A Quantum Qualia hypothesis:
from Quantum Cognition to Quantum Perception

Authors:
Naotsugu Tsuchiya[1,2,3], Peter Bruza[4], Makiko Yamada[5], Hayato Saigo[6], Emmanuel M. Pothos[7]

Corresponding author: naotsugu.tsuchiya@monash.edu
Website: https://sites.google.com/monash.edu/tlab/home
Twitter: @conscious_tlab @naotsuchiya

1. Turner Institute for Brain and Mental Health & School of Psychological Sciences, Faculty of Medicine, Nursing, and Health Sciences, Monash University, Melbourne, Victoria, 3800, Australia

2. Center for Information and Neural Networks (CiNet), National Institute of Information and Communications Technology (NICT), Suita-shi, Osaka 565-0871, Japan

3. Computational Neuroscience Laboratories, Advanced Telecommunications Research Institutes International (ATR), 2-2-2 Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-0288, Japan

4. School of Information Systems, Queensland University of Technology, Australia

5. National Institutes for Quantum and Radiological Science and Technology, Chiba, Japan

6. Nagahama Institute of Bio-Science and Technology, Nagahama, Japan

7. Department of Psychology, City University of London

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Highlights

The recent explosion in theories of consciousness, which aim to link subjectivity and physical substrates, require a better mathematical characterization of the contents of consciousness, or qualia.

In traditional and intuitive models of qualia, a particular quale is assumed to be a point in a high dimensional space.

Such models assume that qualia exist independent of measurements and they are incompatible with the findings that qualia are generally affected by measurements.

To address this issue, we propose to employ a mathematical formalism of quantum theory to account for how qualia (as observables) in a particular state can be measured and how the measurement can affect qualia. We call this novel framework a Quantum Qualia (QQ) hypothesis.

We will outline how QQ can be tested with various experimental paradigms, building on the successful quantum cognition framework.

Abstract

To arbitrate theories of consciousness, scientists need mathematical models of consciousness or qualia. The dominant view regards qualia as points in a dimensional space. Implicit in this view is that qualia can be in principle measured without any effects on it. This contrasts with intuitions and various empirical findings about qualia that they can change when they are measured. How can we deal with entities that are affected by the act of measurement? We argue that mathematical formalisms of quantum theory are precisely developed to deal with such situations. Here, we propose qualia as "observables" (i.e., entities that can be in principle observed), sensory inputs and internal attention as "states" that specify the context that a measurement takes place, and "measurement outcomes" as expected values of qualia observables in a certain state. We propose to call this formalism as the Quantum Qualia (QQ) hypothesis. QQ proposes that qualia observables in a state interact with the world, as if through an interface of sensory inputs and internal attention. This qualia-interface-world scheme has the same mathematical structure as observables-states-environment in quantum theory. Within this framework, mathematical concepts of "instruments" can precisely formalize how measurements can affect (or update) qualia observables and states. QQ naturally explains intuitions about qualia, such as the nature of qualia as indeterminate entities. Finally, QQ leads to empirical predictions that challenge traditional models, such as order effects and violations of Bell inequalities. With further confirmation of such predictions, we believe that QQ will emerge as a viable alternate mathematical model of qualia, offering an important step towards understanding the nature and structures of consciousness.

1. Consciousness theories need updated qualia models

Research on consciousness has recently entered a new phase. A burst of neuroimaging studies on consciousness since 1990 has produced a huge amount of empirical data, requiring a principled explanation for consciousness and its neuronal substrate (Koch et al., 2016; Mashour et al., 2020; Seth & Bayne, 2022). Over the last 20 years, many of the initial ideas about consciousness and brains were abandoned in the face of empirical data. The remaining theories have retained their core principles in the form of variations that have branched out from these theories. Some theories aspire to make quantitative predictions, a few of which are currently pitted against each other in an adversarial way (Melloni et al., 2021). Through empirical tests of rival theoretical predictions, substantial scientific progress is to be expected, as has happened in other fields, such as physics and experimental psychology (Aspect et al., 1982; Bell, 1964; Einstein et al., 1935; Freedman & Clauser, 1972; Kahneman, 2003).

As consciousness science matures, it has become increasingly clear that we lack an understanding of the target phenomenon: consciousness. While “consciousness” can mean the level or presence of consciousness, as in clinical science of coma, general anesthesia, deep sleep (Casarotto et al., 2016), this article focuses on the issue of the contents of consciousness, feelings of what-it-is-like-to-be, or, in short, qualia (Balduzzi & Tononi, 2009; Kanai & Tsuchiya, 2012; Lyre, 2022; Tsuchiya & Saigo, 2021; Tye, 2021). Qualia in consciousness research comes in two senses, broad and narrow. In the broad sense, we use a quale to mean a moment of entire conscious experience across all modalities and thoughts[1], that is, everything being experienced. Qualia in the narrow sense refers to one aspect of the experience, such as the “redness” of the sunset, the particular flavor and taste of tuna sashimi, and so on (Balduzzi & Tononi, 2009; Kanai & Tsuchiya, 2012). This article embraces both senses of qualia. What is not qualia concerns everything that is not any part of our conscious experience.

2. Desired properties of qualia models and traditional default models

Traditional models of qualia are founded on the notion of points in a putative metric space, sometimes called a psychological space (Lee, 2021; Rosenthal, 2015; Shepard & Cooper, 1992) (Figure 1A). Indeed, because there are strong arguments that concepts reside in such a space in the cognitive domain (Gärdenfors, 2000), it is natural to start with the idea to represent qualia as single points in a high dimensional space. Accordingly, a point corresponds to a particular quale (either in the narrow or broad sense). The distance between the two points relates to the “similarity” between the respective qualia (e.g., red and orange are close in similarity, but red and green are dissimilar).

Inspired by early work by Shepard, many variants of such similarity models have been proposed (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991), where visualization techniques such as multidimensional scaling (Borg & Groenen, 2005) have played a central role (Figure 1A). Under this framework, various types of qualia, e.g., color (Bujack et al., 2022; Indow, 1988; Kawakita et al., 2023; Shepard & Cooper, 1992; Zeleznikow-Johnston et al., 2023), sound (Cowen et al., 2020; Shepard, 1982), object (Hebart et al., 2020), emotion (Cowen & Keltner, 2017; Nummenmaa et al.,

[1] We include concepts and thoughts as a kind of qualia as long as they have distinguishable qualitative aspects to have it. See (Kemmerer, 2015; McClelland & Bayne, 2016)

2018), olfaction (Young et al., 2014), art (Graham et al., 2010) etc, have been investigated and visualized based on similarity ratings of pairwise comparisons between the set of qualia under investigation.

Despite widespread use, the psychological space approach to modeling qualia has at least three problems.

Firstly, this approach is unable to capture an intuition that some qualia are indeterminate entities. A system can be indeterminate due to intrinsic randomness. Its property cannot be predicted for a single event. At the level of an ensemble, however, a property can be statistically determined. The indeterminacy of qualia becomes apparent when one introspects on the border of experience in space, or time or the nature of unattended or barely attended experience. To determine the spatial border of experience, one can stretch their arms to estimate the limit of the visual field at the periphery, and experientially confirm that this limit is tenuous. Under complete darkness, it is not clear that any such boundary exists. Time also seems to have indeterminacy. The start and end times of an event often feel unsure and a moment rarely feels point-like, but is typically experienced as having some duration (Filk, 2013). Even when one is focally attending to qualia, one can sense an uncertainty regarding phenomenal appearance. Namely, there is an intuition that the very act of attending can alter the quality of the experience (Schölvinck & Rees, 2009; van Boxtel et al., 2010b). It is questionable whether the psychological space approach can capture the indeterminacy of qualia.

