The Signature of Semantic Structures

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Let us have the compound particle V_{comp} represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:

$stree(V_{comp}) =$

```
            __V1__
           /   |   \
         V2    V5   _V6_
        /  \        / | \
      V3   V4     V7  V8  V9
```

The property tree for each V-particle $V_k, k = 1..9$ are given with the algebraic notation discussed in Semantic Tree Operations.

$ptree(V_k) = \sum_{\boldsymbol{k}\in\mathfrak{T}(V_k), i\in\mathbb{p}(V_k)}(\boldsymbol{k}, P_i)$. Here $\boldsymbol{k}$ denotes the path $\left(k_{l_1}, k_{l_2}, \dots, k_{l_h}\right)$ constructed by branching consecutively along the k_{l_1}-*th* branch from the top level, then the k_{l_2}-th branch from the lower level and finally k_{l_h}-th branch from the h-th level. The set $\mathfrak{T}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of V_k. The set $\mathbb{p}(V_k)$ denotes the indices of the vertices in the property tree of V_k.

```
        V1                          V2 `                  ...         V9

   P1 P2  P3                  P1--- o --- P2                    P1--- o --- P2
      \ | /  ..                    / | \
Pk --- o --- Pi                  P3  P4  P5_
  ..  / | \  ..                  / \      |  \
   Pj+1 Pj  Pj-1               P6  P7     P8   P9
                               |
                               P10
```

Expressing the property tree of V_1 with the algebraic notation discussed earlier:

$ptree(V_1) \;=\; \sum_{k_j\in\mathfrak{T}(V_1), i\in\mathbb{p}(V_1)}\left(k_j, P_j\right)$

Similarly, $ptree(V_2)$ is given with

$$\begin{aligned} ptree(V_2) = {} & (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3k_1, P_6) + (k_3k_2, P_7) \\ & + (k_5k_1, P_8) + (k_5k_2, P_9) + (k_3k_1k_1, P_{10}) \end{aligned}$$

Here P_0 is $text(V_2)$.

Now if we expand the property trees for each V-particle in the semantic tree for the composite particle V_{comp} we will have a larger augmented property tree. This augmented property tree represents the semantic structure of V_{comp} and can be recorded in a matrix form which is the semantic signature of V_{comp}. The semantic signature matrix of V_{comp} will have the following structure:

$$ssig(V_{comp}) = [\boldsymbol{p_0}\;\; \boldsymbol{a_{0,1}}\;\; \boldsymbol{p_1}\;\; \boldsymbol{p_0}\;\; \boldsymbol{a_{0,2}}\;\; \boldsymbol{p_2}\;\; \boldsymbol{p_0}\;\; \boldsymbol{a_{0,3}}\;\; \boldsymbol{p_3}\;\; \dots\;\; \boldsymbol{p_p}\;\; \boldsymbol{a_{p,q}}\;\; \boldsymbol{p_q}]$$

The last matrix can be rewritten in block matrix notation:

$ssig(V_{comp}) = [\mathbf{B_1}\ \mathbf{B_2}\ \mathbf{B_3}\ \ \dots\ \ \mathbf{B}_{\boldsymbol{q}}]$

$\mathbf{B_1} = [\boldsymbol{p_0}\ \ \boldsymbol{a}_{\mathbf{0},\mathbf{1}}\ \ \boldsymbol{p_1}]$, $\mathbf{B_2} = [\boldsymbol{p_0}\ \ \boldsymbol{a}_{\mathbf{0},2}\ \ \boldsymbol{p_2}]$, $\mathbf{B_3} = [\boldsymbol{p_0}\ \ \boldsymbol{a}_{\mathbf{0},\mathbf{2}}\ \ \boldsymbol{p_3}]$, . . . , $\mathbf{B}_{\boldsymbol{q}} = [\boldsymbol{p_0}\ \ \boldsymbol{a}_{\mathbf{0},2}\ \ \boldsymbol{p_q}]$

Here the block matrix $\mathbf{B_1}$fully describes the property P_1 including how it is connected to the property tree $ptree(V_1)$. Similarly, $\mathbf{B_2}$ and $\mathbf{B_3}$ fully describes the properties P_2 and P_3 and their connectivity to $ptree(V_1)$. Finally, $\mathbf{B}_{\boldsymbol{q}}$ fully describes the property $P_q$and its connectivity to $ptree(V_9)$.

