Note on the Fireflies, Flames and Brains

by Paul Thompson, 7/12/2025

🔥 🔥 🔥 WHAT DO FIREFLIES, FLAMES + BRAINS have in common? Check out the videos below.

🔥 🔥 All can be modeled with Kuramoto's PDE, which helps explain how intermediate frequency patterns emerge (like alpha waves on EEG)

🇯🇵 ㊙️ Kuramoto (a Japanese mathematician) went for a walk in the forest and noticed that fireflies that are close to each other start to "synchronize" - their "phase" of their "flashing" becomes coupled. So in the first video ( 9 little circles - https://lnkd.in/grGParKu ) you can see 9 fireflies in a grid, and the little blue dot on the circle moves so that their brightness is sin(theta). Now let each firefly be coupled to its neighbors so that its phase is lagged or sped up to be like it, you can write down Kuramoto's equation. In the steady state, you can see standing waves with a spatial frequency that settles down!

🧠 Why does this pattern emerge? It turns out that Kuramoto's equation has a Laplacian and biharmonic term that come from a Taylor series, and if the interactions are only local (only between very close fireflies), it all smooths out. But you can integrate over all fireflies to get a higher order pattern + these standing waves emerge.

🧯 ❤️‍🔥 🧨 Now Grisha Sivashinsky, a Russian-Israeli physicist expert in Combustion Science (yes there is such a thing!) derived the exact same equation to show you'll see the same intermediate frequency patterns in evolving fronts of fire (video below) where the Laplacian gets rid of very high frequencies but emergy is moved around across frequencies as the fire front propagates

🧠 🧠 🧠 🧠 ... and last of all the neural field theory people (think EEG and MEG) use the exact same +Laplacian -Biharmonic eqn to show that if neurons interact locally and are spiking, you will tend to get standing waves emerge at intermediate frequencies. See video I made here https://lnkd.in/grGParKu

🎇 All these systems have a graph-Laplacian and Fiedler value (2nd lowest Eigenvalue; we used to study algebraic connectivity of brain networks using this [1] paper by Madelaine D. ) that, in dynamical systems, suggests how quickly they may stabilize and settle down. Given initial conditions or stimuli, they settle down to these patterns. You can study the patterns using the spectrum of the governing operator, which helps you model their dynamic behaviour.

🙃 Bonus - they may also help you with Generative AI, where you want to do density estimation by flowing a Gaussian onto your data. Most GenAI models can be a bit slow (like DDPM), but if you add a score function, and a stochastic term, these nonlocal operators could speed up your models!