Inpainting Fundamentals

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# Introduction

Definition of inpainting:

Filling image information on a blank domain or several domains , based on the information available outside of these inpainting domains. On such domains , the original image has been compromised.

But the main question is how to model mathematically the process of inpainting those regions denoted with in the figure above.

inpainting domain

*is given*

Assumption of Locality of the domain

Assumption of Smoothness of the Image Function

To develop a rigorous mathematical framework for inpaintings, we make a simple assumption in which the accuracy of the inpainting can be studied. This is the assumption that the target image function is smooth, that is – the inpainting domain is contained in the interior of a smooth 2D object.

Let be a smooth image function defined on a 2D domain (usually rectangular domain). We denote by the domain to be inpainted and its diameter. We denote the restriction of on by . Then inpainting is the task to find a function defined on such that is a good approximation to .

**Definition** *Linear Inpainting procedure*

An inpainting procedure is linear if for any given smooth image

# Appendix

## Fourier Transform and the Nyquist-Shannon Sampling Theorem

### Fourier Transform

**Definition**: The *Fourier transform pair* is given with

(A.1)

(A.2)

The synthesis formula (A.1) represents as a superposition of infinitesimally small complex sinusoids of the form

with ranging over an interval of length and with determining the relative amount of each complex sinusoidal component.

### Sampling in Frequency Domain

Notation

– continuous signal

– discrete sampled signal

– periodic impulse train

//TODO: finish the Appendix section on Fourier transform and Shannon-Nyquist theorem

## Harmonic Functions

//TODO: finish the Appendix section on Harmonic Functions

## Green’s Identities

Divergence theorem

Let is a subset of which is compact and has a piecewise smooth boundary .

If is a continuously differentiable vector field defined on a neighborhood of , then:

(D.1)

The closed measurable set is oriented by outward pointing normal at almost each point on the boundary .

Figure: A region bounded by the surface with the surface normal .

//TODO: finish the Appendix section on Green’s Identities

## Eikonal Equation

The eikonal equation (from , image) is a non-linear first-order PDE encountered in the problems of wave propagation.

The classical eikonal equation in geometric optics is given as:

(E.1)

where is in an open subset of , is a positive function. The function and we are looking for solutions . More generally, an eikonal equation is an equation of the form

(E.2)

where is a function of two variables. The function is given and is the solution. If , then (E.2) becomes (E.1).

Eikonal equations arise with the application of the WKB method to electro-magnetic wave propagation, providing a link between physical wave propagation / optics and geometric ray optics. A fast computational algorithm to obtain approximate solution to the eikonal equation is the *fast-marching method*.

//TODO: finish the Appendix section on Eikonal Equation

## Poisson Equation

//TODO: finish the Appendix section on Poisson Equation

## Navier-Stokes Equations

//TODO: finish the Appendix section on Naiver-Stokes Equations

# References

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[2] [Mathematical Models for Local Non-Texture Inpaintings, T. Chan and J. Shen, 2002](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/MathematicalL_Models_for_Local_Nontexture_Inpaintings_Chan_2002.pdf)

[3] [Image Inpainting, M. Bertalmio, G. Shapiro, V. Caselles, C. Ballester, 1999](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/Image_Inpainting_bertalmio_1999.pdf)

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[5] [Uncertainty Principles and Signal Recovery, D. Donoho, P. Stark, UC Berkeley, 1987](https://github.com/dimitarpg13/image_processing/blob/main/literature/articles/inpainting_algorithms/UncertaintyPrinciplesAndSignalRecoveryDonoho1987.pdf)

[6] [Discrete Time Processing, Alan Oppenheim, Ronald Schafer, MIT, 2nd edition, 1999](https://github.com/dimitarpg13/image_processing/blob/main/literature/books/Discrete_Time_Signal_Processing_Oppenheim_1999.pdf)

[7] [The Eikonal Equation, Wikipedia](https://en.wikipedia.org/wiki/Eikonal_equation)

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