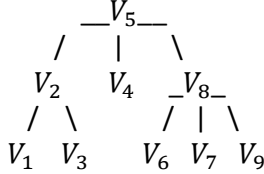# The Signature of Semantic Structures

Let us have the compound particle V_{comp} represented by its elementary particle sequence and semantic tree $stree(V_{comp})$:

$V_{comp} = [V_1 A_1 V_2 A_2 \dots V_9 A_9]$

$stree(V_{comp}) =$

```
           __V5__
          /   |   \
        V2    V4   _V8_
        / \        / | \
      V1   V3    V6  V7  V9
```

The property tree for each V-particle $V_k, k = 1..9$ are given with the algebraic notation discussed in Semantic Tree Operations.

$ptree(V_k) = \sum_{k \in \mathfrak{T}(V_k), i \in \mathbb{p}(V_k)}(k, P_i)$. Here k denotes the path $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$ constructed by branching consecutively along the k_{l_1}-th branch from the top level, then the k_{l_2}-th branch from the lower level and finally k_{l_h}-th branch from the h-th level. The set $\mathfrak{T}(V_k)$ denotes the set of all paths from the root to a leaf in the property tree of V_k. The set $\mathbb{p}(V_k)$ denotes the indices of the vertices in the property tree of V_k.

V_1 V_2 `

```
   P1 P2  P3                        P1--- o --- P2
    \ | / ..                          / | \
 Pk --- o --- Pi                     P3  P4  P5_
  .. / | \ ..                        / \    |  \
  Pj+1 Pj  Pj-1                     P6  P7  P8  P9
                                    |
                                    P10
```

Expressing the property tree of V_1 with the algebraic notation discussed earlier:

$ptree(V_1) = \sum_{k_j \in \mathfrak{T}(V_1), i \in \mathbb{p}(V_1)}(k_j, P_j)$

Similarly, $ptree(V_2)$ is given with

$ptree(V_2) = (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3 k_1, P_6) + (k_3 k_2, P_7)$
$+ (k_5 k_1, P_8) + (k_5 k_2, P_9) + (k_3 k_1 k_1, P_{10})$

Here P_0 is $text(V_2)$.

Now if we expand the property trees for each V-particle in the semantic tree for the composite particle V_{comp} we will have a larger augmented property tree. This augmented property tree represents the semantic structure of V_{comp} and can be recorded in a matrix form which is the semantic signature of V_{comp}. The semantic signature matrix of V_{comp} will have the following structure:

$ssig(V_{comp}) = [\boldsymbol{p_0\ a_{0,1}\ p_1\ p_0\ a_{0,2}\ p_2\ p_0\ a_{0,3}\ p_3\ \dots\ p_p\ a_{p,q}\ p_q}]$

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [B_1 \ B_2 \ B_3 \ \ldots \ B_q]$$

$$B_1 = [p_0 \ a_{0,1} \ p_1], B_2 = [p_0 \ a_{0,2} \ p_2], B_3 = [p_0 \ a_{0,2} \ p_3], \ldots, B_q = [p_0 \ a_{0,2} \ p_q]$$

Here the block matrix B_1 fully describes the property P_1 including how it is connected to the property tree $ptree(V_1)$. Similarly, B_2 and B_3 fully describes the properties P_2 and P_3 and their connectivity to $ptree(V_1)$. Finally, B_q fully describes the property P_q and its connectivity to $ptree(V_9)$.

In the block matrix for $ssig(V_{comp})$ p_0 denotes the signature column vector of the property P_0, $a_{0,1}$ denotes the signature column vector of the arc between property P_0 and property P_1, $a_{p,q}$ denotes the signature column vector of the arc between property P_p and P_q. Let us denote the number of rows of $ssig(V_{comp})$ by N and the number of columns by M.

The semantic signature matrix $ssig(V_{comp})$ can be decomposed as a sum of two intrinsic structural matrices – property signature matrix $psig(V_{comp})$ and connectivity signature matrix $csig(V_{comp})$:

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [p_0 \ 0 \ p_1 \ p_0 \ 0 \ p_2 \ p_0 \ 0 \ p_3 \ \ldots \ p_p \ 0 \ p_q]$$

$$csig(V_{comp}) = [0 \ a_{0,1} \ 0 \ 0 \ a_{0,2} \ 0 \ 0 \ a_{0,3} \ 0 \ \ldots \ 0 \ a_{p,q} \ 0]$$

Let us denote by $psig(P_1, V_{comp})$ the augmented semantic property signature of property P_1 with respect to V_{comp}. It is given with:

$$psig(P_0, V_{comp}) = [p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ \ldots \ 0 \ 0 \ 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0 \ 0 \ p_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ \ldots \ 0 \ 0 \ 0]$$

$$psig(P_q, V_{comp}) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \ldots \ 0 \ 0 \ p_q]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{p}(V_k)} psig(P_i, V_{comp})$$

Here $\mathbb{S}(V_{comp})$ denotes the set of the indices of all semantic particles which the composite V_{comp} is composed from.

Here are some interesting properties of $ssig(V_{comp})$:

The number of rows N in $ssig(V_{comp})$ is $3 \times$ the number of arcs in the augmented property tree of V_{comp}.

The rank of