Modeling attractive and repulsive forces in semantic properties

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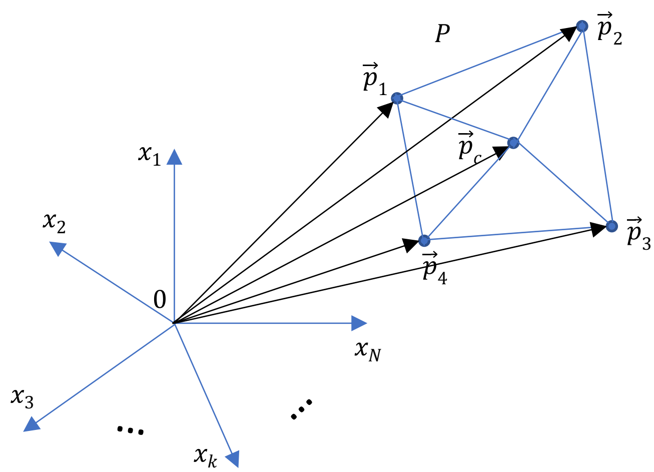
The Concept of Semantic Aspect and modeling interaction between Semantic Aspects

The internal structure of a semantic property P is represented by a set $\mathcal{P}$ of points $\vec{p}_i, i = 1..|\mathcal{P}|$ in semantic space forming a K-polytope which occupies a subset K of the N semantic dimensions i.e. $K \leq N$. On the picture below it is depicted an N-dimensional 4-polytope. With $\vec{p}_i, i = 1..4$ are denoted the vertices of the polytope. With $\vec{p}_c$ we denote the center of mass of the polytope which we will discuss in details later.

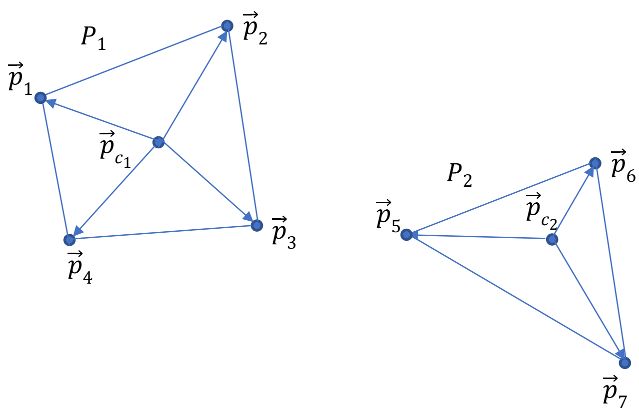


From now on we will denote the polytope associated with property P as P-polytope or *property polytope*. Each vertex of the property polytope models specific semantic aspect of the property.
The distance of each vertex $\vec{p}_i$ from the center of mass $\vec{p}_c$ for the property identifies the type of the semantic aspect this vertex accounts for. The specific position of that vertex $\vec{p}_i$ in semantic space relative to the center of mass $\vec{p}_c$ encodes the current value of the semantic aspect in that property.
In other words the $K-1$-tuple $(\theta_1, \theta_2, \dots, \theta_{K-1})$, where each $\theta_j, j = 1..K-1$ denotes the angle between the aspect tip $\vec{p}_i$ and the semantic axis x_j, uniquely identifies a specific value for the semantic aspect $\vec{p}_i$. For instance, if the property describes the gender of an animal or a human, one particular semantic aspect of this property is if it refers to a subject (a human) or a verb (an action). In both cases the distance of the vertex from the center of mass will be the same but the vertex orientation would be different.

Now imagine the following scenario – we have a primitive semantic particle V_1 which denotes the personal pronoun “she”. We have a second primitive semantic particle V_2 which denotes the verb “to be” in third person, singular “is”. Clearly, the expectation is to be able to combine the two particles as:

$$\begin{array}{l} \quad V_2 \\ \;\,/ \\ V_1 \end{array}$$

The question is how to design the properties those two particles are made of so they will "choose" each other. One way to achieve this is to model some sort of attractive force between the two particles. Each property has a set of semantic aspects which we differentiate by type/kind and by current value. We are going to define an attractive/repulsive force between semantic aspects of the same type.
Let us consider the following example: we have two semantic properties P_1 and P_2 where the first one has 4 vertices while the second one has 3 as shown on the Figure below.



