# Preface

This is a beginning graduate level textbook on applied group theory. Only those aspects of group theory are treated which are useful in the physical sciences, but the mathematical apparatus underlying the applications is presented with a high degree of rigor.

The principal characters in this book are symmetry groups of mathematical physics. The first four chapters are primarily concerned with finite or discrete symmetry groups, e.g., the point, space, and permutation groups. The last six chapters are devoted to Lie groups.

The theory presented here is largely algebraic in nature; the more complicated global topological problems are avoided. Thus topics such as the representation theory of Euclidean, Poincaré, and space groups are omitted. (These topics will be included in a projected second volume by the author which will be primarily devoted to topological aspects of applied group theory.) It is assumed that the reader is proficient in linear algebra and advanced calculus. Such concepts as finite-dimensional vector spaces, linear operators, and Jacobians are used without prior definition. An appendix on Hilbert space lists all the information the reader needs on that topic. There are a few places where greater mathematical sophistication is needed. In Chapter 5 the existence and uniqueness theorem for solutions of ordinary differential equations and some simple properties of power series are employed. In Chapter 6 the Peter–Weyl theorem is stated but not proved.

Most of the theory presented here is applied to quantum mechanics. Thus, it is desirable, though not essential, for the reader to be familiar with the basic concepts of quantum theory, particularly the probabilistic and physical interpretations. To make the applications clearer and to avoid unnecessary detail, the version of quantum theory presented here is slightly oversimplified. (In particular, only a qualitative perturbation theory of energy eigenvalues of the Hamiltonian is presented, and the physical interpretation in terms of spectral lines is omitted.) The author hopes in this way to explain some of the beautiful applications of group theory in atomic and nuclear physics to mathematics students unfamiliar with the physical literature.

There are several features which together differentiate this book from prior works on applications of group theory: (1) A rigorous derivation of point and space groups including a derivation of the fourteen Bravais lattices. (2) A simplified but rigorous presentation of the theory of local linear Lie groups. (3) A construction of the representations of the classical groups using both weights and Young diagrams. (4) An integrated theory which includes applications not only to classical and quantum physics but also to geometry and special function theory.

Finally, the author wishes to acknowledge his debt to those mathematicians and physicists whose writings form the main content of this volume, especially H. Boerner, I. Gel'fand, S. Lie, G. Liubarskii, M. Naimark, N. Vilenkin, H. Weyl, and E. Wigner. (In particular, Chapter 9 is adapted from Weyl's Princeton lecture notes [2].)