## The Signature of Semantic Structures

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Let us have the compound particle $V_{comp}$ represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:

$stree(V_{comp}) =$

```
           __V1__
          /   |   \
        V2    V5   _V6_
       /  \        / | \
     V3   V4     V7  V8  V9
```

The property tree for each $V$-particle $V_k, k = 1..9$ are given with the algebraic notation discussed in Semantic Tree Operations.
$ptree(V_k) \;=\; \sum_{\boldsymbol{k} \in \mathfrak{T}(V_k), i \in \mathbb{p}(V_k)} (\boldsymbol{k}, P_i)$ . Here $\boldsymbol{k}$ denotes the path $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$ constructed by branching consecutively along the $k_{l_1}$-*th* branch from the top level, then the $k_{l_2}$-th branch from the lower level and finally $k_{l_h}$-th branch from the $h$-th level. The set $\mathfrak{T}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of $V_k$. The set $\mathbb{p}(V_k)$ denotes the indices of the vertices in the property tree of $V_k$.

```
        V1                              V2 `                 ...        V9

   P1 P2  P3                       P1--- o --- P2                  P1--- o --- P2
     \ | /  ..                         / | \
Pk --- o --- Pi                       P3  P4  P5_
  ..  / | \  ..                       / \      |  \
  Pj+1 Pj  Pj-1                     P6  P7     P8   P9
                                    |
                                    P10
```

Expressing the property tree of $V_1$ with the algebraic notation discussed earlier:
$ptree(V_1) \;=\; \sum_{k_j \in \mathfrak{T}(V_1), i \in \mathbb{p}(V_1)} (k_j, P_j)$
Similarly, $ptree(V_2)$ is given with

$$ptree(V_2) = (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3 k_1, P_6) + (k_3 k_2, P_7) + (k_5 k_1, P_8) + (k_5 k_2, P_9) + (k_3 k_1 k_1, P_{10})$$

Here $P_0$ is $text(V_2)$.
Now if we expand the property trees for each V-particle in the semantic tree for the composite particle $V_{comp}$ we will have a larger augmented property tree. This augmented property tree represents the semantic structure of $V_{comp}$ and can be recorded in a matrix form which is the semantic signature of $V_{comp}$. The semantic signature matrix of $V_{comp}$ will have the following structure:

$$ssig(V_{comp}) = [\boldsymbol{p_0}\;\; \boldsymbol{a_{0,1}}\;\; \boldsymbol{p_1}\;\; \boldsymbol{p_0}\;\; \boldsymbol{a_{0,2}}\;\; \boldsymbol{p_2}\;\; \boldsymbol{p_0}\;\; \boldsymbol{a_{0,3}}\;\; \boldsymbol{p_3}\;\; \dots\;\; \boldsymbol{p_p}\;\; \boldsymbol{a_{p,q}}\;\; \boldsymbol{p_q}]$$

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [\mathbf{B_1}\ \mathbf{B_2}\ \mathbf{B_3}\ \ \dots\ \ \mathbf{B_q}]$$

$$\mathbf{B_1} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,1}}\ \ \boldsymbol{p_1}]\,, \mathbf{B_2} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,2}}\ \ \boldsymbol{p_2}]\,, \mathbf{B_3} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,2}}\ \ \boldsymbol{p_3}]\,, \dots, \ \mathbf{B_q} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,2}}\ \ \boldsymbol{p_q}]$$

Here the block matrix $\mathbf{B_1}$fully describes the property $P_1$ including how it is connected to the property tree $ptree(V_1)$. Similarly, $\mathbf{B_2}$ and $\mathbf{B_3}$ fully describes the properties $P_2$ and $P_3$ and their connectivity to $ptree(V_1)$. Finally, $\mathbf{B_q}$ fully describes the property $P_q$and its connectivity to $ptree(V_9)$.