Let us assume that vertex $\vec{p}_3$ and vertex $\vec{p}_5$ describe semantic aspects of the same type/kind. This fact is modeled by their relative distances to the corresponding centroid where the following relationship holds:

$$\left|\vec{p}_3 - \vec{p}_{c_1}\right| \sim \left|\vec{p}_5 - \vec{p}_{c_2}\right|$$

Here we will make an important relaxation the need of which will become clear later in the discussion.

We will allow the type of the semantic aspects to be continuous. That is: the types of the semantic aspects are represented by an uncountable set which is modeled with some segment of the real axis $[R_{min}, R_{max}]$. Here the value of R_{min} corresponds to the minimum semantic aspect type and R_{max} corresponds to the maximum semantic aspect type.

Definition: *semantic function* – a function which accepts a set of parameters each of which describes one or more of the following:

a) a specific semantic aspect, property, particle or structure
b) the relative or absolute position of its constituents in semantic space,
c) how the above-mentioned constituents exert influence relative to each other.

Definition: *Local semantic function* – a semantic function which does not have explicit dependence on the absolute position in semantic space of its constituents.

Definition: *Aspect Type Matching function* Φ
We would like semantic aspects of the same type to attract each other or repel each depending on their values. But how to discriminate between different types of semantic aspects? One way to do that is by introduction of a *local semantic* function Φ:

$$\Phi = \Phi(l_1, l_2) \qquad (1)$$

which we will name *aspect type matching function*. Here l_1 and l_2 represent the types of the semantic aspects for which we want to estimate attracting / repulsive force. The aspect type matching function gives an estimate how likely is the two semantic aspects given with their centroid distances l_1 and l_2 to influence each other through attractive or repulsive force. The aspect type matching function has a range $[0,1]$.
In some of our future investigations we will use the following aspect type matching function:

$$\Phi(l_1, l_2) = e^{-c|l_1 - l_2|^2} \quad (2)$$

where c is a constant.

Definition: *Aspect Value Matching function* Θ
Let us have two semantic aspects $\vec{p}_1$ and $\vec{p}_2$, their centroids $\vec{p}_{c_1}$and $\vec{p}_{c_2}$, and their types $l_1 = |\vec{p}_1 - \vec{p}_{c_1}|$, $l_2 = |\vec{p}_2 - \vec{p}_{c_2}|$. Let is denote with $\boldsymbol{\theta}^{(1)} = \left(\theta_1^{(1)}, \theta_2^{(1)}, \dots, \theta_{K-1}^{(1)}\right)$ the coordinates of first centered aspect vector $\vec{p}_1$ and with $\boldsymbol{\theta}^{(2)} = \left(\theta_1^{(2)}, \theta_2^{(2)}, \dots, \theta_{K-1}^{(2)}\right)$ the coordinates of the second centered aspect vector $\vec{p}_2$. Here each of the pairs $\left(l_1, \boldsymbol{\theta}^{(1)}\right)$ and $\left(l_2, \boldsymbol{\theta}^{(2)}\right)$ uniquely identifies the positions of the tip of each of the two aspect values with respect to their corresponding centroids. Let us assume that $\Phi(l_1, l_2)$ is 1 so their types are matching. The question is under what conditions the two aspect values encoded in their corresponding coordinate positions $\boldsymbol{\theta}^{(1)}$ and $\boldsymbol{\theta}^{(2)}$ will attract, repel each other and won't influence each other. The answer to this question is given by the aspect value matching function Θ defined as:

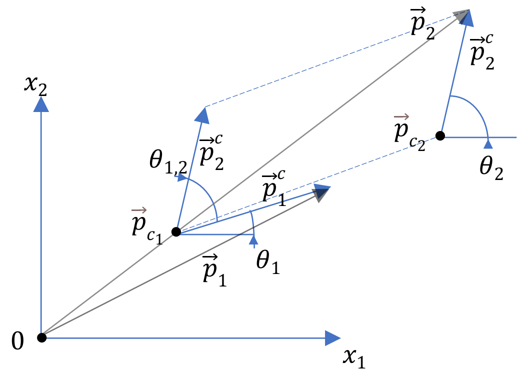
$$\Theta = \Theta\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}\right) \quad (3)$$

The aspect value matching function $\Theta\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}\right)$ has a range of $[-1,1]$.

