# Semantic tree operations

Let us introduce the *tree tuple* where represents certain node - semantic particle or a subtree.

Here is a tree tuple factor which encodes uniquely the position of the node in the tree.

The following operations are defined for tree tuple factors:

We have a base of tree tuple factors which are defined as the digits greater than 1 of -nary number system such that . We define an operation `` denoting digit concatenation . Obviously,

for any pair

Note that the latter implies that

for any tuple where

Encoding a complete -ary tree of height with the algebraic notation above:

. Further we will assume that .

In general we have:

where

Obviously, we have at most distinct terms which represent nodes i.e. semantic values.

The expression for the tree also can be written as:

where and are the *product factors* given with . The node values are the values ordered in increasing order of . This order corresponds to *level order traversal* of the -ary tree. Note that with appropriately defined comparison operation `<` we can model different ways of traversing the -ary tree. For instance, if we define `<` as the comparison for the values of we will have ordering which corresponds to the *preorder traversal* of the tree.

Let us have the tree given with the expression

After bracket expansion becomes: