Signatures of Semantic Structures

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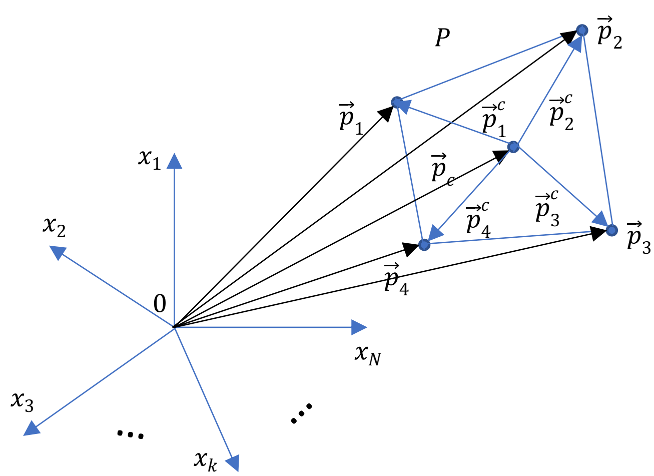
The position, the shape of a Semantic Structure and the volume it takes up in Semantic Space is ever evolving. The Signature of a Semantic Structure represents static view of the position in Semantic Space of that Semantic Structure at specific moment of time.

The Signature of Semantic Properties

Each property can be represented as a collection of $N = N(P)$ points ($N \geq 1$) in Semantic Space plus the position of its centroid. Thus, each property can be represented by the following $(N+1) \times L$ matrix:

$$\mathbf{P} = [\mathbf{p}_c \; \mathbf{p}_1 \; \mathbf{p}_2 \ldots \mathbf{p}_N]$$

Here the column vector which corresponds to the centroid $\vec{p}_c$ will be denoted with subscript c : $\mathbf{p}_c$. Each column vector $\mathbf{p}_i$, $i = 1, N$ represents a point in the L dimensional Semantic Space relative to its centroid $\mathbf{p}_c$. The property P can be visualized as an L-dimensional K-polytope. With $\vec{p}_i, i = 1..4$ are denoted the vertices of the polytope. With $\vec{p}_c$ we denote the center of mass (centroid) of the polytope which we will discuss in detail later.



We have stated that each vertex in the property polytope represents a semantic aspect of the property. The metric $|\vec{p}_i^{\,c}| = |\vec{p}_i - \vec{p}_c|$ represents the type (or kind) of the semantic aspect $\vec{p}_i$. The value of the semantic aspect is represented by the position of $\vec{p}_i$ with respect to the semantic axes $x_1, \ldots, x_N$. In other words the $K-1$-tuple $(\theta_1, \theta_2, \ldots, \theta_{K-1})$ where each $\theta_j, j = 1..K-1$ denotes the angle between $\vec{p}_i$ and the semantic axis x_j uniquely identifies a specific value for the semantic aspect $\vec{p}_i$. For details refer to [Modeling Attractive and Repulsive Forces in Semantic Properties](#).

The Energy Signature of a Semantic Property

With every property there is an associated energy vector which defines the current energy signature of the property P:

$$\vec{E}_P = \left[E_1, E_2, \cdots, E_{N(P)}\right]$$

The energy signature of the property describes how the energy is distributed among the property aspects.

The Signature of Property Association Link

For the structure shown below

$\quad\ \mathbf{w}_{1,2}$

$P_1 - A_{12} - P_2$

Let the dimension of the signature matrix for property P_1 is $K_1 \times N$ and the dimension of the signature matrix for property P_2 is $K_2 \times N$. With $\mathbf{w}$ it is denoted the semantic significance vector pertaining to the association but it is not relevant for the current discussion it will be omitted in the further results.