Example of an aspect value matching function is:

$$\Theta = -\sin\theta_{1,2} \quad (4)$$

where $\theta_{1,2}$ is the angle between the two centered aspect vectors $\vec{p}_1 - \vec{p}_{c_1}$ and $\vec{p}_2 - \vec{p}_{c_2}$.
Here are some special cases for which it is particularly easy to express $\theta_{1,2}$ in terms of $\boldsymbol{\theta}^{(1)}$ and $\boldsymbol{\theta}^{(2)}$.
When $K = 2$ we have $\boldsymbol{\theta}^{(1)} = \theta_1$ and $\boldsymbol{\theta}^{(2)} = \theta_2$. Then $\theta_{1,2} = \theta_1 - \theta_2$ and we have $\Theta = -\sin(\theta_1 - \theta_2)$.



When $K = 3$ the picture is a bit more involved but it is not difficult to find a closed form expression for $\theta_{1,2}$ in terms of $\theta_1^{(1)}, \theta_2^{(1)}, \theta_1^{(2)}$ and $\theta_2^{(2)}$. Here the triplet $\left(l_1, \theta_1^{(1)}, \theta_2^{(1)}\right)$ uniquely identifies the position of the centered semantic aspect vector $\vec{p}_1 - \vec{p}_{c_1}$ and the triplet $\left(l_2, \theta_1^{(2)}, \theta_2^{(2)}\right)$ uniquely identifies the position of the centered semantic aspect vector $\vec{p}_2 - \vec{p}_{c_2}$.

Definition: *Semantic energy density function* Ψ : this is a non-local function giving the strength of the semantic energy field at every point $\mathbf{r} = (x_1, x_2, \dots, x_N)$ in semantic space and for every aspect type l:

$\Psi = \Psi(\mathbf{r}, l)$ (5)

Examples of semantic energy density functions:
Constant semantic energy density for every aspect type:

$\Psi = \Psi(l) = const$ (6)
The energy density Ψ assumes constant value throughout the semantic space for a given aspect type l and is continuous function with respect to l.

Gaussian semantic energy density:

$\Psi(\mathbf{r}, l) = \sum_i A_i \times e^{-|\mathbf{r} - \mathbf{r}_i(l)|^2}$ (7)
Here $\mathbf{r}_i(l)$ represent the positions of a set of points in semantic space which are energy density peaks. The positions of the peaks obviously depend on the type l of the semantic aspect. $A_i = A_i(C)$ are a set of scalars which values depend on the current context C.

Potential Well semantic energy density:

$\Psi(\mathbf{r}, l) = \sum_i A_i - B_i \times e^{-|\mathbf{r} - \mathbf{r}_i(l)|^2}; A_i \geq B_i$ (8)
Here $\mathbf{r}_i(l)$ represent the positions of a set of points in semantic space which are energy density wells. The positions of the wells obviously depend on the type l of the semantic aspect. $A_i(C)$ and $B_i(C) \leq A_i(C)$ represent a set of scalars which values depend on the current context C.

Definition: *closely related semantic aspect types* – two semantic aspect types l_1 and l_2 are closely related if there exist a pair of aspect values having the types l_1 and l_2 for which the product of the

aspect type matching function and the *aspect value matching function* is close enough to 1 by absolute value.
Let us denote with $\vec{p}_1$ and $\vec{p}_2$ two aspect vectors and with $\vec{p}_{c_1}$ and $\vec{p}_{c_2}$ the associated centroids, such that $\left|\vec{p}_1 - \vec{p}_{c_1}\right| = l_1$ and $\left|\vec{p}_2 - \vec{p}_{c_2}\right| = l_2$. Let us denote with $\boldsymbol{\theta}^{(1)} = \left(\theta_1^{(1)}, \theta_2^{(1)}, \dots, \theta_{K-1}^{(1)}\right)$ the coordinates of first centered aspect vector $\vec{p}_1 - \vec{p}_{c_1}$ and with $\boldsymbol{\theta}^{(2)} = \left(\theta_1^{(2)}, \theta_2^{(2)}, \dots, \theta_{K-1}^{(2)}\right)$ the coordinates of the second centered aspect vector $\vec{p}_2 - \vec{p}_{c_2}$.

