Practical Examples Using Semantic Simulation With Reinforcement Learning

D. Gueorguiev 12/4/2022

The Game Addition

Let us consider the game $Addition$ described in *Blackwell's Theory of Games and Statistical Decisions* (Blackwell & Girshik, 1978, p. 14):

I and II alternatively choose integers, each choice being one of the integers $1, \dots, k$ and each choice made with the knowledge of all preceding choices. As soon as the sum of the chosen integers exceeds N, the last player to choose pays his opponent one unit.
The situation at which player I finds himself at his rth move is described by a sequence $s_r = (i_1, i_2, \dots, i_{2r-2})$ with each i_j being one of the integers $1, \dots, k$ and

$\sum_{j=1}^{2r-2} i_j \le N$

Denote by S_r the set of possible sequences s_r where $r = 2, \dots, \left[\frac{N}{2}\right] + 1$ and $[z]$ denotes the closest integer which does not exceed z. A strategy x for I consists of a set of $\left[\frac{N}{2}\right] + 1$ functions $f_1, \dots, f_{\left[\frac{N}{2}\right]+1}$, where f_r is a function defined on S_r assuming only values $1, 2, \dots, k$: f_r specifies I's rth move when the previous history of the play is s_r. Similarly, a strategy y for II is a set of $\left[\frac{(N+1)}{2}\right]$ functions $g_1, \dots, g_{\left[\frac{(N+1)}{2}\right]}$, where g_r is defined for the set T_r of all sequences $t_r = (i_1, \dots, i_{2r-1})$ with each i_j being one of the integers 1,2, …, k and

$\sum_{j=1}^{2r-1} i_j \le N$

Define $i_1(x, y) = f_1$ and inductively for $j > 0$,
$i_{2j}(x, y) = g_j\left(i_1(x, y), \dots, i_{2j-1}(x, y)\right)$
$i_{2j+1}(x, y) = f_{j+1}\left(i_1(x, y), \dots, i_{2j}(x, y)\right)$
(this induction describes the manner in which a referee would carry out the instructions of the players) and let $j^*(x, y)$ be the largest j for which $i_j(x, y)$ is defined. Then

$$M(x, y) = \begin{cases} 1 \text{ if } j^*(x, y) \text{ is even} \\ -1 \text{ if } j^*(x, y) \text{ is odd} \end{cases}$$

Constructing semantic universe for the game $Addition$

Let us consider the following thought experiment – we have two players playing the $Addition$ game described earlier. Each player is represented by semantic simulation which has its own set of semantic structures and semantic template which recognizes the rules of the game. Let us start our experiment by looking in the semantic template which recognizes the rules of the game which we will name *semantic* *recognizer*. That is - we are interested in what the semantic recognizer might be taking as an input and producing as an output and how the semantic recognizer template would be interacting with the rest of the semantic structures running in the simulation.

Let us assume that the semantic simulation corresponding to each of the two players I and II is limited to the simply connected regions R_1 and R_2 in semantic space. Let $\dim(R_1) = \dim(R_2) = L$. Let us assume that $R_1 \cap R_2 = C$ where C is finite, closed and simply connected region of semantic space with the same number of dimensions L. We will denote C as the *common simulation region*.

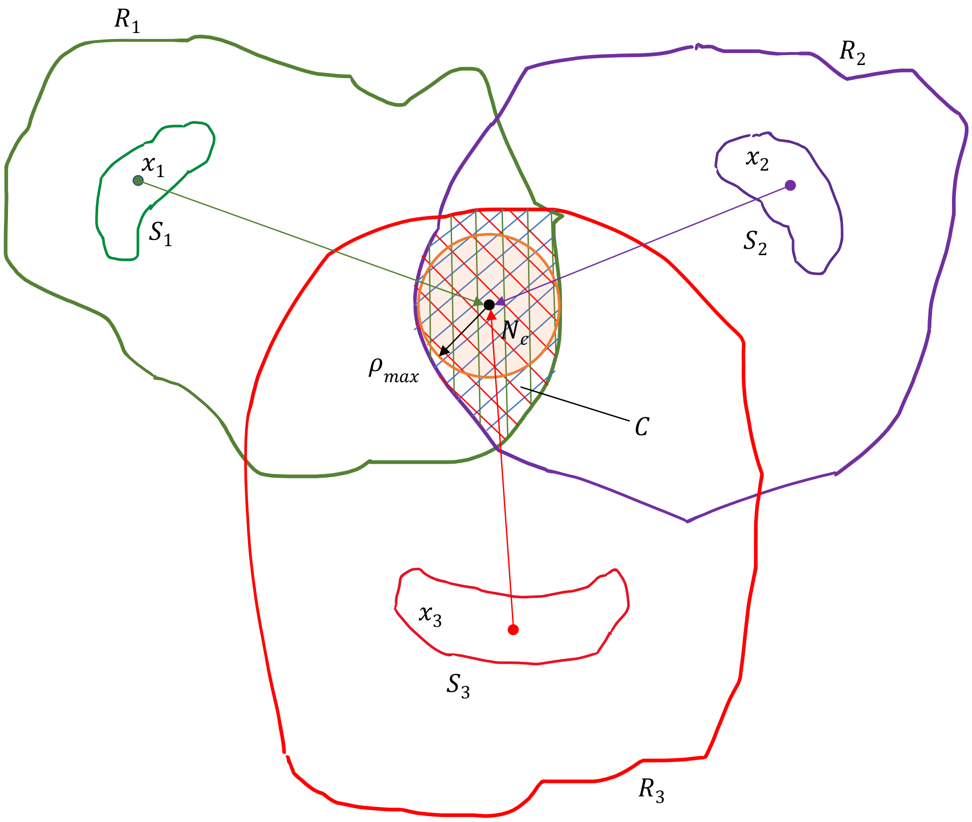

Figure 1: Layout of the simulation space in Blackwell's game *Addition*

Definition: *neutral point* of a simply connected region in metric space
Let C is a simply connected region in some L dimensional metric space. Then the point N_c is a neutral point *iff* it is the center of the largest L dimensional sphere which can fit entirely in the simply connected region C without including any points outside of C. Formally,
$\exists\, N_c \in C \therefore\ \rho_{max} = \max_{\rho} |N_c - x| \leq \rho\ \forall\, x \in C$

With N_c we denote *the neutral point* of the common simulation region C. The neutral point will be the attraction center for all outputs from player I and II's as well as the arbiter simulations. Thus, both players I and II will produce output which will be a semantic particle starting its existence at the point S_0 in C.
Here is how the game simulation will proceed:

First, let us introduce a new entity arbiter in semantic space which will run in a different simply connected region of semantic space R_a such that $C \subset R_a$.

A semantic particle is produced at S_0 in C by the arbiter announcing a proposed value of N.

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