On the signature matrix of semantic property

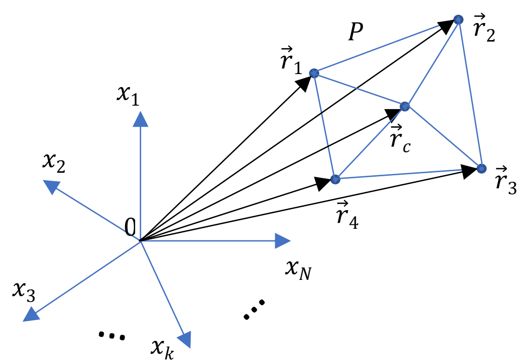
D. Gueorguiev 5/19/2022

Notation

L : the number of semantic dimensions
K : the number of semantic dimensions in a property represented as K-polytope
N : number of semantic aspects in a property
$\mathcal{P}$: set of points forming the K-polytope of a semantic property
A_i : denotes the i-th semantic aspect of a semantic property
P_i : denotes semantic property
V_i : denotes primitive semantic particle
$\vec{r}_c$: in the context of a property: the center of mass of the property
In the context of an ensemble of properties: the center of mass of the ensemble
$\vec{r}_i$: In the context of a property: semantic position of the aspect A_i
In the context of an ensemble of properties: the center of mass of the property
l_i : the type of the aspect A_i
θ_j : angle between the current aspect and semantic axis x_j
$\boldsymbol{\theta}$: a vector with all angular coordinates of the current aspect to the semantic axes

Matrix Representation of Semantic Property

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \mathfrak{m}_i \vec{r}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \qquad (1)$$



If $\mathfrak{m}_l = \mathfrak{m} = const$

$$\vec{r}_c = \frac{\sum_{i=1}^{|\mathcal{P}|} \vec{r}_i}{|\mathcal{P}|} \qquad (2)$$

$$\vec{p}_i = \vec{r}_i - \vec{r}_c \qquad (3)$$

$$\vec{p}_i = \left(1 - \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}\right) \vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \frac{\mathfrak{m}_j}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l} \vec{r}_j \qquad (4)$$

With $\widehat{\mathfrak{m}}_i = \frac{\mathfrak{m}_i}{\sum_{l=1}^{|\mathcal{P}|} \mathfrak{m}_l}$ we write:

$$\vec{p}_i = (1 - \widehat{\mathfrak{m}}_i)\vec{r}_i - \sum_{j=1, j\neq i}^{|\mathcal{P}|} \widehat{\mathfrak{m}}_j \vec{r}_j \qquad (5)$$

In a matrix form:

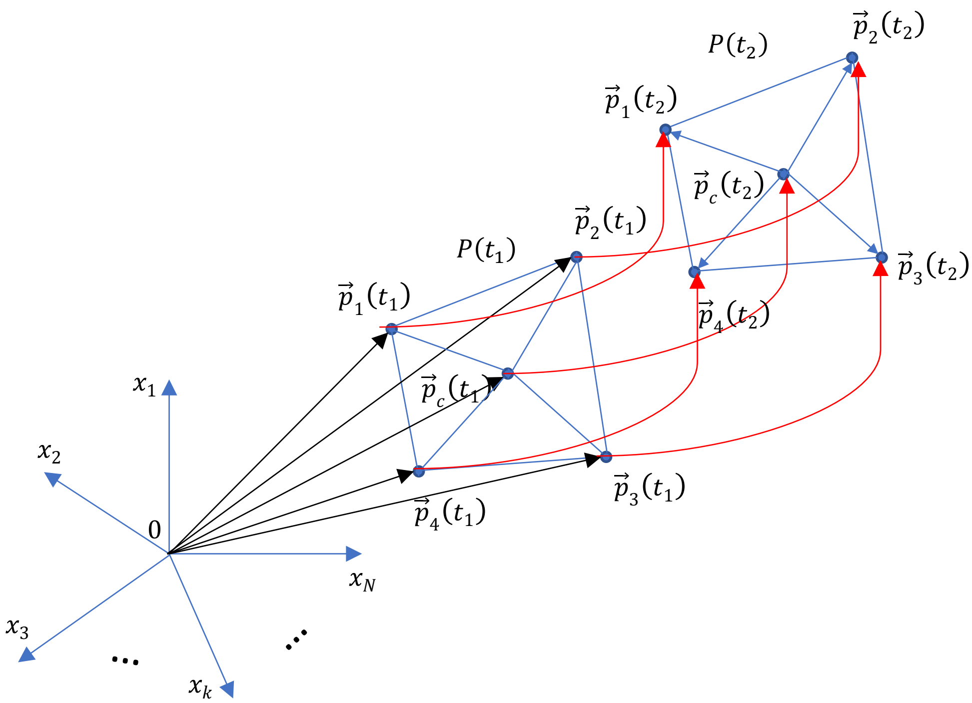
$$P = \begin{bmatrix} 1-\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ -\hat{\mathfrak{m}}_1 & 1-\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & 1-\hat{\mathfrak{m}}_N \end{bmatrix} \begin{Bmatrix} \vec{r}_1 \\ \vec{r}_2 \\ \vdots \\ \vec{r}_N \end{Bmatrix} \qquad (6)$$

or succinctly

$$P = MX \quad (7)$$

where $M = \begin{bmatrix} 1-\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ -\hat{\mathfrak{m}}_1 & 1-\hat{\mathfrak{m}}_2 & \cdots & -\hat{\mathfrak{m}}_N \\ \vdots & \vdots & \vdots & \vdots \\ -\hat{\mathfrak{m}}_1 & -\hat{\mathfrak{m}}_2 & \cdots & 1-\hat{\mathfrak{m}}_N \end{bmatrix}$ $X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,L} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ x_{N,1} & x_{N,2} & \cdots & x_{N,L} \end{bmatrix}$ (8)

While P is traversing the semantic space each aspect A_i retains its properties – this means that each $\vec{p}_i$ is invariant when P traverses semantic space. That is - l_i and $\boldsymbol{\theta}_i$ remain invariant.