Strictly, two semantic types are closely related *iff* for each of the aspect types l_1 and l_2 there exist two coordinate points $\boldsymbol{\theta}^{(1)}$ and $\boldsymbol{\theta}^{(2)}$ and aspect values $\vec{p}_1\left(\boldsymbol{\theta}^{(1)}\right)$ and $\vec{p}_2\left(\boldsymbol{\theta}^{(2)}\right)$ such that:

$$1.0 - \Phi(l_1, l_2) \times \left|\Theta\left(\vec{p}_1\left(\boldsymbol{\theta}^{(1)}\right), \vec{p}_2\left(\boldsymbol{\theta}^{(2)}\right)\right)\right| < \varepsilon \qquad (9)$$

for some small enough $\varepsilon > 0$.
In other words, the types of the semantic aspects need to be matching and there must be a pair of matching aspect values having those types.
In the future we will explore the case where only closely related aspect types can exert attractive or repelling force to each other.

For example, with the example aspect type matching function in (2) a necessary condition for two types l_1 and l_2 to be closely related is that l_1 and l_2 are close enough. That is,
there exists a monotonously increasing function $\delta = \delta(\varepsilon)$ such that

$$|l_1 - l_2| < \delta \qquad (10)$$

Note that with other chosen aspect type matching functions this condition requiring proximity of the aspect types is no longer necessary. In the future discussion we will explore only such aspect type matching functions Φ which require condition (10) for close relatedness of aspect types.

Definition: *Attractive force between semantic aspects*
Let us have the two semantic aspects A_1 and A_2 given with their coordinate tuples $\left(\vec{p}_1, \vec{p}_{c_1}\right)$ and $\left(\vec{p}_2, \vec{p}_{c_2}\right)$. Here $\vec{p}_1$ and $\vec{p}_2$ represent the position of the tip of each of the aspects in semantic space. The vectors $\vec{p}_{c_1}$ and $\vec{p}_{c_2}$ represent the position of each of the centroids. The types of A_1 and A_2 are given with $l_1 = \left|\vec{p}_1 - \vec{p}_{c_1}\right|$ and $l_2 = \left|\vec{p}_2 - \vec{p}_{c_2}\right|$. The positions in semantic space of each of the two aspect types l_1 and l_2 in generalized spherical coordinates with respect to their centroids are denoted with $\boldsymbol{\theta}^{(1)}$ and $\boldsymbol{\theta}^{(2)}$.
Then the attractive/repelling force between the two aspects is given with:

$$f_{12}(A_1, A_2) = \Theta\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}\right)\Phi(l_1, l_2)\Psi(\vec{p}_1, l_1)\Psi(\vec{p}_2, l_2) \qquad (11)$$

Notice that the sign of f_{12} is given with $\operatorname{sgn}\Theta\left(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}\right)$.

Modeling interaction between semantic properties

Let us have two properties P_1 and P_2 given with their centroids $\vec{p}_{c_1}$, $\vec{p}_{c_2}$ and aspect sets $\mathcal{P}_1 = \left\{\vec{p}_1^{(1)}, \vec{p}_2^{(1)}, \dots, \vec{p}_a^{(1)}\right\}$ and $\mathcal{P}_2 = \left\{\vec{p}_1^{(2)}, \vec{p}_2^{(2)}, \dots, \vec{p}_b^{(2)}\right\}$. We would like to model the attractive / repelling force between the two properties. An assumption comes to mind which makes the modeling simple - let us assume that the force between P_1 and P_2 is a linear superposition of the forces acting on every pair of aspects $A_i^{(1)} \in \mathcal{P}_1$ and $A_j^{(2)} \in \mathcal{P}_2$. Then we can write the following expression for the total force between P_1 and P_2:

$$f(P_1, P_2) = \sum_{i,j} f_{12}\left(A_i^{(1)}, A_j^{(2)}\right) \qquad (12)$$

Definition: *relevant aspect pair* $\left(A_i^{(1)}, A_j^{(2)}\right)$ is such pair which has absolute binding force value not in the first ℓ -quantile for some $\ell > 0$. In other words, all region pairs which are *in* the first ℓ-quantile are *irrelevant*

The RHS of the expression above can be split into two sets of terms

$$f(P_1, P_2) = f^+(P_1, P_2) + f^-(P_1, P_2) \qquad (13)$$

where

$f^+(P_1, P_2) = \sum_{i,j} f_{12}^+\left(A_i^{(1)}, A_j^{(2)}\right) > 0$. Here $A_i^{(1)} \in \mathcal{P}_1$ and $A_j^{(2)} \in \mathcal{P}_2$ represent all *relevant* aspect pairs which generate attractive force.

