Journal of the Operational Research Society (1998) 49, 549–557 0160-5682/98 $12.00

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Bounding MOLP objective functions: effect on efficient set size

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In multiple objective linear programming (MOLP) problems the extraction of all the efficient extreme points becomes problematic as the size of the problem increases. One of the suggested actions, in order to keep the size of the efficient set to manageable limits, is the use of bounds on the values of the objective functions by the decision maker. The unacceptable efficient solutions are screened out from further investigation and the size of the efficient set is reduced. Although the bounding of the objective functions is widely used in practice, the effect of this action on the size of the efficient set has not been investigated. In this paper, we study the effect of individual and simultaneous bounding of the objective functions on the number of the generated efficient points. In order to estimate the underlying relationships, a computational experiment is designed, in which randomly generated multiple objective linear programming problems of various sizes are systematically examined.

Keywords: efficient set; multiple objective linear programming

Introduction

The general multiple objective linear programming (MOLP) problem is defined by the following formulation:

$$\max\{\mathbf{Cx} = \mathbf{z} | \mathbf{x} \in \mathbf{S}\}$$

$$S = \{\mathbf{x} \in R^n | \mathbf{Ax} = \mathbf{b},\ \mathbf{x} \geq 0, \mathbf{b} \in R^m\}.$$

Here n is the number of variables, m is the number of constraints, p is the number of objective functions, $\mathbf{C}$ is the $p \times n$ criterion matrix (matrix of objective function coefficients), $\mathbf{z}$ is the criterion vector, $\mathbf{A}$ is the matrix $m \times n$ of technological coefficients, $\mathbf{b}$ the right hand side vector, and $\mathbf{x}$ is the decision variable vector.

A solution $\mathbf{x}'$ is efficient (non-dominated, Pareto optimal) if and only if $\mathbf{x}' \in S$ and there is no other $\mathbf{x} \in S$ such that $\mathbf{c}_k\mathbf{x} \geq \mathbf{c}_k\mathbf{x}'$, $k = 1, 2, \ldots, p$, with at least one strict inequality. Unlike linear programming (LP), in MOLP problems there is no unique optimal solution which simultaneously optimizes all the objectives. Therefore, the aim of MOLP solution methods is to find among the efficient solutions the *most preferred* for the decision maker (DM).

Solution procedures for the MOLP problems can be divided into three categories depending on which stage of the decision process the DM expresses his/her preferences: before, during or after problem solution.[1–3]

In the first category, all the requested information is available before the MOLP solution process (prior articulation of the DMs preference). In this case the preference information has to be so accurate, that it can be quantified into scalar coefficients which express the relative importance of each objective function as perceived by the DM. Therefore, the MOLP problem is reduced to a single objective optimisation LP problem, where the unique objective is the weighted sum of the MOLP objective functions. The optimal solution of the derived LP problem is the most preferred solution sought by the DM. However, it is quite unrealistic to expect the DM to be able to supply such accurate information, especially in the early stages of the decision process. The weighted sum of the objective functions is mostly used as a subroutine with sample weight coefficients in the interactive methods to be described in the next paragraph.

The second category includes methods which articulate the DMs preference during the solution process. The search toward the most preferred solution is driven by the DM who expresses his/her preferences at every stage of the solution process. Therefore, these iterative methods require close interaction between the DM and the analyst in order to converge to the desired efficient solution. There are a lot of so called *interactive methods*[4,5] which are very popular among MOLP specialists. An inevitable drawback of the interactive methods is that the DM does not perform an exhaustive search of the most promising area of the efficient set, because he/she sees and processes only samples or single points. The generation of the relative efficient extreme points allows for a detailed search of the remaining part of the efficient set which reveals useful information (especially in the last stage of the interactive process where the final decision has to be made).

