A Note on Woodbury Matrix Identity

By Andreas Steiner, 6/27/2025

I didn't know that the matrix inversion lemma below has a name: the Sherman-Morrison-Woodbury matrix identity. This can be useful when inverting large covariance matrices, and the entries in that covariance matrix follow a linear factor model. If you do not have an economic factor model, you can use a statistical one like PCA.  
  
The clue of this lemma is that typically, m << n and A a diagonal matrix. This can improve very much the inversion of large covariance matrices. The big assumption here is that the factor model is "good" and its stability over time not worse than the stability of the original covariances.  
  
If I find the time, I might illustrate the PCA calculations by examining the impact of the number of components chosen on the composition of a minimum variance portfolio.  
  
But: given today's computers and algorithms, I have not come across inversion issues anymore in a long time. I can imagine that the Woodbury matrix identity creates a bit of a speed advantage. But who needs "fast" optimizers anyway, really?

References

[1] [Woodbury matrix identity, Wikipedia](https://en.wikipedia.org/wiki/Woodbury_matrix_identity)

[2] [Proof of Woodbury Matrix Identity and Kalman Update Formula, S. Richard, Nagoya U., 2022](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/matrix_algorithms/Woodbury_matrix_identity/Proof_of_Woodbury_Matrix_Identity_and_the_Kalman_Update_Formula_Richard_Nagoya_2022.pdf)

[3] [On Deriving the Inverse of Sum of Matrices, H.V. Henderson, 1980](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/matrix_algorithms/Woodbury_matrix_identity/On_Deriving_The_Inverse_of_Sum_of_matrices_Henderson_1980.pdf)

[4] [Updating the Inverse of a Matrix, William Hager, 1989](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/matrix_algorithms/Woodbury_matrix_identity/Updating_The_Inverse_of_Matrix_Hager_1989.pdf)

[5] [A Note on Stability of the Sherman-Morrison-Woodbury Formula, L. Ma et al, 2025](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/matrix_algorithms/Woodbury_matrix_identity/A_Note_on_the_Stability_of_the_Sherman-Morrison-Woodbury_Formula_Ma_2025.pdf)

A math formula with a few equations

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