Secondly, the psychological space approach is static and does not account for the temporal dynamics of qualia, because it maps sensory input into qualia “at a given time”. The temporal dynamics of qualia, however, are one of the most studied aspects of qualia, from very fine time scales using masking and priming (Bachmann, 2000; Breitmeyer & Ogmen, 2007), to larger time scales involving adaptation, expectation (Melloni et al., 2011), and multistability (Brascamp et al., 2018; Maier et al., 2012). It is unclear how the psychological space approach can account for the spatio-temporal dynamics of qualia.

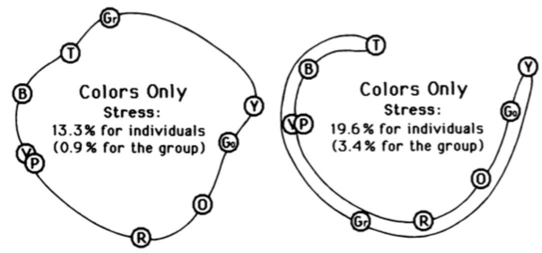
Thirdly, the psychological space approach is silent on how qualia interact with internal mental processes, such as attention. As alluded to above, how we attend to sensory inputs appears to significantly alter what we experience, as implied from change blindness and inattentional blindness demonstrations (Pitts et al., 2018; Simons & Rensink, 2005). However, before we pay attention, we already experience something at the to-be-attended locations, and that is the reason why we can consciously direct attention there. The psychological space model is also mute about how qualia relate to other internal processes, such as memory and expectation.

Of course, any general framework can be in principle extended. Yet, since the pioneering work by Shepard since the 1960s (Shepard, 1970, 1962b, 1962a, 1980, 1987) corresponding extensions (e.g., concerning dynamics) have not been proposed. Note that masking effects have been known for >100 years (Breitmeyer & Ogmen, 2007; Exner, 1868) and >60 years of points-in-high dimension models have had nothing to say about such effects. Even without a properly fleshed out QQ morel, we believe that the outline we present here offers reasonable promise, regarding the explanation of such masking phenomena.

Thus, the psychological space approach to modeling qualia as points in a dimensional space appears deficient in regard to well established intuitions that qualia are indeterminate, dynamic, and interact with other mental processes. But why do researchers continue to adhere to this approach? We surmise that this is due to the combination of the intuitive appeal of such models and lack of compelling alternatives.

Interestingly, a similar situation arose in the field of cognitive science, in particular decision making. In decision making, models based on standard probability theory and logic have been persistently challenged by many (apparently) paradoxical findings in human decision making. Some of these paradoxes in decision making have had fairly natural explanations by means of quantum probability theory, which was introduced in psychology with the quantum cognition framework (Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; A. Y. Khrennikov, 2010; Pothos & Busemeyer, 2022)[2]. Quantum cognition is a research program involving the application of quantum theory in the modeling of cognitive phenomena. It is the basis of the QQ hypothesis.

Decision making and other cognitive processes are inextricably linked to perception and sensation (Barsalou, 2010) and also appear to share basic neural processing architectures. If so, it follows to consider the application of quantum probability theory as an alternate mathematical framework for qualia, in order to address the challenges for the psychological space approach.

A) 

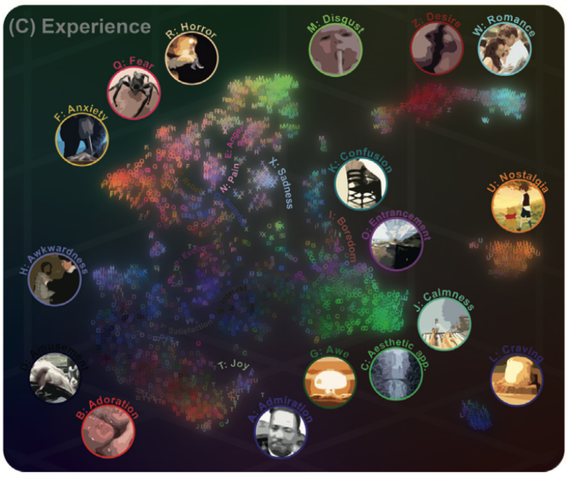
B) 

Figure 1. Traditional psychological space models.

Traditional psychological space models (Lee, 2021; Rosenthal, 2015; Shepard & Cooper, 1992) assume each quale occupies a point in space. “Distances” between two points are assumed to be related to perceived experiential similarity (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991). A) A classic color hue ring model for the representation of similarity relationships among 9 colors for color-typical and red-green color blind individuals(Shepard & Cooper, 1992). B) Similar

[2] Some studies in quantum cognition are highly relevant to ours (Atmanspacher & Müller-Herold, 2016; Filk, 2009; A. Khrennikov, 2015, 2021). While we call our hypothesis the Quantum Qualia hypothesis, our proposal should be distinguished from the so-called Quantum Brain hypothesis, which considers quantum mechanical processes in the brain (Hameroff & Penrose, 2014) and the role of consciousness in quantum collapse (Chalmers & McQueen, 2021). Our proposal is completely consistent with the possibility that all the relevant physical events happening in the brain are purely classical. Our core idea is to utilize the mathematical formalism of quantum theory, as outlined below. For these and other related issues see (Atmanspacher, 2017).

representations (points-in-high dimensional spaces) have been used in the other domain of experience, such as emotional experience (Cowen & Keltner, 2017). Here, 2185 brief videos are represented as points using the tSNE algorithm (van der Maaten & Hinton, 2008).

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3. What is the Quantum Qualia hypothesis?

We believe the essential challenge for existing models is that they treat qualia as something that can be probed, observed, reported or "measured," without any effects on itself. To construct a more general model, it is useful to start with the assumption that such "measurements" can affect qualia by default. (However, as we will discuss later, there are different degrees in which these measurements affect different kinds of qualia.)

Once considered in this way, it is natural to seek inspiration from quantum theory, for constructing formalism that can deal with entities whose properties can change upon measurement. Indeed, by incorporating formalisms from quantum theory, we can obtain a mathematical model that attains the three desired features for qualia. And this is exactly what we call the Quantum Qualia (QQ[3]) hypothesis, which states that qualia are like quantum entities, which are known to be affected by measurement. We first give a broad sketch of QQ (Figure 2), then explain technical concepts with familiar examples from consciousness research.

To account for the indeterminacy of qualia, QQ distinguishes each instance of measured qualia from qualia themselves, with the idea of "observables", that is, intrinsic properties of a system that can be in principle measured. A feature of QQ is that the theory does not presuppose that all aspects of qualia can be simultaneously measured and reported.

While each instance can be indeterminate, under a particular "state", the expected value of a particular quale (as an observable) is given. Formally, states are like functions that return the expected value for a given quale, when a particular observable is measured. For example, in a given experimental condition (e.g., participant A under a particular masking experiment), a "state" gives an expected probability of A's reporting of seeing "a black circle on the left side". Note that this meaning of "state" may not be straightforward, in particular for those who are unfamiliar with quantum theory.

Normally, a conscious "state" refers to a snapshot of an experience. Though it may not be intuitive, our usage of "state" is consistent with such an usage, but is more general than that. In fact, it is perhaps easier to consider state as an interface between observables of a system and the outside environment. Here, we consider that a state would include factors, such as sensory inputs and internal attention. Under a particular state, the expected value of a quale observable (=averaged measured outcomes) can be established. The separation of observables and measured outcomes is fundamental in quantum theory and we propose this to be central in understanding the nature of qualia as well.

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[3] This is different from the quantum question (QQ) equality by Wang and Busemeyer (Z. Wang & Busemeyer, 2013).