In the block matrix for $ssig(V_{comp})$ $\boldsymbol{p_0}$ denotes the signature column vector of the property P_0 , $\boldsymbol{a}_{\mathbf{0},\mathbf{1}}$denotes the signature column vector of the arc between property P_0 and property P_1, $\boldsymbol{a}_{\boldsymbol{p},\boldsymbol{q}}$ denotes the signature column vector of the arc between property $P_p$and P_q. Let us denote the number of rows of $ssig(V_{comp})$ by N and the number of columns by M.
The semantic signature matrix $ssig(V_{comp})$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig(V_{comp})$ and connectivity signature matrix $csig(V_{comp})$:

$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$

$psig(V_{comp}) = [\boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_1}\ \ \boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_2}\ \ \boldsymbol{p_0}\ \ 0\ \ \boldsymbol{p_3}\ \ \dots\ \ \boldsymbol{p_p}\ \ 0\ \ \boldsymbol{p_q}]$

$csig(V_{comp}) = [0\ \ \boldsymbol{a}_{\mathbf{0},\mathbf{1}}\ \ 0\ \ 0\ \ \boldsymbol{a}_{\mathbf{0},\mathbf{2}}\ \ 0\ \ 0\ \ \boldsymbol{a}_{\mathbf{0},\mathbf{3}}\ \ 0\ \ \dots\ \ 0\ \ \boldsymbol{a}_{\boldsymbol{p},\boldsymbol{q}}\ \ 0]$

Let us denote by $psig(P_1, V_{comp})$ the augmented semantic property signature of property $P_1$with respect to V_{comp}. It is given with:

$psig(P_0, V_{comp}) = [\boldsymbol{p_0}\ \ 0\ \ 0\ \ \boldsymbol{p_0}\ \ 0\ \ 0\ \ \boldsymbol{p_0}\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ 0]$

Similarly,

$psig(P_1, V_{comp}) = [0\ \ 0\ \ \boldsymbol{p_1}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ 0]$

$psig(P_q, V_{comp}) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0\ \ 0\ \ \boldsymbol{p_q}]$

Then obviously:

$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$

Here $\mathbb{S}(V_{comp})$ denotes the set of the indices of all semantic particles which the composite V_{comp} is composed from.

Another way to partition the signature matrix into block matrices is:

$ssig(V_{comp}) = [\mathbf{V_1}\ \mathbf{A_{1,2}}\ \mathbf{V_2}\ \mathbf{A_{1,3}}\ \mathbf{V_3}\ \dots\ \mathbf{A_{6,8}}\ \mathbf{V_8}\ \mathbf{A_{6,9}}\ \mathbf{V_9}]$

The block matrix $\mathbf{V_1}$ represents the property tree of the particle V_1 and it is given by:

$\mathbf{V_1} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,1}}\ \ \boldsymbol{p_1}\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,2}}\ \ \boldsymbol{p_2}\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,3}}\ \ \boldsymbol{p_3}\ \ \dots\ \ \boldsymbol{p_0}\ \ \boldsymbol{a_{0,k}}\ \ \boldsymbol{p_k}]$

The block matrix $\boldsymbol{A_{1,2}}$ describes the connection between the particles V_1 and V_2 connecting the root property $\boldsymbol{p_0}$ of V_1 and the root property $\boldsymbol{p_{k+1}}$of V_2. It is given with:

$\mathbf{A_{1,2}} = [\boldsymbol{p_0}\ \ \boldsymbol{a_{0,k+1}}\ \boldsymbol{p_{k+1}}]$

Properties of the signature matrix

Here are some interesting properties of $ssig(V_{comp})$:
The number of rows N in $ssig(V_{comp})$ is 3 × the number of arcs in the augmented property tree of V_{comp}.
The rank of
TO DO: finish the property section

Asymptotic closeness of semantic structures

Let us have two semantic structures S1 and S2.
$ssig(S_1) = [\mathbf{V_{k_1}}\ \mathbf{A_{k_1,k_2}}\ \mathbf{V_{k_2}}\ \mathbf{A_{k_1,k_3}}\ \mathbf{V_{k_3}}\ \dots\ \mathbf{A_{k_p,k_q}}\ \mathbf{V_{k_q}}]$
$ssig(S_2) = [\mathbf{V_{l_1}}\ \mathbf{A_{l_1,l_2}}\ \mathbf{V_{l_2}}\ \mathbf{A_{l_1,l_3}}\ \mathbf{V_{l_3}}\ \dots\ \mathbf{A_{l_r,l_s}}\ \mathbf{V_{l_s}}]$

Uniform asymptotic closeness
K-level uniform asymptotic closeness