Similarly

$f^-(P_1, P_2) = \sum_{i,j} f_{12}^-\left(A_i^{(1)}, A_j^{(2)}\right) < 0$ represent all *relevant* aspect pairs which generate repelling force.

Definition: *In-situ Energy Density of semantic property*
Let us have a semantic property P given with its set of semantic aspects $A_i = (\vec{p}_i, l_i); i = 1..N$. The semantic energy density of the property P *in-situ* is given with the expression:

$$\Psi(P) = \sum_{i=1}^{N} \Psi(\vec{p}_i, l_i) \qquad (14)$$

Semantic Center, Semantic Mass and Semantic Energy of an Assembly of Semantic Properties

On the Semantic Mass of a Property

Each property has specific semantic mass. The path from the root in the particle property tree for a given property is determined based on its semantic mass, attractive or repelling force to other properties in the tree as well as the semantic energy stored in the property.

The semantic mass of a property is determined based on the semantic information content a property conveys:
$Property\ Mass \sim Property\ Valence \times Information\ Content$ or in symbol notation

$M_P = |V_P| \times IC$ where $|V_P|$ is the valence and IC is the information content of the property. Certain properties have the affinity to bind to multiple child properties which reveal additional details for the semantic information provided by the parent. The more child properties a parent property can bind to - the higher will be its property valence.

The carrier of semantic mass in a property is the semantic aspect. We will assume that each semantic aspect in a property carries a unit of mass. Obviously, the more semantic aspects a property is composed of - the higher information content and property valence will be attributed to the property.

On the Semantic Energy of a Semantic Property

Another characteristic of the Semantic Property is the Semantic Energy $E(P)$ stored in it.
The Semantic Energy of a property P is determined by the path the property has travelled in semantic space from its original unbound *in-situ* position until its bound position inside its semantic particle assembly. Every property starts its travel to the centroid $p_c(V)$ of the semantic particle V with zero semantic energy. The semantic energy of P is gradually accumulated along its travel path $S(P)$ and can be computed by the relationship below:

$$E(P) = \sum_k \sum_{i=1}^{N} \Psi(\vec{p}_i + \Delta\vec{p}_k, l_i)\Delta s_k \qquad (15)$$

Definition: *Energy-weighted center of mass (centroid) of a semantic property*
Let us have a semantic property P which is in its original (*in-situ*) position. The set of semantic aspects of P will be denoted with $A_i = (\vec{p}_i, l_i), i = 1..N$. Let us move this property with a small enough incremental step Δs along its path $S(P)$ toward its bound position in the particle V. The new position of each aspect then becomes $\vec{p}_i + \Delta\vec{p}, i = 1..N$. Then the energy of P in its new position will be given with:

$$E(P) = \sum_{i=1}^{N} \Psi(\vec{p}_i + \Delta\vec{p}, l_i)\Delta s \qquad (16)$$

Here we assume that step Δs is small enough so that $\Psi(\vec{p}_i + \Delta\vec{p}, l_i) \sim \Psi(\vec{p}_i, l_i)$.

Then we can compute an approximation of the energy-weighted center of mass of the property P as:

$$\vec{p}_E^{(0)} = \frac{\sum_{i=1}^{N} \vec{p}_i E(\vec{p}_i + \Delta\vec{p}_i, l_i)}{E(P)} \qquad (17)$$