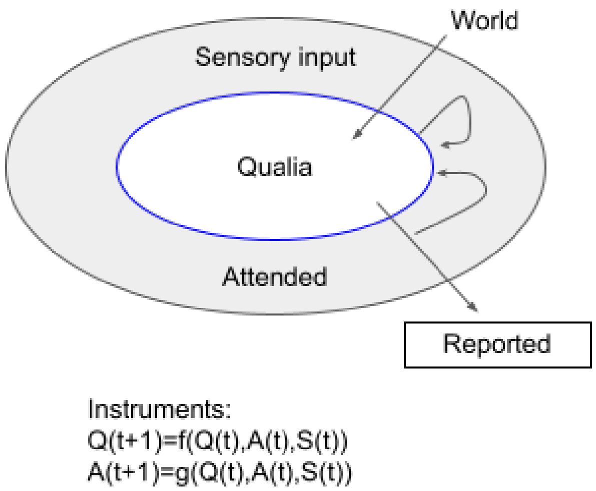

Figure 2. Conceptual framework of the QQ hypothesis.

QQ considers qualia as observables, that are properties of a system that can be in principle "measured", probed and reported. QQ does not presuppose that all aspects of qualia can be simultaneously measured. All the physical or psychological processes that support or underlie qualia can be considered as "states", which can be thought of as a medium or interface between qualia and the world. This membrane-like state is represented by a gray ring interfacing qualia and the external world. We consider sensory inputs and attention as states for now. The interaction between qualia and attention can be further formalized with the theory of instruments (Khrennikov, 2015; Ozawa & Khrennikov, 2019), which describes how qualia (Q), attention (A) and sensory input (S) evolve over time with or without measurement. The theory of instruments generalizes conditional probability. Instruments could formalize how qualia (and attention) at time t+1 depend on qualia and attention at time t. Informally, the putative interaction between the world and qualia, qualia and subjective reports, and how reports alter attention and qualia are depicted by arrows in the figure.

In quantum theory, there are three mathematically equivalent ways to consider the dynamics of observables and states (Sakurai & Napolitano, 2014). QQ considers both observables and states to change over the time. This interpretation is called an interaction picture. As observables are intrinsic properties of the system, we believe the interaction picture is suited for QQ. Most quantum cognition studies assume observables are possible response options, which are fixed, while they treat (mental) states as dynamically changing. This idea of fixed-observables and dynamic-states is called the Schroedinger picture. Some fields of physics (e.g., particle physics) assume states as fixed while observables change, which is called the Heisenberg picture. (See Table 1 for a summary.)

In this paper, we predominantly consider sensory inputs and internal attention as major components of states, but other mental processes, such as memories and expectations, can also correspond to states. Thus, in this interaction picture, QQ explicitly considers how qualia (observables) interact with states (sensory inputs and attention).

Finally, to formalize how qualia interact with other mental processes, we introduce the concept of the theory of instruments (cf. the arrows in Figure 2; Davies & Lewis, 1970). Instruments offer a generalization of conditional probability. In modern measurement theory, any measurement of the system is described by a mathematical structure called an instrument. In standard quantum physics, measurements are considered ‘all or none’. As the theory of quantum measurement matured, researchers arrived at the concept of “instruments” as the most general form of measurement. The instrument formalism offers a bridge from nonlinear wave collapse (which is the result of a measurement in standard quantum theory) to the unitary dynamics of an isolated system and intermediate soft or weak measurements. The theory formalizes how a measurement affects states and observables over time. Instruments are utilized in modern quantum measurement theory and started being applied in the field of quantum cognition (Khrennikov, 2015; Ozawa & Khrennikov, 2019). Instruments can describe how qualia, as observables and states, dynamically develop upon measurements.

While the above descriptions are sufficient to understand the core idea of the QQ, below we supplement with some further technical details and terms.

3.1 What counts as a system?

We defined qualia observables as intrinsic properties of a system. But what do we mean by the term “system”? We consider a system minimally as “that which is experiencing the qualia in question”. It would correspond to “the complex” in Integrated Information Theory (Albantakis et al., 2022) or “Global Neuronal Workspace” in (Mashour et al., 2020). Over time, a system can change but still needs to be identified as a coherent entity or phenomenon. A system is able to experience a certain observable or quale (e.g., a red circle on the left), which can be reported to and measured from the outside environment.

3.2 A state as an interface between qualia and the world

The interrelationship between the system and the environment external to it is represented by the state of the system. In a sense, a state can be considered an interface. This idea may sound strange at first glance, but actually it is equally applicable across classical and quantum theory (Ojima, 2004; Saigo, 2021; Saigo et al., 2019). For example, the temperature of water in a cup as an observable needs to be determined in the context of where and how they are placed. This context is its state. In the case of qualia, such a context would involve at least sensory inputs and internal attention. When one is sitting in the sunset with the mind wandering, the expected value of reporting a particular quale, seeing the color of red for example, can be established. In a more artificial setting, a weak grating stimulus is presented with masking under some task instruction. In such a state, the expected value (= probability) of seeing the grating is statistically determined, even if a single-trial outcome may not be completely determined. The notion of an interface between system and environment is already an important idea, as discussed in many theories of consciousness (e.g., “background conditions” in the Integrated Information Theory of consciousness, Albantakis et al., 2022, “Markov blanket” in the free energy principle, Kirchhoff et al., 2018, and “mediation” in philosophy, (Taguchi, 2019)).

In applying quantum theory, QQ aspires to establish principled associations among observables, states, and their interactions, not necessarily at the level of an individual event but at the level of collections of similar qualia. In fact, every individual event and quale can be considered as unique, especially when its space and time are considered. We are specifically not interested in working at that level, but in constructing laws that emerge when some “similar” qualia are grouped together.

How to construe “similar” is an important question, which the authors have discussed elsewhere, using concepts from category theory (Tsuchiya et al., 2016, 2023). In category theory, it is quite explicit that what to consider as similar is a choice of mathematicians or scientists, not automatically or uniquely ‘given’ by the world (Cheng, 2022). In most theoretical and experimental contexts, qualia are similar as long as they are considered similar in some aspects, by the observing individual, as in the everyday usage of “similar”. In summary, “state” is an interface that assigns a measurable value “on average” to each observable, which may not be measurable from a single event.

3.3 Instrument formalism for dynamics of qualia and states

Let us now explain the dynamics of qualia. For simplicity, consider a discrete time step and consider qualia, sensory input, and attention at time t as Q(t), S(t), and A(t). Their interdependency is illustrated by the arrows in Figure 2. The dynamical update rules are expressed as

$Q(t+1) = f(Q(t), S(t), A(t))$ and
$A(t+1) = g(Q(t), S(t), A(t))$

This simple formulation is a primitive form of an instrument. Currently, we do not have enough data to constrain the form of f and g. While the idea of an instrument was derived in the context of quantum probability theory (Davies & Lewis, 1970), it can be considered as a generalization of conditional probability, that is, a part of classical probability theory as well. Such a concept can generally and naturally formalize how changes of sensory input affect both what we experience and how we attend. It also captures how attending to uncertain aspects of qualia (e.g., a spatial boundary) can change qualia. For specific and empirical applications of instruments in quantum cognition, see Ozawa and Khrennikov (2019).

3.4 A common mathematical and philosophical structure between quantum phenomena and qualia

On the surface, QQ simply introduces several concepts from quantum theory (e.g., separation of observables, states and averaged measured outcomes, and instruments). At a deeper level, however, we surmise that there is a fundamental common mathematical and philosophical structure between quantum phenomena and qualia.

3.4.1 Noncommutativity, complementarity, uncertainty relations of quantum theory, quantum cognition and QQ

One of the foundational ideas behind quantum theory is “complementarity”. In the context of qualia, two qualia are complementary when they cannot be experienced simultaneously, as we consider in more detail below (Bruza et al., 2023). Complementarity is a philosophical concept that one of the founders of quantum theory, Niels Bohr, introduced in physics, indirectly inspired by one of the founders of modern experimental psychology, William James, through Edgar Rubin (Holton, 1988).

