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Awareness is a broad concept, difficult to uniquely identify because it is inextricably related to the human being who is a complex entity. Our specific intent is to propose a mathematical model to explain the interplay between awareness and the decision-making process.

There is now a consistent body of research into the nature of decision-making, particularly into the role of *cognition*, *intuition*, and *emotion* in human decisions [18]. Cognition and intuition refer to different ways of information processing, which we call *analytical* and *intuitive*. Although dual-process theories come in several forms, they reflect the generic distinction between two processes: the first is intuitive, associative, heuristic, tacit, and implicit. In contrast, the second involves conscious, analytical, cognitive, logical, and reason-oriented thinking [9, 10].

When we think about thinking it is easy to assume that “more is better”: the more analytical our reasoning, the better the results. This aspect reflects the past view of scholars and practitioners who agreed that effective choices must occur under only the most rational conditions [17]. In recent works [9, 12], however, there were gradually included in this process aspects like emotions and intuition, developing a richer conception of the decision-maker (DM).

This is also related to the consideration that the increasing availability of data does not necessarily match better decisions: despite their abundance, these data can be inaccurate, incomplete, or confusing, which is evident, for example, in the phenomenon of *infodemiology* [5,6]. If we had copious data drawn from a perfectly representative sample, completely mistake-free and precisely representing what we are trying to evaluate, then using the most

complex model available would indeed be the best approach, but if any of these factors fail to hold, we risk *overfitting* [4]. Successful models in science are based on the clear division of information into a “sloppy” and a “stiff” part [8]: beyond a certain level of detail, we stop considering properties common to a class of problems and begin to model the singularities of the specific reference set. Other aforementioned factors, like intuition, tacit knowledge, and emotions can, in some way, fill in the gaps in information and knowledge [17].

The awareness literature can be organized around three core concepts: *cognitive awareness* [13], which corresponds to the accurate and deep individual’s understanding of one’s perception and thinking. The second perspective argues that awareness is *multilevel* [7], considering both conscious and unconscious, with an end-stage of awareness that results from individual processing of all that is going on in one’s body and mind [19]. The third considers awareness concerning the recognition of the *feelings of others* [2]. About *self-awareness* Carden et al. [3] purpose a definition based on a systematic literature review: *Self-awareness consists of a range of components, which can be developed through focus, evaluation, and feedback, and provides individuals with an awareness of their internal state (emotions, cognitions, physiological responses), that drives their behaviors (beliefs, values, and motivations) and considers how these impact and influence others.*

This work intends to investigate how it is possible to develop a mathematical model filling in the gap of incorporating awareness and self-awareness into decision-making processes. Although limited and imperfect by nature, due to the difficulties in modeling complex phenomena as are human decisions, this study could contribute to introducing new aspects expanding the research in the field of decision-making.

Since awareness is a process with a dynamic nature [16], we move in the framework of *Sequential Decision Models* (SDM), which consider both outcomes of past, current, and future decisions under uncertainty. In an SDM at a specified point in time, the DM observes the system’s state and chooses an action among the available ones. His choice produces two results: receiving a reward, and the system’s evolution to a possibly new state at the next decision epoch, at which he faces a similar problem. A *Markov Decision Process* (MDP) [15] is a specific class of SDM where actions, rewards, and transition probabilities depend on the current action and state, where the last incorporates all past dynamics.

