

## The Signature of Semantic Structures

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Let us have the compound particle  $V_{comp}$  represented by its elementary particle sequence and semantic tree  $stree(V_{comp})$ :

$$stree(V_{comp}) =$$

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      /---V1---\
     /   |   \
    V2  V5  V6-
   / \  / | \
  V3 V4 V7 V8 V9

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The property tree for each  $V$ -particle  $V_k, k = 1..9$  are given with the algebraic notation discussed in [Semantic Tree Operations](#).

$ptree(V_k) = \sum_{k \in \mathfrak{T}(V_k), i \in \mathbb{P}(V_k)} (k, P_i)$ . Here  $k$  denotes the path  $(k_{l_1}, k_{l_2}, \dots, k_{l_h})$  constructed by branching consecutively along the  $k_{l_1}$ -th branch from the top level, then the  $k_{l_2}$ -th branch from the lower level and finally  $k_{l_h}$ -th branch from the  $h$ -th level. The set  $\mathfrak{T}(V_k)$  denotes the set of all paths from the root to a leaf in the property tree of  $V_k$ . The set  $\mathbb{P}(V_k)$  denotes the indices of the vertices in the property tree of  $V_k$ .

|  |   |     |  |
|--|---|-----|--|
| $V_1$  | $V_2$   | ... | $V_9$  |
| $  \begin{array}{c}  P_1 \ P_2 \ P_3 \\  \backslash \   \ / \ \dots \\  P_k \text{---} o \text{---} P_i \\  \dots \ / \   \ \backslash \ \dots \\  P_{j+1} \ P_j \ P_{j-1}  \end{array}  $ | $  \begin{array}{c}  P_1 \text{---} o \text{---} P_2 \\  \ / \   \ \backslash \\  P_3 \ P_4 \ P_5 \text{---} \\  \ / \ \backslash \ \quad   \ \backslash \\  P_6 \ P_7 \ \quad P_8 \ P_9 \\    \\  P_{10}  \end{array}  $ |     | $  \begin{array}{c}  P_1 \text{---} o \text{---} P_2  \end{array}  $ |

Expressing the property tree of  $V_1$  with the algebraic notation discussed earlier:

$$ptree(V_1) = \sum_{k_j \in \mathfrak{T}(V_1), i \in \mathbb{P}(V_1)} (k_j, P_i)$$

Similarly,  $ptree(V_2)$  is given with

$$\begin{aligned}
 ptree(V_2) = & (1, P_0) + (k_1, P_1) + (k_2, P_2) + (k_3, P_3) + (k_4, P_4) + (k_5, P_5) + (k_3 k_1, P_6) + (k_3 k_2, P_7) \\
 & + (k_5 k_1, P_8) + (k_5 k_2, P_9) + (k_3 k_1 k_1, P_{10})
 \end{aligned}$$

Here  $P_0$  is  $text(V_2)$ .

Now if we expand the property trees for each  $V$ -particle in the semantic tree for the composite particle  $V_{comp}$  we will have a larger augmented property tree. This augmented property tree represents the semantic structure of  $V_{comp}$  and can be recorded in a matrix form which is the semantic signature of  $V_{comp}$ . The semantic signature matrix of  $V_{comp}$  will have the following structure:

$$ssig(V_{comp}) = [p_0 \ a_{0,1} \ p_1 \ p_0 \ a_{0,2} \ p_2 \ p_0 \ a_{0,3} \ p_3 \ \dots \ p_p \ a_{p,q} \ p_q]$$

The last matrix can be rewritten in block matrix notation:

$$ssig(V_{comp}) = [\mathbf{B}_1 \ \mathbf{B}_2 \ \mathbf{B}_3 \ \dots \ \mathbf{B}_q]$$

$$\mathbf{B}_1 = [p_0 \ a_{0,1} \ p_1], \mathbf{B}_2 = [p_0 \ a_{0,2} \ p_2], \mathbf{B}_3 = [p_0 \ a_{0,3} \ p_3], \dots, \mathbf{B}_q = [p_p \ a_{p,q} \ p_q]$$

Here the block matrix  $\mathbf{B}_1$  fully describes the property  $P_1$  including how it is connected to the property tree  $ptree(V_1)$ . Similarly,  $\mathbf{B}_2$  and  $\mathbf{B}_3$  fully describes the properties  $P_2$  and  $P_3$  and their connectivity to  $ptree(V_1)$ . Finally,  $\mathbf{B}_q$  fully describes the property  $P_q$  and its connectivity to  $ptree(V_9)$ . From now on we will denote the block matrices  $\mathbf{B}_i$  as *semantic elements* of  $V_{comp}$ .