The idea of complementarity can be mathematically expressed via the concept of noncommutativity (Atmanspacher & Filk, 2018; Streater, 2007). Noncommutativity implies sensitivity to the order of an operation. In general, the effect of processing A then B is not the same as B then A. Noncommutativity is the default for many processes, from cooking to chemical reactions. In the brain, this could correspond to the effect of processing A leaving some trace, in terms of synaptic plasticity or neuronal activity, which impacts on processing B. If this is the case, processes A and B are expected to be noncommutative and likewise for the corresponding qualia.

If observables A and B are noncommutative, A after B typically yields a different outcome to B after A. It is generally accepted that many aspects of human cognition are noncommutative. Even in arithmetic, subtraction and division are noncommutative. Whilst multiplication is commutative for numbers it is not for matrices (matrix operations are fundamental to quantum theory, (Busemeyer & Bruza, 2012). Noncommutative observables can be used to formalize important features of qualia, such as indeterminacy. Starting with the well established noncommutative formalization of quantum theory as a guiding framework, it should be possible to appropriately extend this formalism for QQ and, as we explain later, it should be possible to empirically demonstrate its necessity.

Regarding qualia, in general, when we consider "processes", the order of the process matters. In an example on masking, presenting target T briefly before mask M at a particular interval can make T completely invisible. But swapping the order into M then T, both of them can become highly visible. This is an example of noncommutative observables. Quantitative explanation of the order effects in various cognitive phenomena is one of the hallmarks of the quantum cognition framework (Busemeyer & Bruza, 2012; Busemeyer & Wang, 2017; Pothos & Busemeyer, 2022). Complementarity as noncommutativity is experimentally demonstrated as uncertainty relations (Atmanspacher & Filk, 2018).

Complementarity, noncommutativity and uncertainty relations are the basis of quantum theory, from which quantum cognition arose. Quantum cognition started from explaining enigmatic phenomena in decision making (Aerts et al., 2018; Basieva et al., 2019; Broekaert et al., 2020; Busemeyer et al., 2019; Mistry et al., 2018), concept combination (Aerts & Arguëlles, 2022; Bruza et al., 2015; D. Wang et al., 2021), and judgment (Ozawa & Khrennikov, 2022; Z. Wang et al., 2014; Z. Wang & Busemeyer, 2013; White et al., 2020). It has recently expanded into modeling for language (Surov et al., 2021), emotion (Huang et al., 2022; Khrennikov, 2021), music (beim Graben & Blutner, 2019), and social judgments (Tesař, 2020). It is beginning to be applied to solve real-world problems (Arguëlles, 2018; Song et al., 2022; Wojciechowski et al., 2022) and influence the design of artificial intelligence and robots that aim to interact with the world (Ho & Hoorn, 2022).

To the extent that cognition is continuous with perception (Barsalou, 2010), the quantum cognition approach is relevant to the contents of perceptual consciousness, or qualia. Indeed, certain applications of quantum cognition to perceptual judgements are already emerging (Asano et al., 2014; Atmanspacher & Filk, 2010; Bruza et al., 2023; Conte et al., 2009; Epping et al., 2023; Yearsley et al., 2022), as we will discuss below.

3.4.2 A common philosophical structure between quantum phenomena and qualia

On the philosophical side, both quantum phenomena and qualia have a commonality as phenomena that arise from "interactions". As we pointed above, the philosophical concept of "mediation" (Taguchi, 2019) is almost the same as what we introduced as "a state as an interface". Quantum phenomena arise from interactions between quantum objects, such as photons, and measurement devices (Plotnitsky, 2021).

Notably, Niels Bohr stated that the reality responsible for quantum phenomena is indeterminate and beyond representation (Plotnitsky, 2021). The reality of qualia defies concrete representation in a similar way. Representationalism states that the phenomenal character of experience is reducible to representational content (Block, 1998). By contrast, in the anti-representational view, conscious experience is not reducible to cognitive representations (Gibson, 2014; Schlicht & Starzak, 2021; Varela, Francisco et al., 2017).

Under the QQ hypothesis, some qualia are indeterminate when they are not attended. When attention is directed, such an indeterminate quale can become determinate, which corresponds to an intentional, content-bearing phenomenal object with reducible representational content. This, in turn, means that before attention is directed, some qualia are already there but such qualia do not have reducible representational content.

3.5 Interim summary: What is the Quantum Qualia hypothesis?

In summary, and to add a bit more specificity, QQ hypothesizes the following. First, observables correspond to all possible aspects of experience that a system (e.g., a person) can have. They consist of possible experiences from all sensory modalities, as well as abstract thoughts, concepts, memories and feelings, that is, anything, as long as it is part of an experience (this is qualia in the broad sense). States are a particular arrangement of the system. When the system is in a given state, averaged measurement outcomes, or reports, about qualia observables can be lawfully specified. States represent sensory inputs and any internal condition of the system, including how the system attends to or accesses observables. Second, averaged measurement outcomes are results of interactions between observables and states. Third, observables and states change dynamically and interact with each other, as formalized by quantum instruments theory. From mathematical and philosophical perspectives, qualia have a striking analogical correspondence with quantum phenomena. Table 1 summarizes these terminologies and how they are used in quantum theory, quantum cognition, and QQ.

	Observables	States	Averaged measurement outcomes
Quantum Theory	$\mathcal{A}$	Ψ	$\Psi(a)$, $a \in \mathcal{A}$
Quantum Cognition	Response options (fixed)	Mental states (dynamic)	Responses
Quantum Qualia	Qualia (dynamic)	Sensory inputs, attention (dynamic)	Reportable aspects of qualia

Table 1. Conceptual summary of quantum terminologies (columns: observables, states, averaged measurement outcomes) and how they are used in (rows) quantum theory, quantum cognition, and quantum qualia hypothesis. Each cell entry explains a representative usage of each concept.

4. What are the benefits of the QQ and how can we test its predictions?

As explained above, the QQ hypothesis fits with our intuition about qualia, accommodating their indeterminacy, dynamics, and putative interaction with internal processes. Furthermore, QQ offers some reasonable insights concerning our empirical knowledge about qualia and provides novel perspectives about the nature of qualia. Here we provide some details of three lines of studies, order effects, violation of the Bell inequality, and relationships between qualia and attention, showcasing how to empirically test various predictions from QQ.

4.1 Order effects in similarity judgments among color qualia.

The QQ hypothesis is empirically testable in surprisingly simple ways. One way is to ask if the order of questions or stimuli matters for the resulting reports. Epping and colleagues (Epping et al., 2023) presented a pair of color patches to participants, then asked if the reported similarities are symmetric with respect to the order of color patch presentation.

Since seminal work by Rosch (Rosch, 1975) and Tversky (Tversky, 1977), perceptual similarity judgments about colors, faces and objects have been repeatedly shown to be asymmetric (Best & Goldstone, 2019; Hodgetts & Hahn, 2012; Polk et al., 2002; Roberson et al., 2007). These studies challenge standard points-in-space type models, requiring arguably ad hoc modifications (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991).

Researchers are aware of this asymmetry (reflected by the fact that Tversky's paper is cited over 11,000 times according to Google Scholar as of 2023, May). Yet, it is not common to take asymmetries into account empirically in similarity or perception studies, as this doubles the numbers of trials. Even when different orders are included, researchers invariably do not know how to approach any observed asymmetries. As a result, they "symmetralize" the originally asymmetric similarity matrix, so that they can use popular, existing analytic algorithms, such as multidimensional scaling.

While an isolated instance of asymmetry (e.g., "Is China similar to North Korea" vs. ``Is North Korea similar to China", (Tversky, 1977)) can be explained in many possible models, a collection of perceptual reports for many stimuli, such as color patches, and a particular pattern of asymmetry across the stimuli are not (Figure 3A). Epping et al.'s quantum models, which consider a state as a density matrix (this is a generalization of the idea that a state can be a vector), and similarity as arising from sequential projections (Figure 3B), gave a better fit to the empirical data (Figure 3C), compared to points-in-space models of qualia (Figure 3D, E) with flexibility to accommodate asymmetry when mapping distance between points to similarity.

