Note on binding of match-seeking and match-repelling particles
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Binding between two primitive particles through match-seeking particle

Let us consider two V-particles which are not composite – they are given with their semantic signatures respectively:

$$ssig(V') = \begin{bmatrix} \mathbf{p}'_0 & \mathbf{a}'_{0,1} & \mathbf{p}'_1 & \mathbf{p}'_0 & \mathbf{a}'_{0,2} & \mathbf{p}'_2 & \mathbf{p}'_0 & \mathbf{a}'_{0,3} & \mathbf{p}'_3 & \cdots & \mathbf{p}'_i & \mathbf{a}'_{i,n} & \mathbf{p}'_n \end{bmatrix}$$

$$ssig(V'') = \begin{bmatrix} \mathbf{p}''_0 & \mathbf{a}''_{0,1} & \mathbf{p}''_1 & \mathbf{p}''_0 & \mathbf{a}''_{0,2} & \mathbf{p}''_2 & \mathbf{p}''_0 & \mathbf{a}''_{0,3} & \mathbf{p}''_3 & \cdots & \mathbf{p}''_j & \mathbf{a}''_{j,m} & \mathbf{p}''_m \end{bmatrix}$$

Here each of the quantities $\mathbf{p}$ denotes the property signature vector of the corresponding property P of the V particle. The matrix $\mathbf{a}_{r,s}$ represents the property association particle $A_{r,s}$ which binds to a pair of properties P_r and P_s in the property graph $\mathcal{P}$ of the V particle. Also there is a semantic significance vector $\mathbf{w}_{r,s}$ which is associated the property association particle (a.k.a link) $A_{r,s}$. For details refer to The Signature of Semantic Structures.

Match-seeking particle MA binds to a subgraph $\mathcal{S}$ of the property graph $\mathcal{P}$ of the V particle. There is a closeness condition which needs to be obeyed in order the particle MA to bind to the particle V.

Binding matrix of a match-seeking particle

The match-seeking particle MA exposes a binding matrix $mbind(MA)$:

$$mbind(MA) = [\mathbf{B}^1 \ \mathbf{B}^2 \ \mathbf{B}^3 \ \cdots \ \mathbf{B}^q]$$

$$\mathbf{B}^1 = [\mathbf{p}^0 \ \mathbf{a}^{0,1} \ \mathbf{p}^1], \mathbf{B}^2 = [\mathbf{p}^0 \ \mathbf{a}^{0,2} \ \mathbf{p}^2], \mathbf{B}^3 = [\mathbf{p}^0 \ \mathbf{a}^{0,3} \ \mathbf{p}^3], \ldots, \mathbf{B}^q = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q]$$

Obviously, each of the blocks $\mathbf{B}^i$ is $N \times 4$ matrix where N is the dimension of semantic space. From now on we will denote these blocks of any match-seeking particle as *binding elements* B^i of the match-seeking particle M. Each binding element B^i of a match-seeking particle consists of a couple of property particles P^a and P^b connected with association particle $A^{a,b}$. Each binding element B^i is represented by its binding matrix $\mathbf{B}^i$ and its semantic significance vector $\mathbf{w}^i$.

Note that in each of those blocks having the general form $\mathbf{B}^i = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q]$ it is possible to have $\mathbf{a}^{p,q} = \mathbf{p}^q = \mathbf{0}$ where $\mathbf{0}$ represents the null vector in semantic space. However, $\mathbf{p}^p$ is never close to the null vector i.e. $|\mathbf{p}^p| > \mathbf{0}$.

Binding of match-seeking particle against V-particle formulated as optimization problem

Let a primitive particle V has the following semantic signature:
$$ssig(V) = [\mathbf{B}_1 \mathbf{B}_2 \ldots \mathbf{B}_m]$$

Let us denote by f^i_j the semantic distance between the binding element B^i of MA and the semantic element B_j of V

$$f^i_j = sdist(B^i, B_j), \mathbf{B}^i = [\mathbf{p}^p \ \mathbf{a}^{p,q} \ \mathbf{p}^q], \mathbf{B}_j = [\mathbf{p}_r \ \mathbf{a}_{r,s} \ \mathbf{p}_s]$$

Then we define the following metric:

$$sdist(B^i, B_j) = |\mathbf{p}^p|sdist(P^p, P_r) + |\mathbf{p}^p||\mathbf{p}^q|sdist(A^{p,q}, A_{k,l}) + |\mathbf{p}^q|sdist(P^q, P_s)$$

where

$$sdist(P^p, P_r) = |\mathbf{p}^p - \mathbf{p}_r|, \quad sdist(P^q, P_s) = |\mathbf{p}^q - \mathbf{p}_s|$$
$$sdist(A^{p,q}, A_{r,s}) = |\mathbf{w}^{p,q} - \mathbf{w}_{r,s}| \times sdist(\mathbf{a}^{p,q}, \mathbf{a}_{r,s})$$

The semantic distance of two matrices **a** and **b**, having the same number of columns, is given with:

$sdist(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^{n}|\vec{\mathbf{a}}_i - \vec{\mathbf{b}}_i|$ where $\mathbf{a} = [\vec{\mathbf{a}}_1 \, \vec{\mathbf{a}}_2 \, ... \, \vec{\mathbf{a}}_n]$ and $\mathbf{b} = [\vec{\mathbf{b}}_1 \, \vec{\mathbf{b}}_2 \, ... \, \vec{\mathbf{b}}_n]$.

Notice that if B^i is incomplete that is – contains only a single property not connected to anything then $sdist(B^i, B_j)$ becomes simply the semantic distance between its sole property particle of the binding element B^i and the corresponding property of the semantic element B_j.

//TODO: finish the opmization problem formulation

Closeness condition for a bind between match seeking particle and primitive semantic particle

Let us denote by $sfil(MA, V)$ the following diagonal matrix which will be named *Filter matrix* of the match seeking particle:

$$sfil(MA, V) = \begin{bmatrix} I_1 & & & & & & \\ & 0 & & & & & \\ & & I_2 & & & & \\ & & & 0 & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & I_k \end{bmatrix}$$
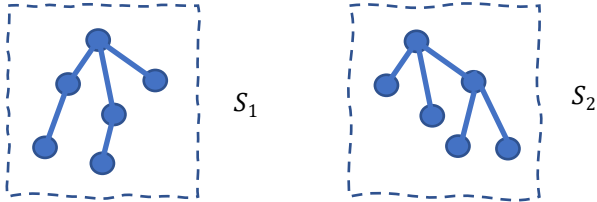
Here $I_i, i = 1, 2, ..., k$ are identity matrices which represent the regions of interest in the semantic signature matrix of V to the match seeking particle MA.

The regions of interest $sreg(MA, V)$ in the semantic signature of V are obtained by multiplying $sfil(MA, V)$ with $ssig(V)$:

$$sreg(MA, V) = sfil(MA, V) \times ssig(V)$$

Between two semantic structures

Let us have two semantic structures S_1 and S_2.

Let the semantic signature of S_1 is given with:

$$ssig(S_1) = \begin{bmatrix} \mathbf{V}_1 \, \mathbf{A}_{1,2} \, \mathbf{V}_2 \, \mathbf{A}_{1,3} \, \mathbf{V}_3 \, ... \, \mathbf{A}_{r,p} \, \mathbf{V}_p \end{bmatrix}$$

and the semantic signature of S_2 is given with:

Between a primitive V particle and a semantic structure S