On the Gaussian Inverse Semantic Energy Well

$y(x) = M_e \cdot v^2 \cdot \left(1 - e^{-\frac{f^2 x^2}{v^2}}\right)$ when $x > 0$ (1)

$\frac{d}{dx}\left(M_e \cdot v^2 \cdot \left(1 - e^{-\frac{f^2 x^2}{v^2}}\right)\right) = 2f^2 e^{-\frac{f^2 x^2}{v^2}} M_e x$ (2)

$\frac{d^2}{dx^2}\left(M_e \cdot v^2 \cdot \left(1 - e^{-\frac{f^2 x^2}{v^2}}\right)\right) = 2a^2 M_e \left(-\frac{2f^2 e^{-\frac{f^2 x^2}{v^2}} x^2}{v^2} + e^{-\frac{f^2 x^2}{v^2}}\right)$ (3)

$\frac{dy}{dx} = 2f^2 M_e x(1-y)$ (4)

$\frac{d^2 y}{dx^2} = \frac{2f^2}{v^2}(M_e v^2 - y)\left(1 - 2\frac{f^2}{v^2}x^2\right)$ (5)

Let $\varkappa = \frac{f}{v}$ (6)

Then (5) becomes:

$\frac{d^2 y}{dx^2} = 2\varkappa^2(M_e v^2 - y)(1 - 2\varkappa^2 x^2)$ (7)

The RHS of (7) can be expanded as:

$\frac{d^2 y}{dx^2} = 2\varkappa^2 M_e v^2(1 - 2\varkappa^2 x^2) - 2\varkappa^2(1 - 2\varkappa^2 x^2)y$ (8)

which is finally rewritten as:

$\frac{d^2 y}{dx^2} + 2\varkappa^2(1 - 2\varkappa^2 x^2)y = 2\varkappa^2(1 - 2\varkappa^2 x^2)M_e v^2$ (9)

Let is denote $K(x) = 2\varkappa^2(1 - 2\varkappa^2 x^2)$ (10)

Then finally:

$\frac{d^2 y}{dx^2} + K(x)y(x) = K(x)M_e v^2$ (11)

Note that $\varkappa$ can be absorbed into x with

$\xi = \varkappa x$ (12)

And thus we come up with the following ODE

$$\frac{1}{2(1-2\xi^2)}\frac{d^2y}{d\xi^2} + y(\xi) = M_e v^2 \qquad (13)$$

with the following boundary conditions

$$\lim_{\xi\to\infty} y(\xi) = M_e v^2 \qquad (14)$$

$$\lim_{\xi\to 0} \frac{dy}{d\xi} = 0 \qquad (15)$$

(13) subject to (14) and (15) gives rise to the *Gaussian Inverse Semantic Energy Well* curve (1) which we will use to compute the bound distance from the semantic mass center of the ensemble for every property in the ensemble.
The dimension of $\varkappa$ is the inverse of a semantic metric unit ($\mathbf{sme}^{-1}$). ξ and K are dimensionless quantities. y has the units of semantic energy ($\mathbf{smu} \times \frac{\mathbf{sme}^2}{\mathbf{stu}^2}$).

From (12) and (13) it follows that

$$\frac{d^2y}{dx^2} = 0 \textit{ iff } x = \frac{v}{\sqrt{2}f} \qquad (16)$$

Thus, the energy curve has inflection point at $x = \frac{v}{\sqrt{2}f}$.

Interpretation of the ODE leading to Gaussian Inverse Semantic Energy Well

Recall that we have the following relation between the net semantic energy E_t of the ensemble, the semantic mass M of the property P and the semantic velocity $v_{p,t}$ of the property P traveling towards its bound state under the energy field of the whole ensemble:

$$v_{p,t} = \sqrt{\frac{E_t}{M_p}} \qquad (17)$$

Let us denote with v_p the velocity of the property P when subjected to its own energy E_p

$$v_p = \sqrt{\frac{E_p}{M_p}} \qquad (18)$$

Since $E_t > E_p$ then obviously $v_{p,t} > v_p$.

The semantic energy is constant away from the center of mass of the semantic particle.
The solution of (13) subject to the boundary conditions (14) and (15) is (1). (1) gives us the distance of the semantic property in its bound state to the mass center of the semantic particle when the total net energy of the ensemble is E_t and the total net energy of the property is E_p. The center of the ensemble has zero energy so only property with zero semantic energy can be at the center of the ensemble.