As noted, in the past, most similarity experiments tend to ignore the effect of order of presentation, using a simultaneous presentation paradigm, or paradigms that allow longer and uncontrolled inspection of the items. This is understandable due to the cost of experiments manipulating order, where the number of the trials increases quadratically with the number of items to examine. Distributing pairs of items across many participants in online samples may solve this issue (Kawakita et al., 2023).

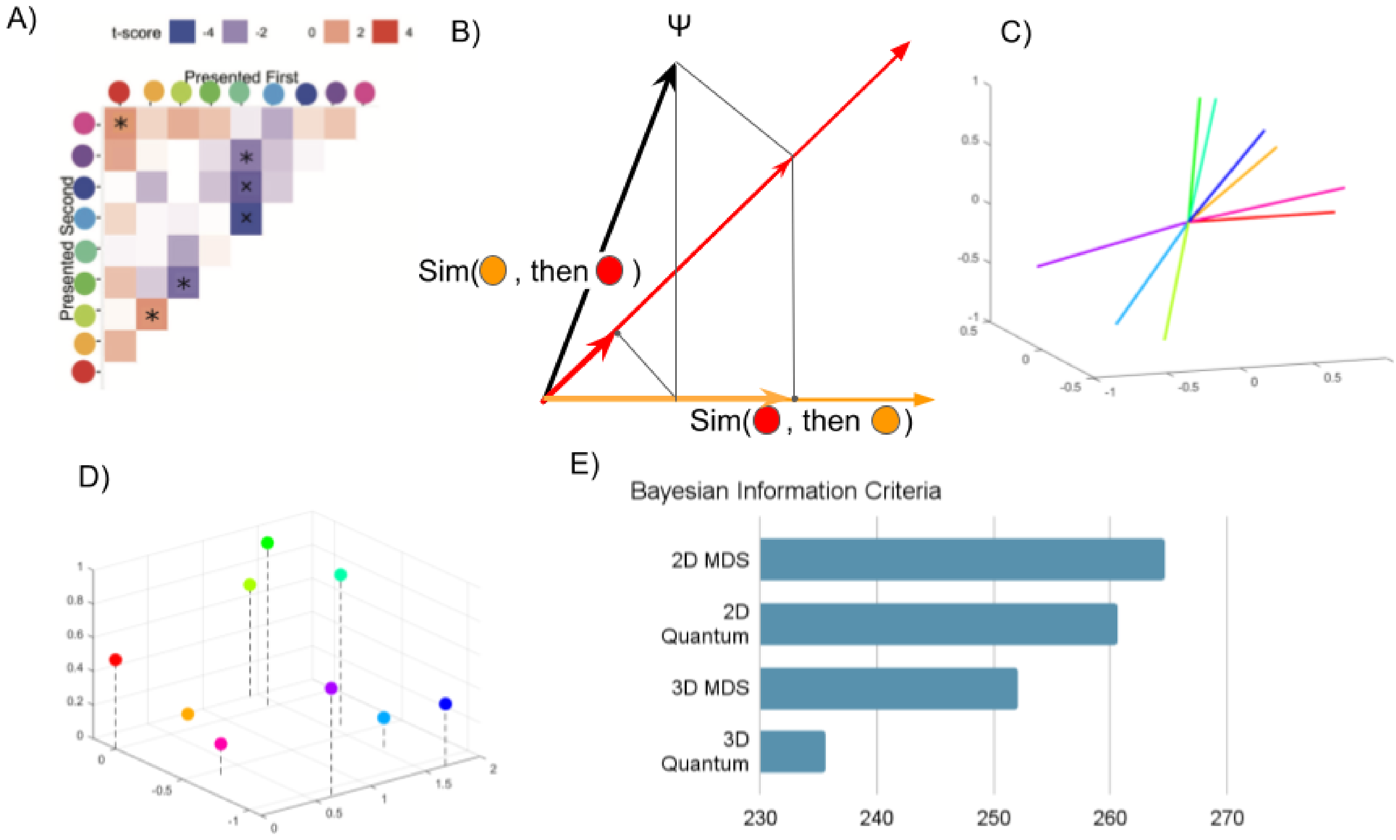

Figure 3. Quantum model of color similarity.

A) Empirical asymmetry matrix. The raw similarity matrix is subtracted from its transpose to reveal the degree of asymmetry in similarity judgements. Taken from (Epping et al., 2023). B) How quantum operations (projections) give rise to perceived similarity (Epping et al., 2023; Pothos et al., 2013; Yearsley et al., 2022). Assume an initial (mental) state as a unit vector Ψ (the black line). The vector is projected onto a subspace representing the color that is first experienced. From there, it is further projected onto the subspace corresponding to the second color. The resulting length of the final projection can be related to the perceived similarity between the two colors. Importantly, the resulting length can depend on the order that the colors are experienced. C) The best fit quantum similarity model for the data in (A) (Epping et al., 2023). In the quantum model, each elementary aspect of experience is modeled as an axis in a space, called a “subspace”. Experienced “similarity” between the two subspaces is related to the square value of the “cosine angle” between them (e.g., the red and the pink subspaces have a narrow angle, but the red and the green subspaces have a near 90 deg angle). D) Traditional 3D MDS representation of 9 colors based on their pairwise similarity. E) Bayesian information criteria (BIC) for best fit 2D and 3D MDS and quantum models. Note that MDS models needed additional free parameters to account for asymmetries in similarity judgements (Nosofsky, 1991), resulting in more complex model. The 3D quantum model offered the best fit to the empirical data.

4.2 Violation of the Bell-type inequality in the domain of qualia.

Another way to test QQ is to test violations of Bell-type inequality in multiple qualia of an object. The next subsection is an introduction to the Bell inequality for those who are not familiar with this concept. Those who are familiar can directly skip to section 4.2.2 for how to test violations empirically.

4.2.1 What is the Bell inequality? And why is it important to demonstrate its violations?

The Nobel Prize for Physics in 2022 was awarded for the demonstration of violations of the Bell inequality. Quantum theory was developed in the 1920s by Bohr, Heisenberg, Shroedinger, Born and others. Quantum theory revised the world view with the key postulate that nature is fundamentally indefinite. In 1935, Einstein, Podolsky and Rosen (Einstein et al., 1935) (EPR) challenged this view, claiming that quantum theory is incomplete. In 1962, Bell discovered one fundamental inequality (Bell, 1964), which had to be satisfied if EPR's view was correct. Subsequently, the violation of the Bell inequality was empirically demonstrated (Aspect et al., 1982; Freedman & Clauser, 1972).

While over the years since the initial EPR experiments there has been debate about loopholes in the experiments that were being conducted, these loopholes have been successively closed. Nowadays, it is generally accepted that the EPR experiments do empirically verify that quantum particles violate the Bell inequalities and are therefore entangled. What this implies about the underlying nature of these particles is, however, still being debated. In parallel, questions about the underlying nature of cognitive phenomena are raised when these violate the Bell inequalities (Bruza et al., 2023).

Bell's inequality can be represented as follows:
S= E(a,b) - E(a,b') + E(a',b) + E(a',b'),
where a and a' are two measurement settings for system A, b and b' for B, and E(:) is the expected value of the corresponding measurements. These expected values have to be measured in separate experimental conditions. In classical systems, $|S|<=2$, unless there are direct influences or signaling. The Bell inequality can be violated if a system operates under quantum theory.

For the QQ hypothesis, demonstrating violations of the Bell inequality in qualia will play a similarly fundamental role. If these types of inequalities are violated, the underlying structure of qualia can be assumed to be quantum-like (which implies additional properties, such as noncommutativity). There are many ways to test Bell-type inequalities psychophysically.