In the block matrix for $ssig(V_{comp})$ $\boldsymbol{p_0}$ denotes the signature column vector of the property $P_0$ , $\boldsymbol{a_{0,1}}$denotes the signature column vector of the arc between property $P_0$ and property $P_1$, $\boldsymbol{a_{p,q}}$ denotes the signature column vector of the arc between property $P_p$and $P_q$. Let us denote the number of rows of $ssig(V_{comp})$ by $N$ and the number of columns by $M$.
The semantic signature matrix $ssig(V_{comp})$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig(V_{comp})$ and connectivity signature matrix $csig(V_{comp})$:

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [\boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_1}\ \ \boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_2}\ \ \boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_3}\ \ \dots\ \ \boldsymbol{p_p}\ \ 0\ \ \boldsymbol{p_q}]$$

$$csig(V_{comp}) = [0\ \ \boldsymbol{a_{0,1}}\ \ 0\ \ 0\ \ \boldsymbol{a_{0,2}}\ \ 0\ \ 0\ \ \boldsymbol{a_{0,3}}\ \ 0\ \ \dots\ \ 0\ \ \boldsymbol{a_{p,q}}\ \ 0]$$

Let us denote by $psig(P_1, V_{comp})$ the augmented semantic property signature of property $P_1$with respect to $V_{comp}$. It is given with:

$$psig(P_0, V_{comp}) = [\boldsymbol{p_0}\ \ 0\ \ 0\ \ \boldsymbol{p_0}\ \ 0\ \ 0\ \ \boldsymbol{p_0}\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0\ \ 0\ \ \boldsymbol{p_1}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ 0]$$

$$psig(P_q, V_{comp}) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ \boldsymbol{p_q}]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$$

Here $\mathbb{S}(V_{comp})$ denotes the set of the indices of all semantic particles which the composite $V_{comp}$ is composed from.

Another way to partition the signature matrix into block matrices is:

$ssig(V_{comp}) = [\mathbf{V_1}\ \mathbf{A_{1,2}}\ \mathbf{V_2}\ \mathbf{A_{1,3}}\ \mathbf{V_3}\ \dots\ \mathbf{A_{6,8}}\ \mathbf{V_8}\ \mathbf{A_{6,9}}\ \mathbf{V_9}]$

The block matrix $\mathbf{V_1}$ represents the property tree of the particle $V_1$ and it is given by:

$$\mathbf{V_1} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,1}}\ \ \boldsymbol{p_1}\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,2}}\ \ \boldsymbol{p_2}\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,3}}\ \ \boldsymbol{p_3}\ \ \dots\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,k}}\ \ \boldsymbol{p_k}]$$

The block matrix $\boldsymbol{A_{1,2}}$ describes the connection between the particles $V_1$ and $V_2$ connecting the root property $\boldsymbol{p_0}$ of $V_1$ and the root property $\boldsymbol{p_{k+1}}$of $V_2$. It is given with:

$$\mathbf{A_{1,2}} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,k+1}}\ \boldsymbol{p_{k+1}}]$$

## Properties of the signature matrix

Here are some interesting properties of $ssig(V_{comp})$:
The number of rows $N$ in $ssig(V_{comp})$ is 3 × the number of arcs in the augmented property tree of $V_{comp}$.
The rank of
TO DO: finish the property section

## Asymptotic closeness of semantic structures

Let us have two semantic structures S1 and S2.
$ssig(S_1) = [\mathbf{V_{k_1}}\ \mathbf{A_{k_1,k_2}}\ \mathbf{V_{k_2}}\ \mathbf{A_{k_1,k_3}}\ \mathbf{V_{k_3}}\ \dots\ \mathbf{A_{k_p,k_q}}\ \mathbf{V_{k_q}}]$
$ssig(S_2) = [\mathbf{V_{l_1}}\ \mathbf{A_{l_1,l_2}}\ \mathbf{V_{l_2}}\ \mathbf{A_{l_1,l_3}}\ \mathbf{V_{l_3}}\ \dots\ \mathbf{A_{l_r,l_s}}\ \mathbf{V_{l_s}}]$

Uniform asymptotic closeness
$K$-level uniform asymptotic closeness