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The third category of MOLP solution procedure assumes that no preference information is available prior to, or during the solution process. The articulation of DMs preference takes place after the multiple objective optimisation (a posteriori articulation of preference). The first stage in these methods, is to generate all (or most) of the efficient solutions to the MOLP problem. Then, the DM is aided to choose his/her most preferred solution among the solutions in the usually quite large efficient set. The task of generating all the efficient extreme points is the most computationally demanding part of such techniques. In many large MOLP problems the method becomes inapplicable because it is practically impossible to extract all the efficient extreme solutions due to restricted computational resources. Sometimes, even if the first stage is successfully completed, the size of the efficient set is so large that it cannot be efficiently investigated by the DM. In order to overcome these difficulties, methods which aim at the reduction of the efficient set have been reported in the literature. The *filtering* of the efficient extreme solutions can take place after the generation of the efficient set in order to produce a representative new set of smaller size.[6] Unfortunately the filtering can only be implemented after the generation process is completed and therefore it does not resolve the problem of memory overflow due to the large number of generated efficient points in large MOLP problems.

In order to effectively overcome this difficulty there are two techniques which can reduce prior to the generation process the number of efficient solutions using information supplied by the DM. The first technique is to restrict the range of the weight coefficients which are applied to the objective functions (interval weight coefficients). The interval weight coefficients is a well known technique,[7] which assigns to some (or all) of the criteria weight coefficients, varying in a sub-interval of [0,1]. The second one (and more widely used) is to restrict the objective functions' range of values by introducing appropriate bounds on them (lower acceptable bounds). After inspecting the payoff table of the MOLP problem the DM may conclude that values below certain levels for some or all of the criteria would not be acceptable under any circumstances (assuming maximisation of the objective functions). Therefore, it is reasonable to avoid the regions of no interest, by imposing bounds to the objective functions. These bounds can be incorporated in the model as usual constraints and the optimisation process can be restarted.

The effect of objective function bounding to the reduction of efficient extreme points can be better illustrated schematically. In Figure 1, the efficient frontier of a bicriteria maximisation MOLP with eight efficient extreme points (A, B, C, D, E, F, G, H) is depicted. The bounds on the two objective functions are shown as dashed lines and the number of the efficient extreme points is reduced from eight to six (B′, C, D, E, F, G′). The points A, B, G and H are excluded from the bounded efficient frontier while the points B and G are introduced as the intersection of the bounding constraints with the efficient frontier.

The bounding of the objective functions during the solution process has a dual effect: It contracts the feasible region of the MOLP problem, but on the other hand, it increases the number of the problem constraints. The contraction of the feasible region, generally, results in the reduction of the efficient extreme points while the addition of constraints results in their increase. Which of the two effects will eventually dominate in the change of the efficient set depends on their relative influence in the solution process.

Although the bounding of the objective functions is widely used in practical MOLP problems, the study of its

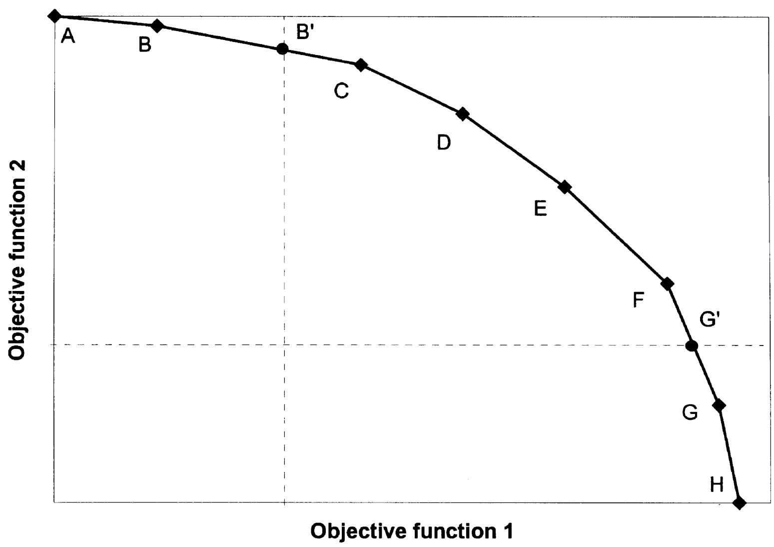

Figure 1 The effect of objective functions' bounding on the number of efficient extreme points in the bicriteria case.