4.2.2 Violations of temporal Bell inequality in multistable perception

Multistable perception is one possibility (Brascamp et al., 2018; Maier et al., 2012). Atmanspacher and Filk (Atmanspacher & Filk, 2010) focused on the number of reversals between three time points of an ambiguous figure. They proposed empirical tests involving the temporal version of the Bell inequality (Yearsley & Pothos, 2014). Specifically, their proposal was to measure perceptual switches between times t1, t2, and t3, where $t1<t2<t3$, selecting two timepoints per condition and for all three possible combinations. The number of perceptual changes between time i and j, n_{ij}, has to follow $n_{12} + n_{23} \geq n_{13}$, if qualia are determinate at all times.

A closely related phenomenon concerns quantum Zeno effects (Atmanspacher et al., 2004; Yearsley & Pothos, 2016). If "measurements" do not affect qualia, any kind of gradual changes in qualia should not be affected by measurements. While multistable percepts change spontaneously, other types of qualia changes, such as morph-induced categorical perception and gradual change blindness, can be used to test if the effects of measurement can be precisely predicted from the quantum formulation of the Zeno effects (Atmanspacher et al., 2004; Yearsley & Pothos, 2016).

4.2.3 Violations of Bell-Boole inequality in multiple qualia about an object

Another way to test the Bell-type inequality is to set up a task with at least three qualia (observables), measuring two observables at a time, but against three different states. If qualia follow classical probability theory and if measurements do not change qualia, then we expect the probability of seeing A, B, and C to follow logical constraints, as exemplified by a Venn diagram (Figure 4A). A simple diagrammatic analysis reveals various inequalities, described by George Boole as "possibility of experience" (Bruza et al., 2023).

Figure 4A demonstrates probability relationships amongst the three questions. Let's say, you are briefly presented with an object, whose color, shape, and location might be red, circular, and on the left. Assume that these properties do not change, regardless of which of two observables you report. If p(A), p(B), p(C) represent the probability that object is red, circular, and on the left, respectively, we obtain that p(A)-p(A,B)-p(B,C)+p(C,A) has to be always positive.

Now, imagine the ball was "masked" to reduce its visibility. These properties can be manipulated randomly. You are then asked to report each property, in a probabilistic manner. P(color=red), P(shape=circle), P(position=left) are all smaller than 1. And this still has to satisfy various constraints, such as that p(R)-p(R, C)-p(R, L)+p(C, L) have to be always above 0, if these three questions are commutative. This is what Boole called "conditions of possible experience". The probability relationships among the questions can be visualized by a Venn diagram – it is worth noting that classical intuitions regarding question outcomes are so entrenched, it is hard to imagine how things could be otherwise.

Bruza and colleagues (Bruza et al., 2023) examined this constraint with three questions about a face (Figure 4B). As questions A, B, and C, participants were asked if the face looked trustworthy (A), dominant (B) or intelligent (C). It turned out that in this case, p(A)-p(A,B)-p(B,C)+p(C,A)<0, implying that the simple classic probabilistic picture in Figure 4A is inappropriate. Violations of such Venn diagram can physically arise and are easy to demonstrate in a classroom using just 3 polarizers (Figure 4C and D, https://www.youtube.com/watch?v=zcqZHYo7ONs). Note this is an excellent demonstration to get familiar with the interesting reality of quantum phenomena, directly observable at the macro level.

Several extensions to the above task are possible. For example, it is plausible that the degree of violation of the Bell inequality may depend on the indeterminacy of qualia. If this were the case, performing the same experiment but with reduced visibility might induce greater violations of the Bell inequality. Visual psychophysics offer a multitude of techniques to reduce visibility of an object (Kim & Blake, 2005; Stein & Peelen, 2021). As mentioned in the opening section, one of the fundamental visibility manipulations is masking. It is interesting to note that masking among three objects (Breitmeyer et al., 1981; Dember & Purcell, 1967) has been reported to be quite complex and might reveal a promising alternative demonstration of Bell inequality violations.

One might argue that properties of faces, such as trustworthiness, dominance, and intelligence are not directly experienced, but rather they are cognitive constructs. It would be a fruitful future experiment to try out if similar conclusions can be obtained when using more perceptual aspects of qualia of an object, such as color, orientation, size, location, and so on.

The Boole-Bell type inequality is a fundamental inequality that has to be satisfied by any classical system. This inequality rules out certain types of correlations between different systems. While superficially simple, definitive tests of such inequalities are subject to several checks and assumptions (Blasiak et al., 2021).

While the fundamental ideas are quite simple, almost no research on qualia has adopted a task design, where three observables are measured under three states. This is understandable given that it would be difficult to motivate such a task or interpret the results, in the absence of a quantum-like theoretical framework. We believe there is a huge opportunity to test novel ideas about consciousness with the QQ formulation involving three or more observables.

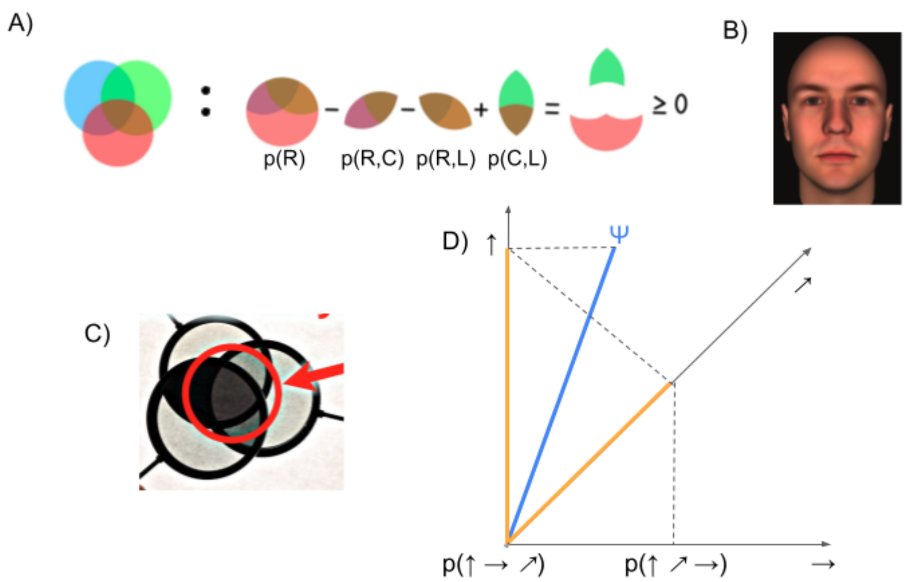

Figure 4. Classical probability predictions and their violations in perceptual and quantum phenomena.

A) Venn's diagram depiction of Boole's idea of possible experience. B) A face used in (Bruza et al., 2023), where the above relationship does not hold for three aspects of the face (dominance, trustworthiness, and intelligence). This implies that the act of reporting some of these properties dynamically interacts with other properties. C) Intuitive physical demonstration of the violation of the Venn diagram constraints using polarizers. See https://www.youtube.com/watch?v=zcqZHYo7ONs. The main idea is this: prepare 3 polarizers. By arranging two of them, you can completely block any light through them. That is, the probability of passing photons across two polarizers can be set to 0. Then, insert a third polarizer between the two. Depending on the angle of the third, the three filters can pass more photons, and thus the output beam would be brighter at the intersection of the three polarizers. D) An explanation of (C) with a quantum projection scheme. Assume the state can be influenced by measurement. After we project the initial state Ψ to the ↑ axis, further projection to the → gives 0 length, which corresponds to a perfect block of photons. However, if we project to the ↗ axis, after the ↑ one, then third projection to the → gives much a non-zero length, explaining why more photons pass through three filters than just the two original ones.

