A Note on Bhattacharyya distance

By Danny Butvinik, 7/1/2025

In statistics, machine learning, and data science, Bhattacharyya distance measures the similarity of two probability distributions. It is closely related to the Bhattacharyya coefficient, which measures the overlap between two statistical samples or populations.

Both measures are named after Anil Kumar Bhattacharya, a statistician who worked in the 1930s at the Indian Statistical Institute.

Bhattacharyya distance, a divergence measure, finds extensive applications in artificial intelligence. This distance metric plays a pivotal role in various AI domains, such as feature extraction and selection research, image processing, speaker recognition, and phone clustering. Notably, researchers have even proposed a unique concept called "Bhattacharyya space," which serves as a technique for texture segmentation. This innovative approach aids in identifying significant features, further enhancing the effectiveness of Bhattacharyya distance in AI applications.

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The Mahalanobis distance used in Fisher's Linear Discriminant Analysis is a particular case of the Bhattacharyya Distance.

The Bhattacharyya coefficient is an approximate measurement of the overlap between two statistical samples. The coefficient can determine the relative closeness of the two samples being considered. Calculating the Bhattacharyya coefficient involves a rudimentary integration of the overlap of the two samples.

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The main difference between the two is that Bhattacharyya is a metric and KL is not, so you must consider what information you want to extract about your data points.

In the context of control theory and the study of the problem of signal selection, the Bhattacharyya distance is superior to the Kullback–Leibler distance.

A diagram of equations and formulas

Description automatically generated

# References

[1] <https://en.wikipedia.org/wiki/Bhattacharyya_distance>

[2] <https://en.wikipedia.org/wiki/Hellinger_distance>

[3] <https://en.wikipedia.org/wiki/Kullback%E2%80%93Leibler_divergence>