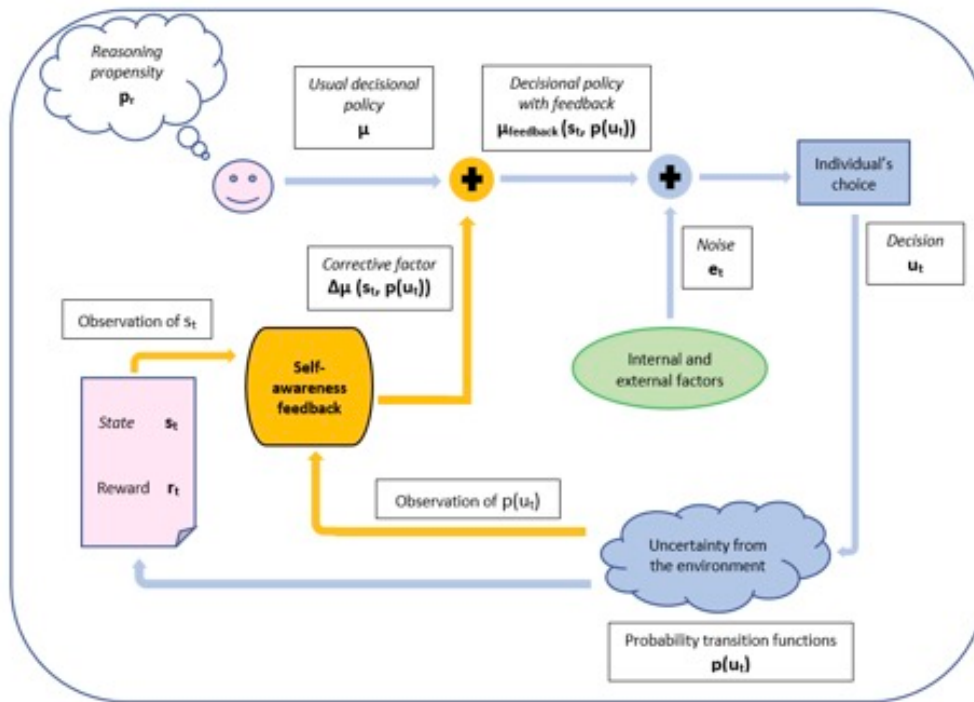


Figure 1. Schematic representation of the model. The blue arrows represent the sequential decision problem of an individual who acts according to a habitual policy μ . The orange lines indicate the presence of self-awareness feedback.

In the model (Figure 1) the *state* (s_t) represents the level of awareness at time t , and the individual has his *reasoning propensity*, $p_r \in (0,1)$, embedding his habitual attitude - character, beliefs, values, and experiences - in processing the information about the problem. p_r takes values in a continuum between two extreme attitudes [1], called 'intuitive' ($p_r=0$) and 'analytical' ($p_r=1$), assuming in this way that both are always involved, with different amounts, in any decision. The individual has his own *policy* that turns out in his *choice*, u_t , which takes values in the continuous interval $(0,1)$, where $u=0$ ($u=1$) holds for an intuitive (analytical) individual, intermediate values represent mixed cases. The *usual policy* μ is defined as a naive policy that makes the DM choose according only to his reasoning propensity, without any dependence on the state nor on the time instants and without considering the presence of uncertainties related to his choice's outcomes. There is, however, some variation of the decisions around the usual ones, given by *internal factors*, such as emotions and tacit knowledge [14], and *external factors*, both depending on interactions with others. These factors represent

a source of uncertainty impacting the decision and can drive the choices distant from the one suggested by the policy. They are considered as a *noise* e_t , and their variance indicates different levels. The decision's *reward*, r_t , is a linear function of the current state and the choice. Although the current version of the model assumes a linear form, $r_t = \alpha s_t - \beta u_t$ with constant and positive coefficients α and β , other functions can be suitable. The higher the individual's level of awareness, the higher his well-being from a whole point of view: physical, mental, and emotional. On the other hand, the more analytical the process resulting in the choice, the higher the cost the DM incurs, including a higher resource consumption.

Another source of *uncertainty* is the transition probability, $P(u_t)$, between two consecutive states (Figure 2) that results from the linear combination (Panel C) of intuitive (Panel A) and analytical (Panel B) reasonings. It arises *from the environment* and impacts the state evolution and the reward received from the DM.

Notice that in this setting the process does not include any component of self-awareness, because the only information available to the individual is reasoning propensity and the state at the previous instant. By additionally observing the transition probability function, the DM would be allowed to modify his usual policy by adding a *corrective factor* $\Delta\mu$. Self-awareness is then modeled as this *feedback* component.

This detached view of himself is represented by the knowledge of the ideal optimal policy suggesting the best action given a certain state, reasoning propensity, and time instant. It can mitigate the habitual tendencies of the individual by modifying his policy. This feedback is embedded in the optimization process, modeling in this way the fact that self-awareness results from a personal effort [11]. Specifically, the optimization consists in maximizing the level of awareness at any time instant using, for example, a Dynamic Programming method [20].

The transition probability, together with the other sources of uncertainty related to internal and external factors, makes the state evolution a not deterministic process.