4.3 Dual-task interference and non-interference between qualia in terms of incompatible and compatible observables.

While QQ was not proposed to explain the relationship between consciousness and attention, this is one of the most debated topics in psychology, neuroscience and philosophy (Block, 2007; Bor & Seth, 2012; Bronfman et al., 2019; Cohen et al., 2012; Dehaene et al., 2006; Hardcastle, 1997; Iwasaki, 1993; Koch & Tsuchiya, 2007; V. A. Lamme, 2003; Maier & Tsuchiya, 2021; Mole, 2008; Pitts et al., 2018; Tallon-Baudry, 2011; van Boxtel et al., 2010a). The QQ appears quite consistent with the known empirical findings. Moreover, it may make further testable predictions about critical empirical research.

Traditionally, sensory inputs are considered to be filtered by attention first (Figure 5A), implying attention is necessary for consciousness. Information selected with attention is experienced as qualia and subsequently reported in a feedforward manner. Only some aspects of sensory input are attended, which ostensibly give rise to particular qualia. Behavioral reports reflect the experienced qualia. In this model, typically, attention is considered as a single limited resource and any task consumes some amount of attention.

This view goes against empirical findings concerning reports of sensory inputs outside of attention. Among many empirical findings, a particularly intriguing one is a pattern of the tasks that consume almost all attention and those that do not consume any attention, as shown in Figure 5B. These properties of task combinations have been documented over the years within the "dual task" research program (Braun & Julesz, 1998; Braun & Sagi, 1990; Bronfman et al., 2019; Fei-Fei et al., 2005; Matthews et al., 2018; Pastukhov et al., 2009; Reddy et al., 2004). For example, conscious experience of genders presented at the periphery do not differ with or without performing a difficult central letter task. Meanwhile, the experience of red/green bisected disks becomes totally unclear under a dual-task with the same central task (Matthews et al., 2018; Reddy et al., 2004, 2006). One possible explanation of this pattern is the existence of attention-free specialized modules in the cortex, possibly due to biological significance or extended training (VanRullen et al., 2004).

There are many alternatives to the traditional view of attention and consciousness. One view that is consistent with empirical findings, including with the dual-task (Figure 5B), considers consciousness and attention to operate independently (Figure 5C) (Koch & Tsuchiya, 2007; V. A. Lamme, 2004). In this scheme, unattended conscious and attended conscious processes are both possible. Attention and consciousness do not proceed in a feedforward manner. While this view is consistent with empirical findings, it does not explain how consciousness and attention interact dynamically.

The QQ hypothesis (Figure 2, Figure 5D) explicitly considers how qualia can be affected by attention through the instruments formalism. But this does not mean that all qualia are equally affected by attention, as demonstrated by the dual task. In fact, the QQ provides two novel explanations about why a given pair of tasks may not interfere with one another.

One explanation has to do with the existence of "commutative" qualia. While any process is expected to result in noncommutativity (See 3.4.1), in quantum theory, some observables, called "centers", are always commutative with any other observables. Such observables include mass, time and space. It is plausible that some types of qualia (e.g., extreme pain, bright light, loud sound, sense of space and time) may also be centers in this sense and so be commutative with (and may not be affected by attention for) other types of qualia. This is an empirical question for future research.

Another explanation relates to the idea of "incompatibility". In quantum theory, when the values of two or more observables cannot not be generally established together, these observables are called "incompatible". According to the QQ, pairs of qualia that cannot be established together may not be "incompatible" in this sense.

From the QQ perspective, it is important to point out that, in many dual tasks, a letter discrimination task was used as the primary difficult fixation task (Matthews et al., 2018; Tsuchiya & Koch, 2015). Thus, the conclusions from these studies may be revealing "incompatibility" of letter qualia and other types of qualia (Busemeyer & Bruza, 2012). In other words, qualia about face gender (Matthews et al., 2018) and the presence of animals in a natural scene (Li et al., 2002) may be just "compatible" with letter qualia. Counterintuitively, however, qualia of rotated letters or bisected colored disk may not be compatible with the letter tasks (Pastukhov et al., 2009). This is an intriguing conjecture requiring further investigation.

Reconsidering the patterns of attentional limits in terms of incompatibilities between observables might allow novel insights into the qualia-attention research. With attention assumed to be a fixed resource (Joseph et al., 1997), which can amplify aspects of qualia, it is hard to explain why in some visual illusions stronger attention leads to poorer visibility of the target (Schölvinck & Rees, 2009; van Boxtel et al., 2010b). Further, it is also hard to understand why distracting participants sometimes leads to better detection performance in attentional blink paradigms (Olivers & Nieuwenhuis, 2006). If attention is considered as a state, whose changes may sometimes decrease the similarity of incompatible observables, this might offer a more coherent explanation on these seemingly odd relationships between qualia and attention.

Unlike the limited resource model, "incompatibility" predicted from the QQ predicts an existence of pairs of compatible qualia, each of which consumes a significant amount of a presumed "resource". Similarly, it predicts pairs of incompatible qualia, each of which does not consume much attention, which cannot be simultaneously established. Discoveries of such pairs of tasks and stimuli would further support the QQ hypothesis.

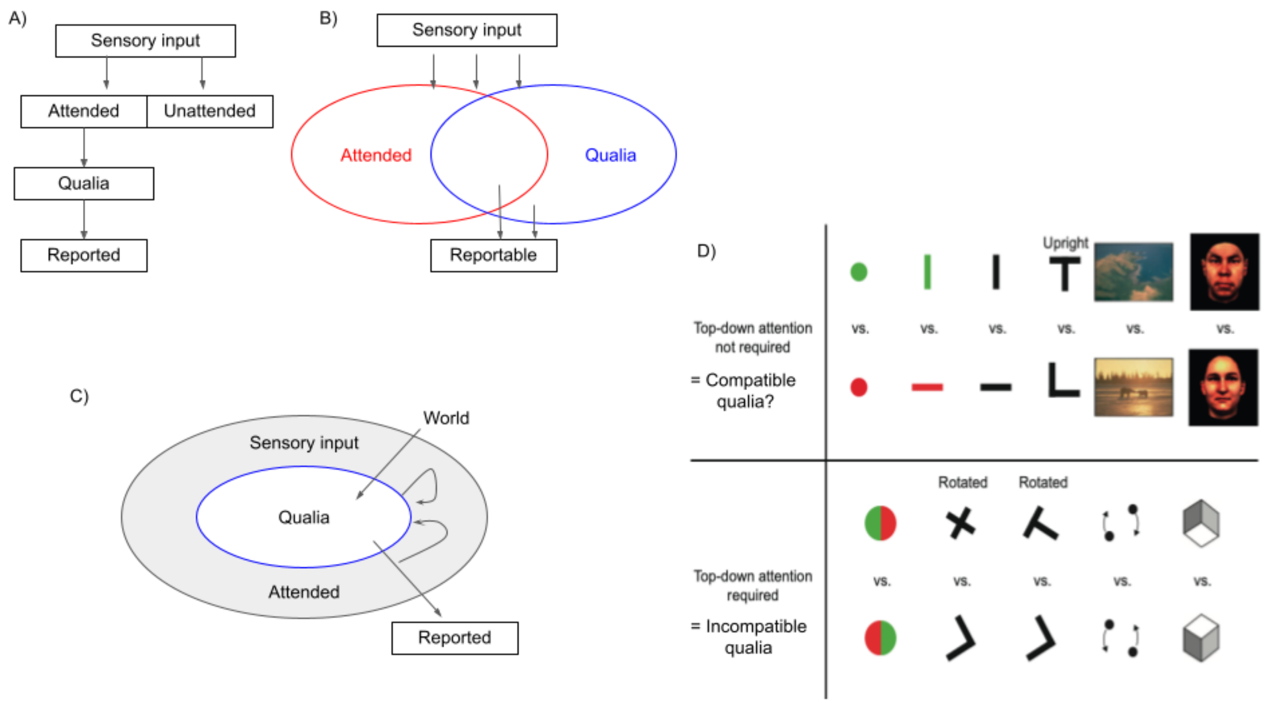

Figure 5. The QQ hypothesis is compatible with the empirical findings about the relationship between attention and consciousness.