The last statement, obviously, implies that there does not exist inverse matrix M^{-1} as the set of $N \times N$ matrices X which map to a given matrix P is a continuum.

Thus, we conclude that each semantic property is uniquely defined by the pair of two quantities: the semantic signature matrix P and the mass vector of the property $\mathfrak{m} = \{\mathfrak{m}_1, \mathfrak{m}_2, \cdots, \mathfrak{m}_N\}$.

In Situ Position of Semantic Property

Definition: *In-situ position of semantic property*
This is the initial position in Semantic Space from which each semantic property starts its travel to bound state.
Each semantic property will be mapped to a portion of semantic space which will contain its initial / in-situ position. In order to determine the portion of semantic space which will map to semantic property we will look into the singular value decomposition of the property signature matrix P given with

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,L} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ p_{N,1} & p_{N,2} & \cdots & p_{N,L} \end{bmatrix} \quad (9)$$

Recall, in (9) each row corresponds to an aspect definition encoding the aspect type l_i and its angular coordinates $\boldsymbol{\theta}_i$. Thus, we will be looking for factorization in the form:

$$P = U\Sigma V^T \quad (10)$$

where U is $N \times N$ orthonormal matrix, V is $L \times L$ orthonormal matrix and Σ is $N \times L$ diagonal matrix with at most N non-zero values on the main diagonal. Let us denote those non-zero values on the main diagonal of Σ with $\sigma_1, \sigma_2, \ldots, \sigma_N$. We can always normalize P such that $\sigma_1 + \sigma_2 + \cdots + \sigma_N = 1$. Then the entropy of the property P is given with:

$$H(P) = -(\sigma_1 \log \sigma_1 + \sigma_2 \log \sigma_2 + \cdots + \sigma_N \log \sigma_N) \quad (11)$$

Here we use $\lim_{\varepsilon \to 0} \varepsilon \log \varepsilon \to 0$.

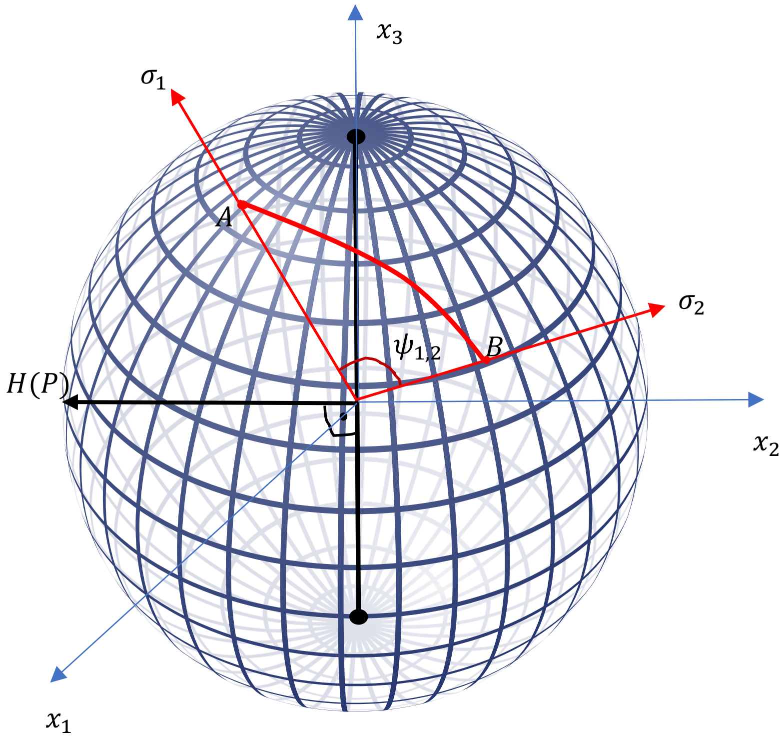
The higher the entropy $H(P)$, the higher the information content of the property P.

One can argue that the in-situ positions of semantic properties with higher information content should be farther from the semantic center compared to the properties with less information content. The argument is that population of properties with high information content is larger than the population of properties with low information content. In fact, we will assume that the population with information content H is proportional to the surface of L dimensional sphere with radius H. Thus, the population increases as the following ratio:

$pop(H_1)$ - population with information content H_1
$pop(H_2)$ - population with information content H_2

$$\frac{pop(H_1)}{pop(H_2)} \sim \left(\frac{H_1}{H_2}\right)^{L-1}$$

Therefore, in order to have maximum utilization of semantic space we will restrict the space of feasible initial (in-situ) positions for a property P with information content $H(P)$ to be the surface of L sphere with radius $H(P)$.



x_3
σ_1
A
σ_2
$\psi_{1,2}$
B
$H(P)$
x_2
x_1