# Notes on Generation of efficient solutions in Multi-objective Mathematical Programming problems with GAMS - Effective implementation of the -constraint method by G Mavrotas

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## Multi-objective Mathematical Programming (MMP) and efficient solutions

In MMP there are more than one objective function and there is no single optimal solution that simultaneously optimizes all the objective functions. We are looking for the “most preferred” solution. The efficient (also known as *Pareto optimal*, *nondominated*, *non-inferior*) solutions are the solutions that cannot be improved in one objective function without deteriorating their performance in at least one of the rest.

Without loss of generality, we assume that all objective functions are for maximization.

**Definition** *efficient solution*:

A feasible solution of a MMP problem is efficient if there is no other feasible solution such that with at least one strict inequality.

**Definition** *non-dominated (or non-improvable) vector*:

Vector in criterion space which corresponds to an efficient solution.

**Definition** *weakly efficient solution*:

A feasible solution of a MMP problem is efficient if there is no other feasible solution such that

**Lemma**: weakly efficient solution can be dominated by an efficient solution

Let us denote criterion space with . Obviously, is a metric space. Let us select a weakly non-dominated point in such that there exists another point with a coordinate **for some** . Note, that in general we can find such point even if there does not exist a point such that **for each** .

Let us consider the special case of linear MMP problems which we will abbreviate as MOLP. In such case is an -dimensional simplex and each is a linear form. Then, all non-dominated points in form the Pareto front of denoted with . There exist multiple algorithms as well as modifications of the simplex algorithm for finding all non-dominated points (the efficient set) in MOLP problems. See (Paul Armand, 1991), (J.P. Evans, 1973), (S.F. Tantawy, 2007).

### Generation methods for tackling general MMP problems

The algorithms for obtaining all efficient solutions are NP complete (J.P. Evans, 1973), (Miettinen, 1998).

The most widely used generation methods are the weighing method and the -constraint method – both provide representative subset of the efficient set.

Let us assume the following MMP formulation:

is the feasible region. are the objective functions.

#### Weighting method

The problem which is optimized via the weighting method is:

By varying the weights, we obtain different efficient solutions.

#### The -constraint method

We optimize one of the objective functions using the other objective functions as constraints as shown below:

and

By parametric variation of right-hand sides of the constrained objective functions efficient solutions of the problem are obtained.

Advantages of the -constraint method over the weighting method:

# Bibliography

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Miettinen, K. (1998). *Nonlinear Multiobjective Optimization.* Jyvaskyla, Finnish Lakeland, Finland: Springer Science+Business Media, LLC.

Paul Armand, C. M. (1991). Determination of the Efficient Set in Multiobjective Linear Programming . *Journal of Optimization Theory and Applications*, pp 467-489.

S.F. Tantawy, R. S. (2007). Using Feasible Direction to Find All Efficient Extreme Points for Multiple Objective Linear Programs (MOLP). *Journal of Applied Sciences Research, 3*(8), pp 662-667.

Links to Literature:

(Mavrotas, 2008): [link](https://nike.box.com/s/e9wryvk9e0pirlfnp5qcgx7tqud3ce7s)

(J.P. Evans, 1973): [link](https://nike.box.com/s/b0d6lqcbxv7xc98k5p9scs6kqya0jwjn)

(Paul Armand, 1991): [link](https://nike.box.com/s/jaxvdfv3lbq465kv0bxpngpluh2dl59c)

(Miettinen, 1998): [link](https://nike.box.com/s/h96xzkvbnl0qp6613pvskkvpidy7lk56)

(S.F. Tantawy, 2007): [link](https://nike.box.com/s/2m6fk1q1q7j0vqvibbpi9ioib51fdr4u)