# Notes on Sparse Nonnegative Solution of Underdetermined Linear Equations by Linear Programming

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## Summary

Let us consider an underdetermined system of linear equations with known matrix and known .

We seek the sparsest nonnegative solution, i.e. the nonnegative with fewest nonzeros satisfying . In general this problem is NP-hard. However, for many matrices A there is a threshold phenomenon: if the sparsest solution is sufficiently sparse, it can be found by linear programming.

In classical convex polytope theory, a polytope is called -*neighborly* if every set of vertices of span a face of . Let denote the -th column of , , let and let denote the convex hull of the .??

//TODO: finish this paragraph

## Introduction

Consider an underdetermined system of linear equations , where , , is a matrix, and is considered known but is unknown. Only non-negative solutions are of interest. We are looking for the *sparsest* solution which is the one with fewest non-zeros. Formally, we write:

subject to ,

## References

[Sparse Nonnegative Solution of Underdetermined Linear Equations by Linear Programming, David L. Donoho and Jared Tanner, 2005](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/periodograms_and_superresolution/SparseNonnegativeSolutionsOfUnderdeterminedLinearEquationsDonoho.pdf)

[An Introduction to Convex Polytopes, Arne Brondsted, Springer, 1982](https://github.com/dimitarpg13/optimization_classification_regression/blob/main/literature/articles/convex_polytopes/Intro_to_convex_polytopes_brondsted.pdf)

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