Note that the LHS of (17) indeed is an approximation because in the energy term on the RHS we have implicit dependency on the centroid $\vec{p}_c$ which was not weighted by the energy. The solution is to continue calculating $\vec{p}_E^{(1)}, \vec{p}_E^{(2)}, \ldots$ iteratively until for some j $\left|\vec{p}_E^{(j)} - \vec{p}_E^{(j+1)}\right|$ gets smaller than some predefined ε. Here are the details:

$$l_i^{(0)} = \left|\vec{p}_i - \vec{p}_E^{(0)}\right| \qquad (18)$$

On the j-th iteration we compute the semantic energy and energy-weighted centroid:

$$E^{(j)} = \sum_{i=1}^{N} \Psi\left(\vec{p}_i + \Delta\vec{p}_i, l_i^{(j-1)}\right) \Delta s \qquad (19)$$

$$\vec{p}_E^{(j)} = \frac{\sum_{i=1}^{N} \vec{p}_i E\left(\vec{p}_i, l_i^{(j-1)}\right)}{E^{(j)}} \qquad (20)$$

We stop the iterations when both are satisfied:

$$\left|\vec{p}_E^{(j)} - \vec{p}_E^{(j+1)}\right| < \varepsilon \text{ and } \left|E^{(j)} - E^{(j+1)}\right| < \varepsilon \qquad (21)$$

Then we denote by $\vec{p}_E = \vec{p}_E^{(j+1)}$ the true energy-weighted centroid of the property on its current position.
Note that if $\Psi(\vec{p}_i, l_i) = const$ for each $i = 1..N$ along the travel path $S(P)$ so far then

$$\vec{p}_E^{(0)} = \vec{p}_E^{(1)} = \cdots = \vec{p}_E^{(j+1)} = \vec{p}_c \qquad (22)$$

Statement: The energy state and semantic position of a semantic property P on every point s along the travel path $S(P)$ toward bound state is represented uniquely by the 4-tuple $\vec{p}_i(s), \vec{p}_E(s), l_i(s), E(s)$.

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Appendix

Constructing the property tree: constraints and inequalities based on binding force

Let us consider the property tree of a V-particle:

V_1

P_0
P_1--- o --- P_2 <----------- P_{11}
/ | \
P_3 P_4 P_5_
/ \ | \
P_6 P_7 P_8 P_9
|
P_{10}

Let us imagine we want to add new P-particle to the property tree. The following steps toward forming a new ensemble take place:

Step 1. All P-particles which are about to participate in the new ensemble become disassociated / disentangled.
Step 2. The center of semantic mass for the new ensemble is determined as we already know the semantic masses of all properties in the new ensemble.
Step 3. The particle with the largest mass will be closest to the common center of mass and will be root to the property tree.

Special Case:
there are two P-particles with the same semantic mass which happens to be the largest mass in the particle tree. Then the particle with the lower semantic energy will be closer to the semantic center than the particle with the higher semantic energy. (not sure about the energy condition). An ensemble in which the heaviest two particles are having the same masses and same energies is ill-formed and one of the two properties has to have a region discarded so it will end up with lower semantic mass.

Step 4. Let us have two particles P-particles P_j and P_k such that $M_{P_j} \geq M_{P_k}$. Let us denote with P_i the particle with the closest but larger semantic mass than that of P_j and P_k. Thus $M_{P_i} > M_{P_j} \geq M_{P_k}$. Then each one of the following configurations are possible:

a)	*b)*	*c)*	*d)*	*e)*	*f)*	*g)*
P_i	P_i	P_i	P_i	P_i	P_i	P_i
/ \	/ \	$\vert$	$\vert$	$\vert$	$\vert$	x x
P_j P_k	P_k P_j	P_j	P_k	P_j	P_k	P_j P_k
		$\vert$	$\vert$	x	x	
		P_k	P_j	P_k	P_j	

Case a) will occur when there is non-zero binding force $f_{i,j}^{+} = f^{+}(P_i, P_j) > 0$ between P_i and P_j and also between P_i and P_k - $f_{i,k}^{+} = f^{+}(P_i, P_k) > 0$. In this case either $M_{P_j} > M_{P_k}$or $M_{P_j} = M_{P_k}$and $f_{i,j}^{+} > f_{i,k}^{+}$.

Case b) will occur when there is non-zero binding force $f^{+}(P_i, P_j) > 0$ between P_i and P_j and also between P_i and P_k - $f^{+}(P_i, P_k) > 0$. In this case $M_{P_j} = M_{P_k}$and $f_{i,k}^{+} > f_{i,j}^{+}$.

