On the Gaussian Inverse Semantic Energy Well

$$y(x) = M \cdot v^2 \cdot \left(1 - e^{-\frac{f^2x^2}{v^2}}\right) \quad \text{when } x > 0 \quad (1)$$

$$\frac{d}{dx}\left(M \cdot v^2 \cdot \left(1 - e^{-\frac{f^2x^2}{v^2}}\right)\right) = 2f^2 e^{-\frac{f^2x^2}{v^2}} Mx \quad (2)$$

$$\frac{d^2}{dx^2}\left(M \cdot v^2 \cdot \left(1 - e^{-\frac{f^2x^2}{v^2}}\right)\right) = 2a^2 M\left(-\frac{2f^2 e^{-\frac{f^2x^2}{v^2}} x^2}{v^2} + e^{-\frac{f^2x^2}{v^2}}\right) \quad (3)$$

$$\frac{dy}{dx} = 2f^2 Mx(1-y) \quad (4)$$

$$\frac{d^2y}{dx^2} = \frac{2f^2}{v^2}(Mv^2 - y)\left(1 - 2\frac{f^2}{v^2}x^2\right) \quad (5)$$

Let $\varkappa = \frac{f}{v}$ (6)

Then (5) becomes:

$$\frac{d^2y}{dx^2} = 2\varkappa^2(Mv^2 - y)(1 - 2\varkappa^2x^2) \quad (7)$$

The RHS of (7) can be expanded as:

$$\frac{d^2y}{dx^2} = 2\varkappa^2 Mv^2(1 - 2\varkappa^2x^2) - 2\varkappa^2(1 - 2\varkappa^2x^2)y \quad (8)$$

which is finally rewritten as:

$$\frac{d^2y}{dx^2} + 2\varkappa^2(1 - 2\varkappa^2x^2)y = 2\varkappa^2(1 - 2\varkappa^2x^2)Mv^2 \quad (9)$$

Let is denote $K(x) = 2\varkappa^2(1 - 2\varkappa^2x^2)$ (10)

Then finally:

$$\frac{d^2y}{dx^2} + K(x)y(x) = K(x)Mv^2 \quad \text{when } x > 0 \quad (11)$$

Note that $\varkappa$ can be absorbed into x with

$$\xi = \varkappa x \quad (12)$$

$$\frac{1}{K(\xi)}\frac{d^2y}{d\xi^2} + y(\xi) = Mv^2 \qquad (13)$$

$$K(\xi) = 2(1 - 2\xi^2) \qquad (14)$$

With the constraint $\xi > 0$ (15)

The dimension of $\varkappa$ is the inverse of a semantic metric unit ($\mathbf{sme}^{-1}$). ξ and K are dimensionless quantities. y has the units of semantic energy ($\mathbf{smu} \times \frac{\mathbf{sme}^2}{\mathbf{stu}^2}$).

From (12) and (14) it follows that

$$\frac{d^2y}{dx^2} = 0 \textit{ iff } x = \frac{v}{2f} \qquad (16)$$

Interpertation of the ODE leading to Gaussian Semantic Energy Well

The semantic energy is in steady state away from the center of mass of the semantic particle