# Finding the optimal cropped image for a given aspect ratio

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## The image cropping problem with given aspect ratio as an optimization problem

Inputs

We are given:

1 ) a source image with size and and given pixel density.

2 ) target aspect ratio – one of the set .

### Obtaining Segmentation Mask by using Depth-based Segmentation Algorithm

We would like to identify segments of interest which will need to be included in the target (cropped) image.

Using the given source image by using appropriate depth-based segmentation algorithm (see [1]) we obtain a gray-scale segmentation mask .

### Obtaining the Segments of Interest by Training a Pixel-based Segmentation Algorithm

Since the mask is gray-scale the boundary between the segments of interest and the background which we do not care about is not clearly defined. In order to obtain clear boundary we run pixel segmentation algorithm using [RandomForestClassifier](https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.RandomForestClassifier.html) as shown on the example in [2].

Let us assume that after the execution of the pixel segmentation algorithm on the gray-scale segmentation mask we do get segment of interest denoted with . Each segment can be closed (Figure 1) or open (Figure 2).Note that in general we can have more than one segment of interest – that is we have (see Figure 3).

Figure 1: Source image with a segmentation mask having single closed segment

Figure 2: Source image with a segmentation mask having a single open segment

Figure 3: Source image with mask having multiple segments

Outputs

Target image with dimensions and and position of top left corner in the frame of reference of the source image. The target image has the same pixel density as the source image.

Constraints

We want the following restrictions to be in place:

0 ) no rotation of the source image is allowed in order to fit the given aspect ratio

1 ) ,

2 )

3 ) soft constraint : : if possible



Figure 4: Graphical illustration of the image cropping problem with single closed segment



Figure 5: Graphical illustration of the image cropping problem with 3 closed segments

Objective terms of the multi-criteria objective

What we want optimize:

1 ) Minimize the area of the segments which is outside of the bounds of the target image ; that is:

(1)

Here is the number of close/open segments found in the source image.

Equivalently, we want to maximize the area of the segments which are inside the bounds of the target image:

s.t. (2)

Note that (1) and (2) can be refined by giving additional weight on minimizing those segments with the larger areas compared to the smaller segments in size. So we rewrite (1) and (2) as:

(3)

Here is appropriately chosen constant which might have dependency on as well.

s.t. (4)

2 ) Minimize the distance between the center of the target image and the mass center of the segments

(5)

Here are the coordinates of the top left corner of the target image , are the coordinates of the mass center of the segments .

Note that (5) can be linearized by converting it into two separate terms , within the multi-criteria objective.

3 ) Maximize the area of the target image

(6)

where is the target aspect ratio

## Multi-criteria constrained optimization formulation of the Image cropping problem

Free variables:

: the coordinate of the top left corner of the target image (see Figure 4 and Figure 5)

: the coordinate of the top left corner of the target image

: the height of the target image

### Formulating the multi-criteria objective

1 ) the area of the segments which is out-of-bounds from the target image – denoted with

criterion (i)

2 ) the distance between the center of the target image and the mass center of the segments – denoted with

criterion (ii)

3 ) the area of the source image sans the area of the target image – denoted with

criterion (iii)

Constraints:

The triplet represents a coordinate in the *feature space* of the problem while the triplet represents a coordinate in *objective space*.

Note that the objective function in general is not convex due to the non-convexity of .

obviously is not convex either.

### Splitting the search space into two decoupled search spaces: and

**Simplification**: we can consider to be an independent variable and the minimization of can happen after we complete the minimization in the space .

So the algorithm for image cropping becomes:

1 ) we set

2 ) find the set of [*efficient* (*non-dominated*)](https://en.wikipedia.org/wiki/Pareto_efficiency) points for the two criteria and .

The set of *efficient* (*non-dominated*) points in is defined as:

and such that and where at least one of is strictly positive. This set of points represent [the Pareto front](https://en.wikipedia.org/wiki/Pareto_front) for .

3 ) find the efficient (non-dominated) point which minimizes and

## References

[1] [Highly Accurate Dichotomous Image Segmentation, Xuebin Qin et al, 2022](https://github.com/dimitarpg13/deep_learning_for_image_processing/blob/main/literature/articles/semantic_segmentation/Highly_Accurate_Dichotomous_Image_Segmentation_Qin_2022.pdf)

[2] [Trainable pixel segmentation with scikit-image using RandomForestClassifier](https://scikit-image.org/docs/stable/auto_examples/segmentation/plot_trainable_segmentation.html)