The signature of the property association A_{12} is

$\mathbf{A}_{12} = \left[\mathbf{p}_1^1 \mathbf{p}_2^1 \dots \mathbf{p}_k^1 | \mathbf{p}_1^2 \mathbf{p}_2^2 \dots \mathbf{p}_l^2\right]$

where $k < K_1$ and $l < K_2$. Here the points in semantic space $\mathbf{p}_1^1, \mathbf{p}_2^1, \dots, \mathbf{p}_k^1$ are a subset of the semantic aspects of property P_1 and the points $\mathbf{p}_1^2, \mathbf{p}_2^2, \dots, \mathbf{p}_l^2$ are a subset of the semantic aspects of property P_2. In a sense the property association acts as a filter which exposes only certain aspects of each property to generate binding force. In the expression for $\mathbf{A}_{12}$ above the symbol | denotes a column separator separating the points pertaining to the property P_1 and those pertaining to the property P_2. The last expression can be written concisely in block matrix notation as

$\mathbf{A}_{12} = [\mathbf{P}^1 | \mathbf{P}^2]$

The Signature of Primitive Semantic Particles

Below are shown two examples of primitive semantic particles with their property trees:

V_1

P_1 P_2 P_3
\ | / ..
P_k --- o --- P_i
.. / | \ ..
P_{j+1} P_j P_{j-1}

V_2

P_1--- o --- P_2
/ | \
P_3 P_4 P_5_
/ \ | \
P_6 P_7 P_8 P_9
|
P_{10}

The Signature of Compound Semantic Particles

Let us have the compound particle V_{comp} represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:

$stree(V_{comp}) =$

```
          __V1__
         /   |   \
       V2    V5   _V6_
      /  \        / | \
    V3    V4    V7  V8  V9
```

The property tree for each V-particle $V_k, k = 1..9$ are given with the algebraic notation discussed in [Semantic Tree Operations](#).
$ptree(V_k) \;=\; \sum_{\boldsymbol{k}\in\mathfrak{T}(V_k), i\in\mathbb{p}(V_k)}(\boldsymbol{k}, P_i)$. Here $\boldsymbol{k}$ denotes the path $\left(k_{l_1}, k_{l_2}, \dots, k_{l_h}\right)$ constructed by branching consecutively along the k_{l_1}-*th* branch from the top level, then the k_{l_2}-th branch from the lower level and finally k_{l_h}-th branch from the h-th level. The set $\mathfrak{T}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of V_k. The set $\mathbb{p}(V_k)$ denotes the indices of the vertices in the property tree of V_k.

```
     V1                               V2 `                 ...       V9

   P1 P2  P3                      P1--- o --- P2                 P1--- o --- P2
     \ | /  ..                         / | \
Pk --- o --- Pi                      P3   P4  P5_
  ..  / | \  ..                      / \       |   \
   Pj+1 Pj  Pj-1                    P6   P7     P8   P9
                                    |
                                    P10
```

Expressing the property tree of V_1 with the algebraic notation discussed earlier:
$ptree(V_1) \;=\; \sum_{k_j\in\mathfrak{T}(V_1), i\in\mathbb{p}(V_1)}\left(k_j, P_j\right)$
Similarly, $ptree(V_2)$ is given with

$$ptree(V_2) = (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3k_1, P_6) + (k_3k_2, P_7) + (k_5k_1, P_8) + (k_5k_2, P_9) + (k_3k_1k_1, P_{10})$$

Here P_0 is $text(V_2)$.
Now if we expand the property trees for each V-particle in the semantic tree for the composite particle V_{comp} we will have a larger augmented property tree. This augmented property tree represents the semantic structure of V_{comp} and can be recorded in a matrix form which is the semantic signature of V_{comp}. The semantic signature matrix of V_{comp} will have the following structure in block matrix notation:

$ssig(V_{comp}) = \left[\mathbf{P}_0 \mid \mathbf{A}_{0,1} \mid \mathbf{P}_1 \mid \mathbf{P}_0 \mid \mathbf{A}_{0,2} \mid \mathbf{P}_2 \mid \mathbf{P}_0 \mid \mathbf{A}_{0,3} \mid \mathbf{P}_3 \mid \;\dots\; \mid \mathbf{P}_p \mid \; \mathbf{A}_{p,q} \mid \mathbf{P}_q\right]$
Here the matrices $\mathbf{P}_i$ represent the signatures of the properties P_i. The size of $\mathbf{P}_i$ is $K_i \times N$.
Here the symbol | denotes a separator column.

