# Hidden Markov Models

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## Markov Processes

Consider an -valued stochastic process i.e. each is an -valued random variable on a common underlying probability space where is some measure space. We think of as the state of a model at time : for example, could represent

* the price of a stock at time with
* the position and momentum of a particle at time with
* the operating status of an industrial process with

will be denoted as the *state space* of the process .

The process is said to possess the *Markov property* if

for all

### The transition kernel

**Definition**: *transition kernel*

A kernel from a *measurable space* to a measurable space is a map such that

i) for every , the map is a *measure* on ;

and

ii) for every , the map is *measurable*.

If for every , the kernel is denoted as a *transition kernel*.

Let us return back to the Markov process. We will call the stochastic process on the state space a *homogeneous Markov process* if there exists a transition kernel from to itself such that

for all .

represents the probability that the process will be in the set in the next time step, when it is currently in the state . ‘Homogeneous’ refers to the fact that this probability is the same at every time .

Example: let be an i.i.d. sequence of real-valued random variables with law , and define recursively the -valued random variables

, ,

where is a measurable function and . Then is a homogeneous Markov process on the state space if there exists a transition kernel

, .

Here the operator is equal to when and equal to otherwise.

Indeed, note that is independent of

## References

[Hidden Markov Models Lecture Nodes, Ramon van Handel, 2008](https://github.com/dimitarpg13/dynamical_systems_and_ergodicity/blob/main/literature/articles/hidden_markov_models/Hidden_Markov_Models_vanHandle_LectureNotes_2008.pdf)