Cellular Automata Introduction

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# Introductory Notes

Cellular automata are mathematical idealizations of physical systems in which space and time are discrete, and physical quantities take on a finite set of discrete values. A cellular automaton consists of a regular uniform ***lattice*** ( or ***array***), usually extending infinitely with discrete variable at each ***site*** (or ***cell***). The ***state*** of a cellular automaton is completely specified by the values of the variables at each site. A cellular automaton evolves in discrete time steps, with the value of the variable at one site being affected by the values of variables at sites in its ***neighborhood*** on the previous time step. The neighborhood of a site is typically taken to be the site itself and all immediately adjacent sites. The variables at each site are updated ***simultaneously*** (or ***synchronously***), based on the values of the variables in their neighborhood at the preceding time step, and according to a definite set of ***local rules***.

***Elementary cellular automata***: one dimensional cellular automata with two possible values of the variable at each site (base 2) and in which the neighborhood of a given site is simply the site itself and the sites immediately adjacent to it on the left and on the right.

Figure 1: Example of a set of local rules for the time evolution of one-dimensional elementary cellular automaton.

111

0

110

1

101

0

100

1

011

1

010

0

001

1

000

0

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
|  | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |  |

Figure 2: Evolution of a configuration in 1D cellular automaton for one time step according to the module-2 rule given on Figure 1

The local rules for 1D neighborhood-three cellular automaton are described by an eight-digit binary number, as in Figure 1. In specifying cellular automata we use this binary number interchangeably with its decimal equivalent.

Since any eight-digit binary number specifies a cellular automaton, there are possible distinct cellular automaton rules in one dimension with a three-site neighborhood. Two inessential restrictions will usually be imposed on these rules:

**R1**): a cellular automaton rule will be considered “illegal” unless a “null” or “quiescent” initial state consisting solely of remains unchanged. This forbids rules whose binary specification ends with a 1 (and removes symmetry in the treatment of 0 and 1 sites).

**R2**): the rules must be reflection symmetric so that the pairs (100, 001) or (110,011) would yield identical value for each part of the pair.

R1 and E2 leave 32 possible cellular automaton rules of the form

# References

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