Note on modeling binding and repulsion force in semantic properties

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We already have stated that the internal structure of a semantic property can be represented by a set of semantic regions occupying a subset of semantic dimensions. Each region denotes a specific semantic aspect of the property. Thus, the total binding / repulsion force is equal to the sum of the of the binding forces between all relevant region pairs minus the sum of the repulsion forces between all relevant region pairs $(\mathbf{r}_a, \mathbf{r}_b)$:

$$f(\mathbf{p}_1, \mathbf{p}_2) = \sum_{a,b} f^+(\mathbf{r}_a, \mathbf{r}_b) + \sum_{c,d} f^-(\mathbf{r}_c, \mathbf{r}_d)$$

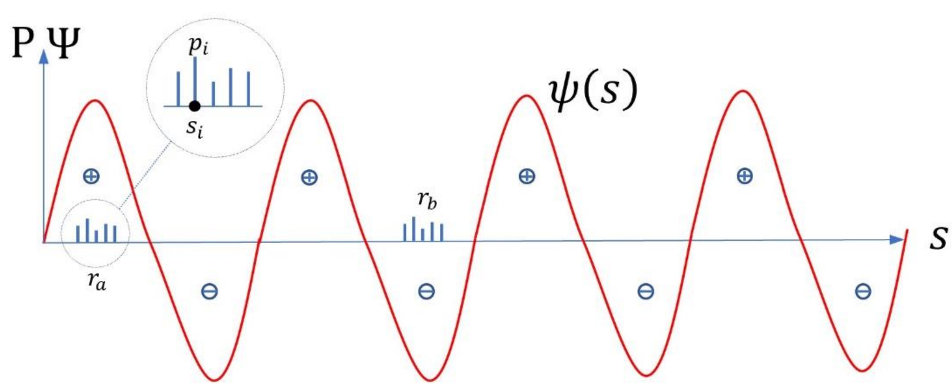
The relevant region pairs $(\mathbf{r}_a, \mathbf{r}_b)$ are defined as follows. Let us sort the pairs of regions from $\mathbf{p}_1$ and $\mathbf{p}_2$ by the absolute value of the binding force.

Definition: *relevant region pair* $(\mathbf{r}_a, \mathbf{r}_b)$ is such pair which has absolute binding force value ***not in*** the ℓ-th quantile for some $\ell > 0$. In other words, all region pairs which are in the ℓ-th quantile are *irrelevant*.

The question now is how do we want to model the binding / repulsion force between a pair of regions. Here we are proposing a possible way to calculate the binding and repulsion forces and will discuss why it is useful to be done this way.
A pair of regions $(\mathbf{r}_a, \mathbf{r}_b)$ from the properties $\mathbf{p}_1$ and $\mathbf{p}_2$ are depicted on the discrete horizontal axis s on the Figure below. The horizontal axis s is discrete in nature and represents the entire set of semantic dimensions for every point in Semantic Space. Let us imagine that region $\mathbf{r}_a$, composed of a set of semantic values p_i, $i = 1..\dim(\mathbf{r}_a)$, will somehow *generate an energy wave* ψ_a which will span the entire horizontal axis s. This wave is depicted in red in the Figure below. Each region will encode the parameters of the energy wave $\psi_a(s)$ in its values p_i. $\psi_a(s)$ will, in general, span all dimensions of the semantic space i.e. the integer coordinate s. Obviously, ψ_a will be periodic function along the semantic dimensions axis s. As we said the amplitude A, the frequency ω and the phase φ of the energy wave generating binding force are somehow encoded in a portion of each region values. Hence, we can write:

$\psi_a = \psi_a(s; A, \omega, \varphi)$ and $\mathbf{r}_a = \mathbf{r}_a(A, \omega, \varphi)$

Now let us introduce the second region $\mathbf{r}_b$ coming from the other property $\mathbf{p}_2$.



The region $\mathbf{r}_b$ is composed of a set of semantic values p_j, $i = 1..\dim(\mathbf{r}_b)$ which *generate an energy wave* ψ_b.

We postulate that the two regions will interact with each other through binding or repulsive force only if the frequencies of the corresponding energy waves *are the same* i.e. $\omega_a = \omega_b = \omega$. For simplicity and as we will see later - without a loss of generality, we will assume that the amplitudes of the two energy waves are the same and they are in phase i.e. $A_a = A_b = A$ and $\varphi_a = \varphi_b = \varphi$.

We postulate that region $\mathbf{r}_a$ will attract region $\mathbf{r}_b$ *iff*:

1. The frequencies of the corresponding to each region energy wave are the same i.e. $\omega_a = \omega_b = \omega$
2. $f_a = \sum_{i \in a} \psi_a(s_i) > 0$; $\psi_a(s_i)$ is the value of the energy at the i-th dimension wave of region $\mathbf{r}_a$
3. $f_b = \sum_{j \in b} \psi_b(s_j) < 0$; $\psi_b(s_j)$ is the value of the energy at the j-th dimension wave of region $\mathbf{r}_b$

Then the attraction force between the two regions $\mathbf{r}_a$ and $\mathbf{r}_b$ will be given by the product of the absolute values:

$$f^+(\mathbf{r}_a, \mathbf{r}_b) = |f_a||f_b|$$

If $sign(f_a) = sign(f_b)$ then we have a repulsive force instead of attracting one:

$$f^-(\mathbf{r}_a, \mathbf{r}_b) = -|f_a||f_b|$$

If we have more than one region with the same frequency ω in one of the properties we sum them up and then multiply with the sum of the regions of the other property:

$$f(\mathbf{p}_1, \mathbf{p}_2; \omega) = \sum_{a,b} f^+\big(\mathbf{r}_a(\omega), \mathbf{r}_b(\omega)\big) + \sum_{c,d} f^-\big(\mathbf{r}_c(\omega), \mathbf{r}_d(\omega)\big)$$

Finally, the total binding/repulsive force between the two properties is given as the sum of all binding/repulsive forces on all frequencies:

$$f(\mathbf{p}_1, \mathbf{p}_2) = \sum_{\omega} f(\mathbf{p}_1, \mathbf{p}_2; \omega)$$

//TODO: finish this