# Notes on dynamical systems and their applicability in semantic analysis

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In physics *dynamical system* is described as:

*An ensemble of particles whose state varies over time and obeys equations of evolution involving derivatives. The evolution of the whole system is dictated by the solution of the equations of the evolution of the system.*

To be a bit more precise we recognize that dynamical system includes the following concepts:

1. The concept of *Phase Space* , whose elements or “points” represent possible states of the system
2. The concept of *Time* which may be discrete or continuous. It may extend either only in the future (irreversible or noninvertible processes) or into the past as well as the future (reversible / invertible processes). The sequence of time moments for a reversible discrete-time process is in a natural correspondence to the set of all integers; irreversibility corresponds to considering only nonnegative integers. For continuous-time processes, time is represented by the set of all real numbers in the reversible case and by the set of nonnegative real numbers for the irreversible case
3. *Equations of evolution aka time evolution laws*. Most generally, this is a rule that allows us to determine the state of the system at each moment of time from its states at all previous times. Thus, the most general time-evolution law is time-dependent and has infinite amount of memory. Most popular time evolution laws are those that allow us to define all future states (and for the reversible system also past) states given a single state at any particular moment.

Further assumption which is made is that the law of time evolution does not change with time. In other words, the result of time evolution will depend only on the initial position of the system and on the length of the evolution but not on the moment of time when the state of the system was initially set up. Thus, if our system was initially at state , it will find itself after time at a new state which is uniquely determined by and , and thus can be denoted by . Fixing , we obtain a transformation of the phase space onto itself. These transformations for different are related to each other. Namely, the evolution of the state for time can be accomplished by first applying the transformation to and then by applying to the new state . Thus, we have or equivalently, the transformation is equal to the composition of and : . In other words, the transformations form a semigroup. For a reversible system the transformations are defined for both positive and negative values of and each is invertible. Thus, a reversible discrete-time dynamical system is represented by a cyclic group of one-to-one transformations of the phase space onto itself. Similarly, a reversible continuous-time dynamical system determines one-parameter group of one-to-one transformations of onto itself.