Inpainting Fundamentals

compiled by D.Gueorguiev 9/27/2024

# Introduction

Definition of inpainting:

Filling image information on a blank domain or several domains , based on the information available outside of these inpainting domains. On such domains , the original image has been compromised.

But the main question is how to model mathematically the process of inpainting those regions denoted with in the figure above.

inpainting domain

*is given*

Assumption of Locality of the domain

Assumption of Smoothness of the Image Function

To develop a rigorous mathematical framework for inpaintings, we make a simple assumption in which the accuracy of the inpainting can be studied. This is the assumption that the target image function is smooth, that is – the inpainting domain is contained in the interior of a smooth 2D object.

Let be a smooth image function defined on a 2D domain (usually rectangular domain). We denote by the domain to be inpainted and its diameter. We denote the restriction of on by . Then inpainting is the task to find a function defined on such that is a good approximation to .

**Definition** *Linear Inpainting procedure*

An inpainting procedure is linear if for any given smooth image

# Appendix

## Fourier Transform and the Nyquist-Shannon Sampling Theorem

### Fourier Transform

**Definition**: The *Fourier transform pair* is given with

(A.1)

(A.2)

The synthesis formula (A.1) represents as a superposition of infinitesimally small complex sinusoids of the form

with ranging over an interval of length and with determining the relative amount of each complex sinusoidal component.

### Sampling in Frequency Domain

Notation

– continuous signal

– discrete sampled signal

– periodic impulse train

//TODO: finish the Appendix section on Fourier transform and Shannon-Nyquist theorem

## Harmonic Functions

//TODO: finish the Appendix section on Harmonic Functions

# References

[1] [Mathematical Models for Local Deterministic Inpaintings, T. Chan and J. Shen, 2000](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/Mathematical_Models_for_Local_deterministic_Inpaintings_Chan_2000.pdf)

[2] [Mathematical Models for Local Non-Texture Inpaintings, T. Chan and J. Shen, 2002](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/MathematicalL_Models_for_Local_Nontexture_Inpaintings_Chan_2002.pdf)

[3] [Image Inpainting, M. Bertalmio, G. Shapiro, V. Caselles, C. Ballester, 1999](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/Image_Inpainting_bertalmio_1999.pdf)

[4] [Nonlinear Total Variation Based Noise Removal Algorithms, L. Rudin, S. Osher, E. Fatemi, 1992](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/Nonlinear_total_variation_based_noise_removal_algo_Rudin_1999.pdf)

[5] [Uncertainty Principles and Signal Recovery, D. Donoho, P. Stark, UC Berkeley, 1987](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/UncertaintyPrinciplesAndSignalRecoveryDonoho1987.pdf)

[6] [Discrete Time Processing, Alan Oppenheim, Ronald Schafer, MIT, 2nd edition, 1999](https://github.com/dimitarpg13/image_processing/blob/main/literature/books/Discrete_Time_Signal_Processing_Oppenheim_1999.pdf)