# Pooling Layer Basics

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## Preliminaries

### Convolution

Continuous space convolution (abbrev. continuous convolution)

(1)

Shorthand notation for the convolution operator is :

(2)

Think of the function as an *input* and the second function as a *kernel*. The output is sometimes referred to as the *feature map*.

Discrete space convolution (abbrev. discrete convolution)

(3)

With discrete convolution the input is multi-dimensional array of data, and the kernel is a multi-dimensional array of parameters related to the specific learning algorithm being used. These multi-dimensional arrays will be referred to as *tensors*.

Often, we will use discrete convolutions over more than one axis. For example for two-dimensional image as an output we would use two-dimensional kernel in general:

(4)

(4) can be rewritten as:

(5)

Thus we conclude that discrete convolution is commutative i.e.

(6)

Flipping operator :

(7)

Using the notation introduced with (7) we rewrite (4) and (5) as

(8)

(9)

Cross-correlation

The same as discrete convolution but the kernel is not *flipped*:

(10)

We will denote the *cross-correlation* operator with .

Discrete convolution can be viewed as a multiplication by a matrix.

Pooling Layers

## Appendix

### Toeplitz matrix (aka diagonal-constant matrix)

A matrix in which each descending diagonal from left to right is constant. For example:

A matrix constructed such that is another example of Toeplitz matrix.

## References

[Pooling Layers for Convolutional Neural Networks, Jason Brownlee, July 5, 2019, online tutorial](https://machinelearningmastery.com/pooling-layers-for-convolutional-neural-networks/)

[Deep Learning for Computer Vision: Image Classification, Object Detection, Face Recognition, Jason Brownlee, 2019](https://github.com/dimitarpg13/deep_learning_for_image_processing/blob/main/literature/books/Deep_Learning_for_Computer_Vision-Image_Classification_Object_Detection_and_Face_Recognition_in_Python_by_Jason_Brownlee.pdf)

[Deep learning, Ian Goodfellow, Yoshua Bengio, Aaron Courville, 2016](file:///Users/dimitargueorguiev/git/ml/deep_learning_for_image_processing/literature/books/deeplearning_latest_edition.pdf)