The Signature of Semantic Structures

Let us have the compound particle V_{comp} represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:
$V_{comp} \;=\; [V_1\, A_1\, V_2\, A_2\; \dots\; V_9\, A_9]$
$stree(V_{comp}) =$

```
              __V5__
             /   |   \
           V2    V4   _V8_
          /  \        / | \
        V1   V3     V6  V7  V9
```

The property tree for each V-particle $V_k, k = 1..9$ are given with the algebraic notation discussed in Semantic Tree Operations.
$ptree(V_k) \;=\; \sum_{\boldsymbol{k}\in\mathfrak{I}(V_k), i\in\mathbb{p}(V_k)}(\boldsymbol{k}, P_i)$. Here $\boldsymbol{k}$ denotes the path $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$ constructed by branching consecutively along the k_{l_1}-*th* branch from the top level, then the k_{l_2}-th branch from the lower level and finally k_{l_h}-th branch from the h-th level. The set $\mathfrak{I}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of V_k. The set $\mathbb{p}(V_k)$ denotes the indices of the vertices in the property tree of V_k.

```
       V1                                   V2 `

   P1 P2 P3                           P1--- o --- P2
     \ | /  ..                             / | \
Pk --- o --- Pi                          P3  P4  P5_
  ..  / | \  ..                          / \       |   \
  Pj+1 Pj  Pj-1                        P6  P7     P8   P9
                                       |
                                       P10
```

Expressing the property tree of V_1 with the algebraic notation discussed earlier:
$ptree(V_1) \;=\; \sum_{k_j\in\mathfrak{I}(V_1), i\in\mathbb{p}(V_k)}(k_j, P_j)$
Similarly, $ptree(V_2)$ is given with

$$ptree(V_2) = (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3k_1, P_6) + (k_3k_2, P_7) + (k_5k_1, P_8) + (k_5k_2, P_9) + (k_3k_1k_1, P_{10})$$

Here P_0 is $text(V_2)$.
Now if we expand the property trees for each V-particle in the semantic tree for the composite particle V_{comp} we will have a larger augmented property tree. This augmented property tree represents the semantic structure of V_{comp} and can be recorded in a matrix form which is the semantic signature of V_{comp}. The semantic signature matrix of V_{comp} will have the following structure:

$$ssig\left(V_{comp}\right) = \begin{Bmatrix} P_0 \\ A_{0,1} \\ P_1 \\ P_0 \\ A_{0,2} \\ P_2 \\ P_0 \\ A_{0,3} \\ P_3 \\ \vdots \\ P_p \\ A_{p,q} \\ P_q \end{Bmatrix}$$

. Here $A_{0,1}$denotes the arc between property P_0 and property P_1. $A_{p,q}$denotes the arc between property $P_p$and P_q. Let us denote the number of rows of $ssig\left(V_{comp}\right)$ by N and number of columns by M.

The semantic signature matrix $ssig\left(V_{comp}\right)$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig\left(V_{comp}\right)$ and connectivity signature matrix $csig\left(V_{comp}\right)$:

$$ssig\left(V_{comp}\right) = psig\left(V_{comp}\right) + csig\left(V_{comp}\right)$$

$$psig\left(V_{comp}\right) = \begin{Bmatrix} P_0 \\ 0 \\ P_1 \\ P_0 \\ 0 \\ P_2 \\ P_0 \\ 0 \\ P_3 \\ \vdots \\ P_p \\ 0 \\ P_q \end{Bmatrix},\ csig\left(V_{comp}\right) = \begin{Bmatrix} 0 \\ A_{0,1} \\ 0 \\ 0 \\ A_{0,2} \\ 0 \\ 0 \\ A_{0,3} \\ 0 \\ \vdots \\ 0 \\ A_{p,q} \\ 0 \end{Bmatrix}$$

Here are some interesting properties of $ssig\left(V_{comp}\right)$:

The number of rows N in $ssig\left(V_{comp}\right)$ is 3 $\times$ the number of arcs in the augmented property tree of V_{comp}.

The rank of