A) Traditional feedforward models of sensory input, attention, qualia, and reports (taken from [Lamme, 2004)](). B) A static view of consciousness and attention that is consistent with dissociations between qualia and attention (Maier & Tsuchiya, 2021). C) Quantum qualia hypothesis (reproduced from Figure 2). D) Top row: a list of peripheral perceptual discriminations that can be conducted simultaneously with difficult letter discrimination tasks at the fixation. For example, conscious experience of genders presented at the periphery does not differ with or without performing a difficult central letter task (Matthews et al., 2018). Bottom row: a list of tasks that cannot be done concurrently with the letter task. One novel interpretation of such results is using the notion of incompatibility. Incompatibility is the inability to jointly establish the values of two or more observables. Modified from (Tsuchiya & Koch, 2015).

5. Conjecture: What are the potential neural mechanisms that support QQ?

The potential neural mechanisms for the QQ and quantum cognition can overlap. There have already been some proposals for quantum cognition (Acacio de Barros & Suppes, 2009; Busemeyer et al., 2017; A. Khrennikov et al., 2018; Stewart & Eliasmith, 2013). Here we discuss an interesting perspective to this debate, relating to recurrent neural network architectures, which links to the various proposals from the qualia/consciousness side (Boly et al., 2011; V. A. Lamme & Roelfsema, 2000; Malach, 2021; Seth & Bayne, 2022).

In a recurrent neural network (RNN), processing stimulus A leaves some trace in its subsequent network state. Thus, when RNNs are presented with A then B, they output differently from the case of B then A, especially when the delay between A and B is short. That is, outputs from RNNs are generally noncommutative; they exhibit order effects and can produce violations of the Bell inequality (Atmanspacher & Filk, 2006; Filk, 2016).

The issue of recurrency is interesting from the QQ viewpoint, raising at least 3 questions.

First, can QQ effects serve as a probe for conscious experience? Let us assume that QQ effects arise from a RNN-type structure in the brain. Most theories of consciousness agree that recurrent processes in the cerebral cortex are strongly correlated with qualia, while feedforward processes, such as those in the cerebellum and basal ganglia, are not (Albantakis et al., 2022; V. A. F. Lamme, 2015; Malach, 2021; Mashour et al., 2020; Seth & Bayne, 2022). If recurrency is considered as a proxy marker of consciousness at the neuronal level, can QQ effects serve as a behavioral marker? In turn, with respect to nonconscious processing, can invisible stimuli ever induce QQ effects? If nonconscious processes are supported by feedforward processing, they would not be expected to show any QQ effects.

Second, why have QQ effects not been reported widely so far? If recurrency is sufficient for QQ, should there not have been more reports, given how prevalent recurrency is in the brain? Indeed, relevant phenomena might have been noticed and recorded, but under different names. Sensory effects of adaptation/priming, spatio-temporal masking, context/surround effects and various illusions related to attention/expectation may be potentially all explained within the quantum framework for qualia.

This leads to the third question. Are there any neural mechanisms that reduce noncommutative effects in RNNs? There may be such mechanisms, for example, involving resetting RNNs to decay quickly to some baseline. Either explicitly or implicitly, experimenters may have invoked such mechanisms to reduce the QQ effects. Alternatively, active mechanisms may exist to minimize the effects of processing of previous stimuli. Large-amplitude oscillations and global transients would saturate the activity ranges of neurons and might serve this purpose. Cognitive processes, such as attention and memory, may be related to these mechanisms, so that they can select and protect information for a current stimulus from being influenced by prior stimuli. If these possibilities are true, can a demonstration of QQ effects be easier, through manipulations to reduce quantum-like effects?

While the theoretical link between recurrency/feedback and quantum-like effects makes sense, their relations are not yet clear, especially with a realistic large-scale network. Can we use a large-scale recurrent network to investigate quantum-like effects in behavior? Would embodied robots, that are internally controlled by recurrent networks (Park & Tani, 2015; Tani, 2017), demonstrate quantum-like effects?

6. Further future questions

In this last section, we further outline some of theoretical and empirical future research directions.

First, with respect to the theoretical concepts we introduced in this paper, we need to clarify what would count as a "state". In the main text, we considered mainly sensory inputs and internal attention. Should more global properties, including arousal, emotion, mood and physical states, be included? Let us say we consider such cognitive components as states (or parts of a state). Then, when we manipulate these states, can we render incompatible observables into compatible ones? Such possibilities can be tested using the dual tasks (Figure 6).

In fact, something close to this has been implied in the attentional blink phenomenon (Chun & Potter, 1995; Raymond et al., 1992). In attentional blink, people often do not see the second target after detecting the first target, when both are embedded in a rapid stream of distractors (e.g., one target per 100ms) and if the second target follows the first at around 200-300 ms. However, when participants engage in distracting, but positive, thoughts, the attentional blind disappears (Olivers & Nieuwenhuis, 2005, 2006). It is hard to reconcile this finding with the known models of attention and consciousness (Figure 6), but the QQ may be able to explain it, as a reduction of incompatibility between observables under a particular state.

Second, would babies and children, who presumably have less mature cortex, demonstrate stronger quantum cognition / QQ effects? We expect they would, especially if attention and working memory reduce noncommutative quantum effects and if these cognitive aspects take some time to develop. What about animals? Also, would these cognitive characteristics explain individual differences in quantum-like effects in psychophysical tests (Trueblood et al., 2017)? What are adaptive or evolutionary reasons for the origins of QQ? Is it the case that a quantum-like architecture for consciousness confers some kind of adaptive advantage, relative to one based on classical theory?

Finally, can the QQ hypothesis explain the sense of self and identity, which we associate with conscious experience? A different way to consider this question is whether quantum-like states can encompass the extent of information which endows us with a sense that our decisions, opinions, judgments etc. are informed by our knowledge and experience (Fodor, 1983).

7. Conclusion

We proposed a Quantum Qualia (QQ) hypothesis based on a quantum theoretical framework (e.g., noncommutative observables, states, and instruments; Figure 2, Table 1). QQ explains intuitive and known properties of qualia, such as indeterminacy, dynamics, and interaction with attention. Predictions from QQ can be empirically tested with demonstrations of asymmetry in perceptual similarity judgements, violations of the Bell inequality, and apparent incompatibilities between particular qualia. Amongst these, particularly powerful are demonstrations of Bell inequality violations. To test them, we need to measure three observables, two at a time across three different states (Figure 4). Such experiments have been rarely conducted systematically, due to the lack of theoretical background and motivation. Additionally, there are subtle loopholes which need to be considered, before a fairly definitive case that qualia are indefinite is made (Atmanspacher & Filk, 2019; Basieva et al., 2019; Emary, 2017). In physics, it took more than twenty years from the theoretical proposal by Bell through to the initial experiment by Clauser and then to the compelling demonstration by Aspect (Section 4.2.1). Will a similar pathway await the Quantum Qualia hypothesis in the future? Only time will tell.

QQ might show how seemingly unrelated phenomena can be brought together in a unified manner, as has been partially achieved in decision making and causal inference with quantum cognition (Mistry et al., 2018; Trueblood et al., 2017; Z. Wang et al., 2014). Such a research program has some advantages compared to ad hoc explanations of each phenomenon in isolation. A coherent body of findings would present a critical challenge for many extant theoretical proposals about consciousness (Seth & Bayne, 2022).

Regarding neurophysiology, it is interesting to note that recurrent computation in the brain is expected to be noncommutative and so might give rise to QQ effects at the behavioral level (Atmanspacher & Filk, 2006; Filk, 2016) (Section 5). It would be very surprising if qualia does not manifest any noncommutative properties. With increasing evidence that QQ provides a coherent explanation on the mathematical structure of qualia, QQ may well emerge as a promising framework to link qualia and the brain.

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