}_{p,q}$ have the following structure:

$$\mathbf{a}_{p,q} = \begin{bmatrix} \mathbf{r}_1^p & \mathbf{r}_1^q \\ \mathbf{0} & \mathbf{0} \\ \mathbf{r}_2^p & \mathbf{r}_2^q \\ \mathbf{0} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{0} \\ \mathbf{r}_k^p & \mathbf{r}_l^q \end{bmatrix}$$

So $\mathbf{a}_{p,q}$ is a two-column matrix of size $N \times 2$ with non-zero regions in each column denoted by the vectors $\mathbf{r}_i$ where $\sum_{i=1}^{k} \text{size}(\mathbf{r}_{i=1}^p) \le N$ and $\sum_{j=1}^{l} \text{size}(\mathbf{r}_j^q) \le N$. The non-zero regions $\mathbf{r}_i^p$ and $\mathbf{r}_j^q$ are also known as the ***active regions*** of the association link between the two properties $P_p \in ptree(V_i)$ and $P_q \in ptree(V_j)$ at time t. For details refer to Note On Binding Of An Association Property to Semantic Properties.

The binding force between the two V-particles is conveyed through the Association Particle which exposes the active regions which are to be considered. The binding force is given with the expression:

$$F^b(V_i, V_j | A_{i,j}) = \sum_{P_k \in \{V_i \cap A_{i,j}\}, P_l \in \{V_j \cap A_{i,j}\}} \sum_{a \in P_k, b \in P_l} f(\mathbf{r}_a^k, \mathbf{r}_b^l)$$

The set $\{V_i \cap A_{i,j}\}$ denotes all properties of V_i which are included in $\mathbf{A}_{i,j}$. Similarly, the set $\{V_j \cap A_{i,j}\}$ denotes all properties of V_j which are included in $\mathbf{A}_{i,j}$. The notation $V_i, V_j|A_{i,j}$ reflects the fact that the property pairs contributing to the total binding force is filtered by the chosen in $\mathbf{A}_{i,j}$ property pairs.
In other words, the Association Particle is acting as a filter which selects which property pairs are relevant and will contribute to the binding force between V_i and V_j.
Obviously $F^b(V_i, V_j|A_{i,j})$ will be smaller or equal than the binding force $F^b(V_i, V_j)$ created by considering all property pairs without any Association Particle acting as a filter:

$F^b(V_i, V_j|A_{i,j}) \leq F^b(V_i, V_j)$ where $F^b(V_i, V_j)$ is given as

$F^b(V_i, V_j) = \sum_{P_k \in \{V_i\}, P_l \in \{V_j\}} \sum_{a \in P_k, b \in P_l} f(\mathbf{r}_a^k, \mathbf{r}_b^l)$