# Notes on Testing Statistical Hypotheses by Lehmann and Romano

compiled by D.Gueorguiev, 4/30/24

## Statistical Inference, Statistical Decisions, and the Purpose of Statistics

Each statistical observation has a set of observations. The values in this set are the values taken by a random variable whose distribution is at least partly unknown. The parameter represents a label of the distribution which can uniquely identify the latter, but the parameter value is unknown; it is known only that lies in a certain set which will be denoted as *parameter space*. Generally, *Statistical inference* is concerned with the methods of using the given observation data to obtain additional information about the distribution of X or the parameter with which it is labeled.

The need for statistical analysis stems from the fact that the distribution of , and hence some aspect of the situation underlying the mathematical model, is not known. The consequence of such lack of knowledge is uncertainty as to the best mode of behavior. To formalize this, suppose that a choice has to be made between a number of alternative actions. The observations, by providing information about the distribution from which they came, also provide guidance as to the best decision. The problem is to determine a rule which, for each set of values of the observations, specifies what decision should be taken. Mathematically, such a rule is a function , which to each possible value of the random variables assigns a decision , that is, a function whose domain is the set of values of and whose range is the set of possible decisions.

In order to see how should be chosen, one must compare the consequences of using different rules. To this end suppose that the consequence of using different rules. To this end, suppose that the consequence of taking decision when the distribution of is is a *loss*, which can be expressed as a nonnegative real number . Then the long-term average loss that would result from the use of in a number of repetitions of the experiment is the expectation evaluated under the assumption that is the true distribution of . This expectation, which depends on the decision rule and the distribution is named as the *risk function* of and will be denoted by . By basing the decision on the observations, the original problem of choosing a decision with loss function is thus replaced by that of choosing , where the loss is now .

The above statement suggests that the aim of statistics is the selection of a decision function which minimizes the resulting risk. As will be seen later, this statement is not sufficiently precise to be meaningful; its proper interpretation is in fact one of the basic problems of the theory. (*Note to myself*: very nice introductory discussion about the goal of Statistics as a theoretical instrument - I typed it one-on-one because I liked it so much).

## Specification of a Decision Problem

The methods required for the solution of a specific statistical problem depend quite strongly on the three elements that define it: the class to which the distribution of is assumed to belong; the structure of the space of possible decisions ; and the form of the loss function . In order to obtain concrete results it is therefore necessary to make specific assumptions about these elements. On the other hand, if the theory is to be more than a collection of isolated results, the assumptions must be broad enough either to be of wide applicability or to define classes of problems for which a unified treatment is possible.

Consider the specification of the class . Precise numerical assumptions concerning probabilities or probability distributions are usually not warranted. However, it is frequently possible to assume that certain events have equal probabilities, and that certain other are statistically independent. (*Note to myself*: very nice general assumption in sync with the earlier statements of the authors - indeed this assumption does not limit the scope of the investigation considerably due to its generality).

Another type of assumption concerns the relative order of certain infinitesimal probabilities, for example the probability of occurrences in an interval, for example the probability of occurrences in an interval of time or space as the length of the internal tends to zero. The following classes of distributions are derived on the basis of only such assumptions and are therefore applicable in a great variety of situations.

The *binomial distribution* with

, , (1)

Another related distribution is the Poisson distribution with

, , (2)

This is the distribution of the number of events occurring in a fixed interval of time or space in case

i ) the probability of more than one occurrence in a very short interval is of smaller order of magnitude than that of a single occurrence

and

ii ) the number of events in nonoverlapping intervals are statistically independent

Under the assumptions i ) and ii ) the process generating these events is called Poisson process.

Example:

Let us denote the number of events occurring from to moment with .

Then the assumptions i ) and ii ) can be expressed mathematically as:

when is small enough.

The normal distribution with probability density

, , (3)

Under general conditions, made precise by the central limit theorem this is the approximate distribution of the sum of a large number of independent random variables when the relative contribution of each term to the sum is small.

## References

[Testing Statistical Hypotheses, E.L. Lehmann, J.P. Romano, Third Edition, Stanford U., Springer, 2005](https://github.com/dimitarpg13/generalized_synthetic_control_for_testops/blob/main/articles/hypothesis_testing/Lehmann_and_Romano-TestingStatisticalHypotheses.pdf)