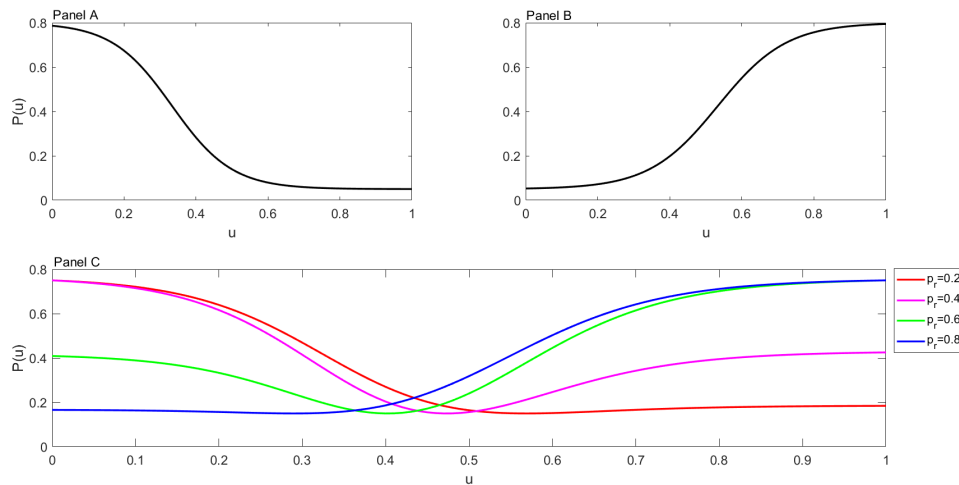


Figure 2. Transition probability functions. Panels A and B indicate the transition probabilities of an intuitive and an analytical individual, respectively. They are modeled as sigmoid functions and in the first case, the maximum probability of increasing the state is when the reasoning is intuitive, and then, the bigger u the lower is the probability. The second case is the opposite. These two functions are linearly combined using the specific individual's reasoning propensity, p_r . Some examples of transition probability functions for different p_r are shown in Panel C.

Some numerical simulations to show the resulting model dynamics have been developed using Matlab. In Figure 3, four different reasoning propensities are considered: a predominant intuitive (red), a predominant analytical (blue), and two mixed cases (magenta and green). The numerical results highlight that the feedback (continuous lines) has an improving action. One can notice, for example, that in all cases the level of awareness st , is monotonically increasing, differently from the ones without feedback (dotted lines). Moreover, the level of awareness of the individuals operating without feedback is lower in all cases.

The results provide examples of different types of individuals: Panel A considers an individual with a high initial level of awareness, who strongly weighs the possibility of decreasing his state and is subject to the low noise variance. For any reasoning propensity, the feedback produces a faster increase of awareness state than the case without feedback, for which a decrease of awareness over time could happen (magenta and green dotted lines). Panel B assumes an individual with characteristics like the previous one but with a high variance noise. While the feedback situation is not very different from the previous case, the behavior without

feedback can benefit from the noise since it allows the level of awareness to increase.

The results are similar in the case of an individual with a low initial state and a low penalty (Panel C and D). Even if this similarity in behaviors with the feedback could suggest a reduction of the heterogeneity of the individuals, it must be remembered that the homogeneity concerns the evolution of the level of awareness and not the choices. Rather it could be seen as a comforting factor: independently of the ideal reasoning, everyone has the - theoretical - opportunity to reach the same level of awareness over time.

Reasonably, the initial condition influences the trend of the state's increasing to the maximum. Moreover, the state evolution, in the case of feedback, is more robust to the magnitude of the external noise.