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [\mathbf{B}_1|\mathbf{B}_2|\mathbf{B}_3| \ \ \dots \ \ |\mathbf{B}_q]$$

$$\mathbf{B}_1 = [\mathbf{P}_0 \mid \mathbf{A}_{0,1} | \mathbf{P}_1] \text{ , } \mathbf{B}_2 = [\mathbf{P}_0 \mid \mathbf{A}_{0,2} \mid \mathbf{P}_2] \text{ , } \mathbf{B}_3 = [\mathbf{P}_0 \mid \mathbf{A}_{0,3} \mid \mathbf{P}_3] \text{ , } \dots \text{ , } \mathbf{B}_q = [\mathbf{P}_p \mid \mathbf{A}_{p,q} | \mathbf{P}_q]$$

Here the block matrix $\mathbf{B}_1$fully describes the property P_1 including how it is connected to the property tree $ptree(V_1)$. Similarly, $\mathbf{B}_2$ and $\mathbf{B}_3$ fully describes the properties P_2 and P_3 and their connectivity to $ptree(V_1)$. Finally, $\mathbf{B}_q$ fully describes the property P_q and its connectivity to $ptree(V_9)$. From now on we will denote the block matrices $\mathbf{B}_i$ as *semantic elements* of V_{comp}.

Statement: Every semantic particle, primitive or composite, can be represented as a sequence of *semantic elements*.

Definition: Semantic distance between two semantic elements B_1 and B_2
The semantic element B_1 represents two properties - P_i and P_j connected through association link $A_{i,j}$. The properties P_i and P_j are represented by their property signature matrices $\mathbf{P}_i$ and $\mathbf{P}_j$. The association link $A_{i,j}$ is represented with its association matrix $\mathbf{A}_{i,j}$ and semantic significance vector $\mathbf{w}_{i,j}$. (*Note*: Sometimes for clarity all vectors in a block matrix representing semantic element will be denoted with the vector symbol $\overrightarrow{}$ when clear distinction needs to be made). For details refer to the document Note On Binding of Association Property to Semantic Properties. Similarly, the semantic element B_2 represents the properties P_k and P_l connected through association link $A_{k,l}$. As before the properties P_k and P_l are represented by their property signatures $\mathbf{P}_k$ and $\mathbf{P}_l$. The association link $A_{k,l}$ is represented with its association matrix $\mathbf{A}_{k,l}$ and semantic significance vector $\mathbf{w}_{k,l}$.
Let $\mathbf{B}_1$ denotes the matrix of the first semantic element B_1 given with $\mathbf{B}_1 = [\mathbf{P}_i \ \ \mathbf{A}_{i,j} \ \ \mathbf{P}_j]$
Let $\mathbf{B}_2$ denotes the matrix of the second semantic element B_2 given with $\mathbf{B}_2 = [\mathbf{P}_k \ \ \mathbf{A}_{k,l} \ \mathbf{P}_l]$
Then the semantic distance between the two is given with:

$$sdist(B_1, B_2) = sdist(P_i, P_k) + sdist(A_{i,j}, A_{k,l}) + sdist(P_j, P_l)$$

where

$$sdist(P_i, P_k) = |\mathbf{p}_i - \mathbf{p}_k|, \ \ sdist(P_j, P_l) = |\mathbf{p}_j - \mathbf{p}_l|$$
$$sdist(A_{i,j}, A_{k,l}) = |\mathbf{w}_{i,j} - \mathbf{w}_{k,l}| \times sdist(\mathbf{a}_{i,j}, \mathbf{a}_{k,l})$$

Definition: The semantic distance of two semantic matrices $\mathbf{a}$ and $\mathbf{b}$ which have the same number of columns is given with:
$sdist(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^{n} |\vec{\mathbf{a}}_i - \vec{\mathbf{b}}_i|$ where $\mathbf{a} = [\vec{\mathbf{a}}_1 \ \vec{\mathbf{a}}_2 \dots \vec{\mathbf{a}}_n]$ and $\mathbf{B} = [\vec{\mathbf{b}}_1 \ \vec{\mathbf{b}}_2 \dots \vec{\mathbf{b}}_n]$.

In the block matrix for $ssig(V_{comp})$ $\mathbf{p}_0$ denotes the signature column vector of the property P_0 , $\mathbf{a}_{0,1}$denotes the association matrix of the arc between property P_0 and property P_1, $\mathbf{a}_{p,q}$ denotes the association matrix of the arc between property P_p and P_q. Let us denote the number of rows of $ssig(V_{comp})$ by N and the number of columns by M.

