

The Signature of Semantic Structures

Let us have the compound particle V_{comp} represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:

$$stree(V_{comp}) =$$

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      V_1
     / | \
    V_2 V_5 V_6
   / \ / | \
  V_3 V_4 V_7 V_8 V_9
  
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The property tree for each V -particle $V_k, k = 1..9$ are given with the algebraic notation discussed in [Semantic Tree Operations](#).

$ptree(V_k) = \sum_{k \in \mathfrak{T}(V_k), i \in \mathbb{P}(V_k)} (\mathbf{k}, P_i)$. Here \mathbf{k} denotes the path $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$ constructed by branching consecutively along the k_{l_1} -th branch from the top level, then the k_{l_2} -th branch from the lower level and finally k_{l_h} -th branch from the h -th level. The set $\mathfrak{T}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of V_k . The set $\mathbb{P}(V_k)$ denotes the indices of the vertices in the property tree of V_k .

V_1	V_2	...	V_9
$ \begin{array}{c} P_1 \ P_2 \ P_3 \\ \backslash \ \ / \ \dots \\ P_k \ \dots \ o \ \dots \ P_i \\ \dots \ / \ \ \backslash \ \dots \\ P_{j+1} \ P_j \ P_{j-1} \end{array} $	$ \begin{array}{c} P_1 \ \dots \ o \ \dots \ P_2 \\ \backslash \ \ / \\ P_3 \ P_4 \ P_5 \ \dots \\ \backslash \ / \ \backslash \ \ \backslash \\ P_6 \ P_7 \ P_8 \ P_9 \\ \\ P_{10} \end{array} $		$ \begin{array}{c} P_1 \ \dots \ o \ \dots \ P_2 \end{array} $

Expressing the property tree of V_1 with the algebraic notation discussed earlier:

$$ptree(V_1) = \sum_{k_j \in \mathfrak{T}(V_1), i \in \mathbb{P}(V_1)} (k_j, P_j)$$

Similarly, $ptree(V_2)$ is given with

$$\begin{aligned}
 ptree(V_2) = & (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3 k_1, P_6) + (k_3 k_2, P_7) \\
 & + (k_5 k_1, P_8) + (k_5 k_2, P_9) + (k_3 k_1 k_1, P_{10})
 \end{aligned}$$

Here P_0 is $text(V_2)$.

Now if we expand the property trees for each V -particle in the semantic tree for the composite particle V_{comp} we will have a larger augmented property tree. This augmented property tree represents the semantic structure of V_{comp} and can be recorded in a matrix form which is the semantic signature of V_{comp} . The semantic signature matrix of V_{comp} will have the following structure:

$$ssig(V_{comp}) = [p_0 \ a_{0,1} \ p_1 \ p_0 \ a_{0,2} \ p_2 \ p_0 \ a_{0,3} \ p_3 \ \dots \ p_p \ a_{p,q} \ p_q]$$

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [B_1 \ B_2 \ B_3 \ \dots \ B_q]$$

$$B_1 = [p_0 \ a_{0,1} \ p_1], B_2 = [p_0 \ a_{0,2} \ p_2], B_3 = [p_0 \ a_{0,2} \ p_3], \dots, B_q = [p_0 \ a_{0,2} \ p_q]$$

Here the block matrix B_1 fully describes the property P_1 including how it is connected to the property tree $ptree(V_1)$. Similarly, B_2 and B_3 fully describes the properties P_2 and P_3 and their connectivity to $ptree(V_1)$. Finally, B_q fully describes the property P_q and its connectivity to $ptree(V_9)$.

In the block matrix for $ssig(V_{comp})$ p_0 denotes the signature column vector of the property P_0 , $a_{0,1}$ denotes the signature column vector of the arc between property P_0 and property P_1 , $a_{p,q}$ denotes the signature column vector of the arc between property P_p and P_q . Let us denote the number of rows of $ssig(V_{comp})$ by N and the number of columns by M .

The semantic signature matrix $ssig(V_{comp})$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig(V_{comp})$ and connectivity signature matrix $csig(V_{comp})$:

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [p_0 \ 0 \ p_1 \ p_0 \ 0 \ p_2 \ p_0 \ 0 \ p_3 \ \dots \ p_p \ 0 \ p_q]$$

$$csig(V_{comp}) = [0 \ a_{0,1} \ 0 \ 0 \ a_{0,2} \ 0 \ 0 \ a_{0,3} \ 0 \ \dots \ 0 \ a_{p,q} \ 0]$$

Let us denote by $psig(P_1, V_{comp})$ the augmented semantic property signature of property P_1 with respect to V_{comp} . It is given with:

$$psig(P_0, V_{comp}) = [p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0 \ 0 \ p_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

$$psig(P_q, V_{comp}) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ p_q]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$$

Here $\mathbb{S}(V_{comp})$ denotes the set of the indices of all semantic particles which the composite V_{comp} is composed from.

Another way to partition the signature matrix into block matrices is:

$$ssig(V_{comp}) = [V_1 A_{1,2} V_2 A_{1,3} V_3 \dots A_{6,8} V_8 A_{6,9} V_9]$$

The block matrix V_1 represents the property tree of the particle V_1 and it is given by:

$$V_1 = [p_0 \ a_{0,1} \ p_1 \ p_0 \ a_{0,2} \ p_2 \ p_0 \ a_{0,3} \ p_3 \ \dots \ p_0 \ a_{0,k} \ p_k]$$

The block matrix $A_{1,2}$ describes the connection between the particles V_1 and V_2 connecting the root property p_0 of V_1 and the root property p_{k+1} of V_2 . It is given with:

$$A_{1,2} = [p_0 \ a_{0,k+1} \ p_{k+1}]$$

Properties of the signature matrix

Here are some interesting properties of $ssig(V_{comp})$:

The number of rows N in $ssig(V_{comp})$ is $3 \times$ the number of arcs in the augmented property tree of V_{comp} .

The rank of

TO DO: finish the property section

Asymptotic closeness of semantic structures

Let us have two semantic structures S_1 and S_2 .

$$ssig(S_1) = [V_{k_1} A_{k_1,k_2} V_{k_2} A_{k_1,k_3} V_{k_3} \dots A_{k_p,k_q} V_{k_q}]$$

$$ssig(S_2) = [V_{l_1} A_{l_1,l_2} V_{l_2} A_{l_1,l_3} V_{l_3} \dots A_{l_r,l_s} V_{l_s}]$$

Uniform asymptotic closeness

k-level uniform asymptotic closeness