Case c) will occur when $f_{i,j}^{+} > 0$, $f_{i,k}^{+} = 0$, and $f_{j,k}^{+} > 0$ when either $M_{P_j} > M_{P_k}$or $M_{P_j} = M_{P_k}$.

Case d) will occur when $f_{i,k}^{+} > 0$, $f_{i,j}^{+} = 0$, and $f_{j,k}^{+} > 0$ when $M_{P_j} = M_{P_k}$.

Case e) will occur when $f_{i,j}^{+} > 0$ and $f_{i,k}^{+} = 0$

Case f) will occur when $f_{i,j}^{+} = 0$ and $f_{i,k}^{+} > 0$

Case g) will occur when $f_{i,j}^{+} = 0$ and $f_{i,k}^{+} = 0$

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Construction of semantic properties

There are set of optimization problems which are related to the construction of new properties. Those are posed below:

Let us have a given set of semantic aspect types $l_1, l_2, \ldots, l_k$. We would like to construct a new property P which can be bound to a given set of primitive semantic particles $V_1, V_2, \ldots, V_m$ in an enclosing semantic structure S. When adding a new property to a subset of primitive particles of S then obviously this will cause a displacement of the centroid of S. Let us denote the displacement of the centroid of S as a result of the introduction of the new property P with $\Delta\vec{r}_c(S,P)$. Let us denote by $\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \dots, \boldsymbol{\theta}^{(k)}$ the positions of the k semantic aspect values which correspond to the new aspect types $l_1, l_2, \dots, l_k$. We would like to obtain the values $\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \dots, \boldsymbol{\theta}^{(k)}$ of the k semantic aspects based on the minimization of certain cost function.

Problem 1: Determine the values $\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \dots, \boldsymbol{\theta}^{(k)}$ of the semantic aspects given by their types $l_1, l_2, \dots, l_k$ in the new property such that the centroid of the enclosing semantic structure S is moved by the least amount from its original position before the introduction of the new property. Thus, we have:

$$\min_{\boldsymbol{\theta}^{(1)},\dots,\boldsymbol{\theta}^{(k)}} |\Delta\vec{r}_c| \qquad \text{(A.1)}$$

Problem 2: Determine the values $\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \dots, \boldsymbol{\theta}^{(k)}$ of the semantic aspects given by their types $l_1, l_2, \dots, l_k$ in the new property such that the centroid of the enclosing semantic structure S is moved in a direction which is as close as possible to some given direction $\vec{D}_0$ in semantic space. Then, we want to minimize the angle between $\Delta\vec{r}_c$ and $\vec{D}_0$ which is equivalent to maximizing $\frac{\vec{D}_0\,\Delta\vec{r}_c}{|\vec{D}_0||\Delta\vec{r}_c|}$:

$$\max_{\boldsymbol{\theta}^{(1)},\dots,\boldsymbol{\theta}^{(k)}} \frac{\vec{D}_0\,\Delta\vec{r}_c}{|\vec{D}_0||\Delta\vec{r}_c|} \qquad \text{(A.2)}$$

Related to the second optimization problem is maximization of the displacement of the centroid of the enclosing semantic structure S in the given direction $\vec{D}_0$. In this case we want to maximize:

$$\max_{\boldsymbol{\theta}^{(1)},\dots,\boldsymbol{\theta}^{(k)}} \frac{\vec{D}_0\,\Delta\vec{r}_c}{|\vec{D}_0|} \qquad \text{(A.3)}$$

A more general weighted objective function, linear combination of (A.2) and (A.3) would be

$$\max_{\boldsymbol{\theta}^{(1)},\dots,\boldsymbol{\theta}^{(k)}} W_1 \frac{\vec{D}_0\,\Delta\vec{r}_c}{|\vec{D}_0||\Delta\vec{r}_c|} + W_2 \frac{\vec{D}_0\,\Delta\vec{r}_c}{|\vec{D}_0|} \qquad \text{(A.4)}$$

where $W_1 = W_1(C)$ and $W_2 = W_2(C)$ depend on the enclosing the semantic structure S context C.

And here there is a variation of the previous problems:
In all of the following cases we have a given set of aspect types $l_1, l_2, \dots, l_k$ and we want to find an extra set of types $l_{k+1}, l_{k+2}, \dots, l_m$ which

Problem 3:

//TODO: finish this