*Statement:* Every semantic particle, primitive or composite, can be represented as a sequence of *semantic elements*.

*Definition:* Semantic distance between two semantic elements

Let  $\mathbf{B}_1$  denotes the matrix of the first semantic element and is given with  $\mathbf{B}_1 = [p_i \ a_{i,j} \ p_j]$

Let  $\mathbf{B}_2$  denotes the matrix of the second semantic element and is given with  $\mathbf{B}_2 = [p_k \ a_{k,l} \ p_l]$

Then the semantic distance between the two is given with:

$$sdist(\mathbf{B}_1, \mathbf{B}_2) = |p_i - p_k| + |a_{i,j} - a_{k,l}| + |p_j - p_l|$$

In the block matrix for  $ssig(V_{comp})$   $p_0$  denotes the signature column vector of the property  $P_0$ ,  $a_{0,1}$  denotes the signature column vector of the arc between property  $P_0$  and property  $P_1$ ,  $a_{p,q}$  denotes the signature column vector of the arc between property  $P_p$  and  $P_q$ . Let us denote the number of rows of  $ssig(V_{comp})$  by  $N$  and the number of columns by  $M$ .

The semantic signature matrix  $ssig(V_{comp})$  can be decomposed as a sum of two intrinsic structural matrices – property signature matrix  $psig(V_{comp})$  and connectivity signature matrix  $csig(V_{comp})$ :

$$ssig(V_{comp}) = psig(V_{comp}) + csig(V_{comp})$$

$$psig(V_{comp}) = [p_0 \ 0 \ p_1 \ p_0 \ 0 \ p_2 \ p_0 \ 0 \ p_3 \ \dots \ p_p \ 0 \ p_q]$$

$$csig(V_{comp}) = [0 \ a_{0,1} \ 0 \ 0 \ a_{0,2} \ 0 \ 0 \ a_{0,3} \ 0 \ \dots \ 0 \ a_{p,q} \ 0]$$

Let us denote by  $psig(P_1, V_{comp})$  the augmented semantic property signature of property  $P_1$  with respect to  $V_{comp}$ . It is given with:

$$psig(P_0, V_{comp}) = [p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ p_0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

Similarly,

$$psig(P_1, V_{comp}) = [0 \ 0 \ p_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ 0]$$

$$psig(P_q, V_{comp}) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 0 \ p_q]$$

Then obviously:

$$psig(V_{comp}) = \sum_{k \in \mathbb{S}(V_{comp})} \sum_{i \in \mathbb{P}(V_k)} psig(P_i, V_{comp})$$

Here  $\mathbb{S}(V_{comp})$  denotes the set of the indices of all semantic particles which the composite  $V_{comp}$  is composed from.

Another way to partition the signature matrix into block matrices is:

$$ssig(V_{comp}) = [V_1 \ A_{1,2} \ V_2 \ A_{1,3} \ V_3 \ \dots \ A_{6,8} \ V_8 \ A_{6,9} \ V_9]$$

The block matrix  $V_1$  represents the property tree of the particle  $V_1$  and it is given by:

$$V_1 = [p_0 \ a_{0,1} \ p_1 \ p_0 \ a_{0,2} \ p_2 \ p_0 \ a_{0,3} \ p_3 \ \dots \ p_0 \ a_{0,k} \ p_k]$$

The block matrix  $A_{1,2}$  describes the connection between the particles  $V_1$  and  $V_2$  connecting the root property  $p_0$  of  $V_1$  and the root property  $p_{k+1}$  of  $V_2$ . It is given with:

$$A_{1,2} = [p_0 \ a_{0,k+1} \ p_{k+1}]$$

### Properties of the signature matrix

Here are some interesting properties of  $ssig(V_{comp})$ :

The number of rows  $N$  in  $ssig(V_{comp})$  is  $3 \times$  the number of arcs in the augmented property tree of  $V_{comp}$ .

The rank of

TO DO: finish the property section

### Asymptotic closeness of semantic structures

Let us have two semantic structures  $S_1$  and  $S_2$ .

$$ssig(S_1) = [V_{k_1} \ A_{k_1,k_2} \ V_{k_2} \ A_{k_1,k_3} \ V_{k_3} \ \dots \ A_{k_p,k_q} \ V_{k_q}]$$

$$ssig(S_2) = [V_{l_1} \ A_{l_1,l_2} \ V_{l_2} \ A_{l_1,l_3} \ V_{l_3} \ \dots \ A_{l_r,l_s} \ V_{l_s}]$$

Uniform asymptotic closeness

$K$ -level uniform asymptotic closeness