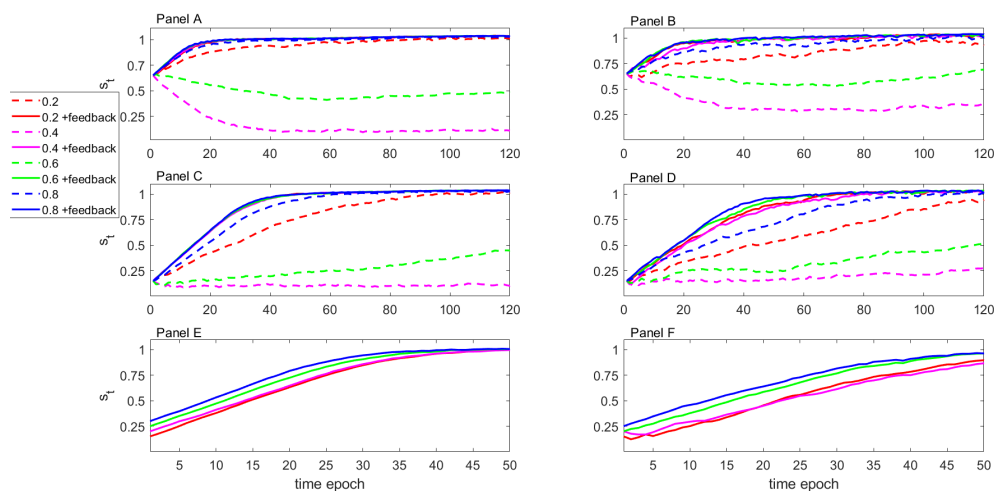


Figure 3. Time evolution of state for four different reasoning propensities ($pr = 0.2, 0.4, 0.6, 0.8$, from more intuitive to more analytical), with (continuous lines) and without (dotted lines) the feedback term. In Panels A, C, and E the level of noise is assumed low (normally distributed with mean u_t and standard deviation 0.08), while in Panels B, D, and F, it is high (normally distributed with mean u_t and standard deviation 0.27). In Panels A and B, the initial level of awareness and the penalty related to the decreasing awareness are high, while in Panels C and D they are low. Panels E and F report detail of state evolution shown in Panels C and D with feedback.

In the end, it must be noticed that the evolution of awareness with feedback could also be not monotonically increasing, and present some momentary decreases, as drawn in Figure 4.

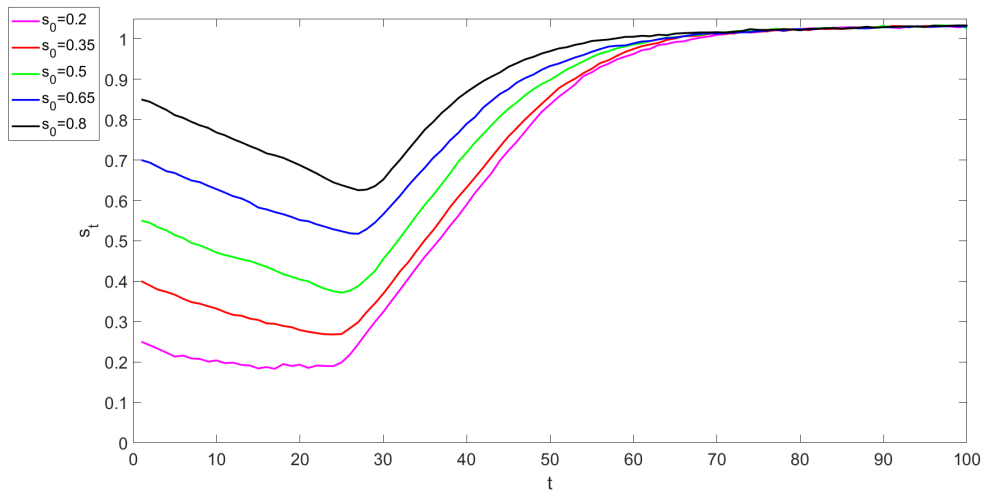


Figure 4. Non-monotonic dynamics in presence of feedback for individuals with reasoning propensity $pr = 0.6$, a low penalty of awareness decreasing, and a low level of noise. The different colors indicate five different initial conditions.

In this article, a mathematical model of awareness, still under study, is introduced, and some numerical results are discussed. The planned next steps will consist of performing a model validation through the testing on a large sample of real cases, aimed to identify specific reasoning propensities, transition probability functions, penalties for awareness decreasing, noise level, and all other parameters. The model will be extended by considering independently and separately, as far as possible, the impact of the different factors - emotions, tacit knowledge, and so on. Moreover, a network of interactions will be included so that the choice will explicitly depend on the influences of other individuals.

In the end, it would be interesting to use the model to investigate the effects of practices devoted to increasing awareness, such as Mindfulness and meditation, on individuals' behavior.

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