The semantic signature matrix $ssig(V_{comp})$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig(V_{comp})$ and connectivity signature matrix $csig(V_{comp})$:

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [\mathbf{p}_0 \;\; 0 \;\; \mathbf{p}_1 \;\; \mathbf{p}_0 \;\; 0 \;\; \mathbf{p}_2 \;\; \mathbf{p}_0 \;\; 0 \;\; \mathbf{p}_3 \;\; \dots \;\; \mathbf{p}_p \;\; 0 \;\; \mathbf{p}_q]$$

$$csig(V_{comp}) = [0 \;\; \mathbf{a}_{0,1} \;\; 0 \;\; 0 \;\; \mathbf{a}_{0,2} \;\; 0 \;\; 0 \;\; \mathbf{a}_{0,3} \;\; 0 \;\; \dots \;\; 0 \;\; \mathbf{a}_{p,q} \;\; 0]$$

Let us denote by $psig(P_1, V_{comp})$ the augmented semantic property signature of property P_1 with respect to V_{comp}. It is given with:

$$psig(P_0, V_{comp}) = [\mathbf{p}_0 \;\; 0 \;\; 0 \;\; \mathbf{p}_0 \;\; 0 \;\; 0 \;\; \mathbf{p}_0 \;\; 0 \;\; 0 \;\; \dots \;\; 0 \;\; 0 \;\; 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0 \;\; 0 \;\; \mathbf{p}_1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \dots \;\; 0 \;\; 0 \;\; 0]$$

$$psig(P_q, V_{comp}) = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \dots \;\; 0 \;\; 0 \;\; \mathbf{p}_q]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$$

Here $\mathbb{S}(V_{comp})$ denotes the set of the indices of all semantic particles which the composite V_{comp} is composed from.

Another way to partition the signature matrix into block matrices is:

$$ssig(V_{comp}) = [\mathbf{V}_1 \; \mathbf{A}_{1,2} \; \mathbf{V}_2 \; \mathbf{A}_{1,3} \; \mathbf{V}_3 \; \dots \; \mathbf{A}_{6,8} \; \mathbf{V}_8 \; \mathbf{A}_{6,9} \; \mathbf{V}_9]$$

The block matrix $\mathbf{V}_1$ represents the property tree of the particle V_1 and it is given by:

$$\mathbf{V}_1 = [\mathbf{p}_0 \;\; \mathbf{a}_{0,1} \;\; \mathbf{p}_1 \;\; \mathbf{p}_0 \;\; \mathbf{a}_{0,2} \;\; \mathbf{p}_2 \;\; \mathbf{p}_0 \;\; \mathbf{a}_{0,3} \;\; \mathbf{p}_3 \;\; \dots \;\; \mathbf{p}_0 \;\; \mathbf{a}_{0,k} \;\; \mathbf{p}_k]$$

The block matrix $\mathbf{A}_{1,2}$ describes the connection between the particles V_1 and V_2 connecting the root property $\mathbf{p}_0$ of V_1 and the root property $\mathbf{p}_{k+1}$ of V_2. It is given with:

$$\mathbf{A}_{1,2} = [\mathbf{p}_0 \;\; \mathbf{a}_{0,k+1} \; \mathbf{p}_{k+1}]$$ //TODO: expand it – the matrix structure is more complicated!

Properties of the signature matrix

Here are some interesting properties of $ssig(V_{comp})$:

The number of rows N in $ssig(V_{comp})$ is 3 × the number of arcs in the augmented property tree of V_{comp}.

The rank of

TO DO: finish the property section

Asymptotic closeness of semantic structures

Let us have two semantic structures S1 and S2.

$$ssig(S_1) = \left[\mathbf{V}_{k_1}\ \mathbf{A}_{k_1,k_2}\ \mathbf{V}_{k_2}\ \mathbf{A}_{k_1,k_3}\ \mathbf{V}_{k_3}\ \ldots\ \mathbf{A}_{k_p,k_q}\ \mathbf{V}_{k_q}\right]$$

$$ssig(S_2) = \left[\mathbf{V}_{l_1}\ \mathbf{A}_{l_1,l_2}\ \mathbf{V}_{l_2}\ \mathbf{A}_{l_1,l_3}\ \mathbf{V}_{l_3}\ \ldots\ \mathbf{A}_{l_r,l_s}\ \mathbf{V}_{l_s}\right]$$

Uniform asymptotic closeness

K-level uniform asymptotic closeness