Notes on Convex Optimization by Steven Boyd and Lieven Vandenberghe

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# Notation

– the set of symmetric matrices

– the set of symmetric positive semidefinite matrices

– the set of symmetric positive definite matrices

# Affine and Convex sets

**Definition** *Affine set*

A set is *affine* if the line through any two distinct points in lies in , i.e. if for any and , we have . In other words, contains the linear combination of any two points in , provided the coefficients of the linear combination sum to one.

**Definition** *Affine combination*

A point defined as , where , , and is affine combination of the points , .

**Corollary**: An affine set contains every affine combination of its points

The Positive Semi-Definite Cone

We use the notation to denote the set of symmetric matrices,

which is a vector space with dimension . We use the notation to denote the set of symmetric positive semidefinite matrices:

and the notation to denote the set of symmetric positive definite matrices:

A graph of a function

Description automatically generated

Figure : Boundary of the positive semidefinite cone in given with

**Statement**: The set is a convex cone

If and , then . This can be verified directly from the definition of positive semi definiteness: for any , we have

if , and

# References

[1] [Convex Optimization, Steven Boyd, Lieven Vandenberghe, 2009](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/books/ConvexOptimization_Boyd_2004.pdf)