# The Normal Form

The description of a game can be viewed as a listing of the strategies of the players and the outcome of any set of choices of strategies, without regard to the attitudes of the players toward various outcomes. We now indicate how the final simplification of the game – the normal form – is obtained, by taking into account the preferences of the players.

The result of any set of strategies is a probability distribution over the set of possible outcomes. It would be particularly convenient if a given player could express his/her preference pattern in by a bounded numerical function defined on , such that he or she prefers to *iff* . Note that denotes indifference between to . Also, the function is such that if for any probability distgribution over we define as the expected value of computed with respect to as

the player prefers to *iff* .

It is remarkable fact that, under extremely plausible hypothesis concerning the preference pattern such function exists.

***Definition*** (*utility function*): The function defined for all probability distributions over , is called the player’s ***utility function***.

is unique, for a given preference pattern up to a linear transformation. We will assume that each player has such utility function.

The aim of each player in the game is to maximize his/her expected utility. If is the utility function of player , his/her aim is to make as large as possible where is the probability distribution for fixed over determined by the overall chance move.

We are in a position to give a description of the normal form of a game:

***Definition*** (*normal form of a game*): A game consists of spaces and bounded numerical functions defined on the space of all -tuples , . The game is played as follows: Player chooses an element of , the choices being made simultaneously and independently; player then receives the amount .

TODO: finish it

***Example*** (*two player game involving coin-toss and a number choice*):

Player moves first and selects one of the two integers . The referee then tosses a coin and if the outcome is “head”, he informs player of player ’s choice and not otherwise. Player then moves and selects one of two integers . The fourth move is again a chance move by the referee and consists of selecting one of three integers with respective probabilities . The numbers selected in the first, third and the fourth move are added and the amount of dollars is paid by to if the sum is even and by to if the sum is odd. Note that .

Here is the set of possible outcomes for this game where I denotes player , 0 denotes the referee and II denotes player :

I –> 2 – 0 –> Head – II –> 4 – 0 –> 3 = 9, probability , strategies

I –> 2 – 0 –> Head – II –> 4 – 0 –> 2 = 8, probability , strategies

I –> 2 – 0 –> Head – II –> 4 – 0 –> 1 = 7, probability , strategies

I –> 2 – 0 –> Tail – II –> 4 – 0 –> 3 = 9, probability , strategies

I –> 2 – 0 –> Tail – II –> 4 – 0 –> 2 = 8, probability , strategies

I –> 2 – 0 –> Tail – II –> 4 – 0 –> 1 = 7, probability , strategies

I –> 2 – 0 –> Head – II –> 3 – 0 –> 3 = 8, probability , strategies

I –> 2 – 0 –> Head – II –> 3 – 0 –> 2 = 7, probability , strategies

I –> 2 – 0 –> Head – II –> 3 – 0 –> 1 = 6, probability , strategies

I –> 2 – 0 –> Tail – II –> 3 – 0 –> 3 = 8, probability , strategies

I –> 2 – 0 –> Tail – II –> 3 – 0 –> 2 = 7, probability , strategies

I –> 2 – 0 –> Tail – II –> 3 – 0 –> 1 = 6, probability , strategies

I –> 1 – 0 –> Head – II –> 4 – 0 –> 3 = 9, probability , strategies

I –> 1 – 0 –> Head – II –> 4 – 0 –> 2 = 8, probability , strategies

I –> 1 – 0 –> Head – II –> 4 – 0 –> 1 = 7, probability , strategies

I –> 1 – 0 –> Tail – II –> 4 – 0 –> 3 = 9, probability , strategies

I –> 1 – 0 –> Tail – II –> 4 – 0 –> 2 = 8, probability , strategies

I –> 1 – 0 –> Tail – II –> 4 – 0 –> 1 = 7, probability

I –> 1 – 0 –> Head – II –> 3 – 0 –> 3 = 8, probability

I –> 1 – 0 –> Head – II –> 3 – 0 –> 2 = 7, probability

I –> 1 – 0 –> Head – II –> 3 – 0 –> 1 = 6, probability

I –> 1 – 0 –> Tail – II –> 3 – 0 –> 3 = 8, probability

I –> 1 – 0 –> Tail – II –> 3 – 0 –> 2 = 7, probability

I –> 1 – 0 –> Tail – II –> 3 – 0 –> 1 = 6, probability

Here are the two strategy spaces:

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