Semantic tree operations

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Semantic Tree Notation

Let us introduce the *tree tuple* (k, v) where v represents certain node - semantic particle or a subtree.
Here k is a tree tuple factor which encodes uniquely the position of the node v in the tree.
The following operations are defined for tree tuple factors:
We have a base of m tree tuple factors $k_1, k_2, \ldots k_m$ which are defined as the digits greater than 0 of $(m + 1)$-nary number system such that $0 = k_0 < k_1 < k_2 < \ldots < k_m$. We define an operation `*` denoting digit concatenation $k_i * k_j = (m + 1)k_i + k_j$. Obviously,
$k_i * k_j > k_i$ for any pair $i, j = 1..m$
Note that the latter implies that
$k_{i_1} * k_{i_2} * \ldots * k_{i_n} > k_{j_1} * k_{j_2} * \ldots * k_{j_{n-1}}$ for any tuple where $i_p, j_q = 1..m, p = 1..n, q = 1..n - 1$

$$\left(k_i, \left(k_j, v_j\right)\right) = (k_i * k_j, v_j)$$

Encoding a complete m-ary tree T of height h with the algebraic notation above:
$T = (k_0, v_0^0)$. Further we will assume that $k_0 = 0$.
$$v_0^0 = (k_0, v_0^1) + (k_1, v_1^1) + (k_2, v_2^1) + \ldots + (k_m, v_m^1)$$

In general, we have:
$$v_q^p = \left(k_0, v_{(q-1)m}^{p+1}\right) + \left(k_1, v_{(q-1)m+1}^{p+1}\right) + \left(k_2, v_{(q-1)m+2}^{p+1}\right) + \ldots + \left(k_m, v_{qm}^{p+1}\right)$$
where $q = 1..m^h, p = 1..h$
Obviously, we have at most $\frac{(m^{h+1}-1)}{m-1}$ distinct terms v_q^p which represent nodes i.e. semantic values.
The expression for the tree also can be written as:
$T = \sum_{i=0}^{N}(k_i^*, v_i)$ where $N \leq \frac{(m^{h+1}-1)}{m-1}$ and k_i^* are the *node factors* given with $k_i^* = k_{i_1} * k_{i_2} * \ldots * k_{i_n}; n \leq h$. The node values v_i are the values v_q^p ordered in increasing order of k_i^*. This order corresponds to *level order traversal* of the m-ary tree. Note that with appropriately defined comparison operation `<` we can model different ways of traversing the m-ary tree. For instance, if we define `<` as the comparison for the values of $\left(m + 1 - k_{i_1}\right)m^{n-1} + \left(m + 1 - k_{i_2}\right)m^{n-2} + \cdots + \left(m + 1 - k_{i_n}\right)$ we will have ordering which corresponds to the *preorder traversal* of the tree.

Example
Peter is Dimitar's son.
Dimitar's son has a friend in the neighborhood and his friend's name is James.
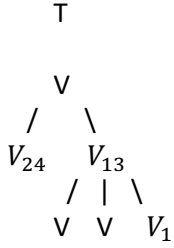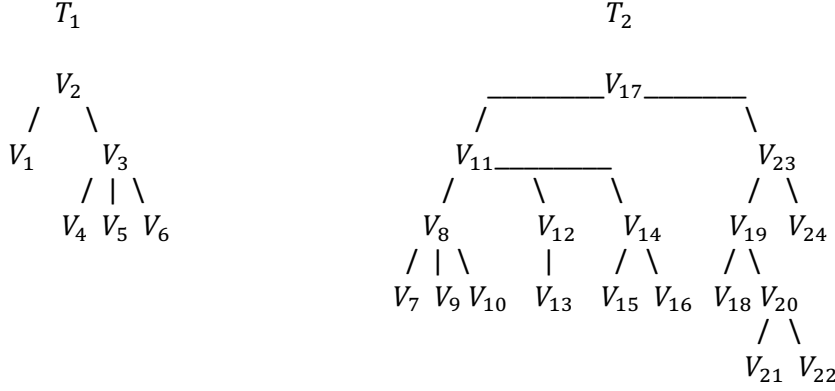   ⇨ James is Peter's friend

Peter is the son of Dimitar.
  V_1 V_2 V_3 V_4 V_5 V_6
The son of Dimitar has   a   friend in the neighborhood and the name of   his friend is James.
V_7 V_8 V_9 V_{10} V_{11} V_{12} V_{13} V_{14} V_{15} V_{16} V_{17} V_{18} V_{19} V_{20} V_{21} V_{22} V_{23} V_{24}

```
        T₁                                    T₂

       V₂                          _____V₁₇_____
      /  \                        /                      \
    V₁    V₃                    V₁₁_____              V₂₃
         / | \                 /      \     \            /  \
       V₄ V₅ V₆              V₈      V₁₂   V₁₄        V₁₉  V₂₄
                            / | \      |    / \       / \
                          V₇ V₉ V₁₀   V₁₃ V₁₅ V₁₆  V₁₈ V₂₀
                                                         / \
                                                      V₂₁  V₂₂
```

```
      T

      V
    /   \
  V₂₄    V₁₃
        / | \
       V  V   V₁
```

Expressing T_1 with the algebraic notation discussed earlier:

$$T_1 = (k_0, v_2) + (k_1, v_1) + \left(k_2, \left((k_0, v_3) + (k_1, v_4) + (k_2, v_5) + (k_3, v_6)\right)\right)$$

which is expanded to:

$$T_1 = (k_0, v_2) + (k_1, v_1) + (k_2 k_0, v_3) + (k_2 k_1, v_4) + (k_2 k_2, v_5) + (k_2 k_3, v_6)$$

Expressing T_2 with the algebraic notation yields:

$$T_2 = (k_0, v_{17}) + \left(k_1, \left((k_0, v_{11}) + \left(k_1, \left((k_0, v_8) + (k_1, v_7) + (k_2, v_9) + (k_3, v_{10}) \right) \right) + \right.\right.$$

$$\left. \left(k_2, \left((k_0, v_{12}) + (k_1, v_{13}) \right) \right) + \left(k_3, \left((k_0, v_{14}) + (k_1, v_{15}) + (k_2, v_{16}) \right) \right) \right) \right) + \left(k_2, \left((k_0, v_{23}) + \right.\right.$$

$$\left. \left(k_1, \left((k_0, v_{19}) + (k_1, v_{18}) + \left(k_2, \left((k_0, v_{20}) + (k_1, v_{21}) + (k_2, v_{22}) \right) \right) \right) \right) + (k_2, v_{24}) \right) \right)$$

which is expanded to:

$$\begin{aligned}
T_2 = \ & (k_0, v_{17}) + (k_1 k_0, v_{11}) + (k_1 k_1 k_0, v_8) + (k_1 k_1 k_1, v_7) + (k_1 k_1 k_2, v_9) + (k_1 k_1 k_3, v_{10}) \\
& + (k_1 k_2 k_0, v_{12}) + (k_1 k_2 k_1, v_{13}) + (k_1 k_3 k_0, v_{14}) + (k_1 k_3 k_1, v_{15}) + (k_1 k_3 k_2, v_{16}) \\
& + (k_2 k_0, v_{23}) + (k_2 k_1 k_0, v_{19}) + (k_2 k_1 k_1, v_{18}) + (k_2 k_1 k_2 k_0, v_{20}) + (k_2 k_1 k_2 k_1, v_{21}) \\
& + (k_2 k_1 k_2 k_2, v_{22}) + (k_2 k_2, v_{24})
\end{aligned}$$

Semantic Tree Difference