Notes on Information Theory and Statistical Mechanics by ET Jaynes

compiled by D. Gueorguiev, 12/31/24

# Introductory Notes

Information theory provides a constructive criterion for setting up probability distributions on the basis of partial knowledge and leads to a type of statistical inference – the *maximum entropy estimate*. The maximum entropy estimate is the least biased estimated possible on the given information – it is maximally noncommittal with regard to missing information. If one considers statistical mechanics as a form of statistical inference rather than as a physical theory, it is found that the usual computational rules, starting with the determination of the partition function, are an immediate consequence of the maximum-entropy principle.

In the resulting “subjective statistical mechanics”, the usual rules are thus justified independently of any physical argument, and in particular independently of experimental verification; whether or not the results agree with experiment, they still represent the best estimates that could have been made on the basis of the information available.

# Maximum-Entropy Estimates

The quantity is capable of assuming the discrete values . We are not given the corresponding probabilities ; all we know is the expectation value of the function :

(1)

On the basis of this information, what is the expectation value of the function ? At first glance, the problem seems insoluble because the given information is insufficient to determine the probabilities . (1) and the normalization condition

(2)

would have to be supplemented by more conditions before could be found.

The problem of specification of probabilities in case where little or no information is available was attempted to be resolved through a criterion of choice, in which one said that two events are to be assigned equal probabilities if there is no reason to think otherwise. However, except in cases where there is an evident element of symmetry that clearly renders the events “equally possible”, this assumption may appear just as arbitrary as any other that might be made. Furthermore, it has been very fertile in generating paradoxes in the case of continuously variable random quantities, since intuitive notions of “equally possible” are altered by a change of variables. Hence this way of resolving probabilistic distributions in problem has been abandoned as lacking constructive principle.

Again, our problem is to find a probability assignment which avoids bias, while agreeing with whatever information is given. Using information theory we can devise a unique, unambiguous criterion for the amount of uncertainty represented by a discrete probability distribution, which agrees with our intuitive notions that a broad distribution represents more uncertainty than does a sharply peaked one.

## Entropy of a Probability Distribution

The variable can assume the discrete values . Our partial understanding of the processes which determine the value of can be represented by assigning corresponding probabilities .

Question: Is it possible to find any quantity which measures in a unique way the amount of uncertainty represented by this probability distribution. The three conditions on are

1. is a continuous function of the .
2. If all are equal, the quantity is a monotonic increasing function of .
3. The composition law: instead of giving the probabilities of the events directly, we might group the first of them together as a single event, and give its probability ; then the next m possibilities are

# References

[1] [Information Theory and Statistical Mechanics, E.T. Jaynes, Department of Physics, Stanford University, 1957](https://github.com/dimitarpg13/information_theory_and_statistical_mechanics/blob/main/literature/articles/Information_theory_and_statistical_mechanics_part1_Jaynes_1957.pdf)

[2] [Information Theory and Statistical Mechanics, E.T. Jaynes, Department of Physics, Stanford University II, 1957](https://github.com/dimitarpg13/information_theory_and_statistical_mechanics/blob/main/literature/articles/Information_theory_and_statistical_mechanics_part2_Jaynes_1957.pdf)