# Notes on Gaussian Models

compiled by D.Gueorguiev 4/4/2024

## Introductory Notes

Gaussian models are probabilistic models which involve the multivariate Gaussian (Normal) distribution usually denoted with MVN.

### Notation

*Matrices* will be denoted with uppercase bold letters, e.g. .

*Matrix entries* will be denoted by subscripted uppercase non-bold letters, e.g.

*Vectors* are assumed to be column vectors unless stated otherwise.

denotes column vector created by stacking scalars.

Similarly, will denote column vector created by concatenating vectors. Alternatively we can write:

where the notation denotes dimensional row vector formed by the scalars .

The pdf for MVN in dimensions is defined as follows:

(1)

A diagram of a red circle with arrows

Description automatically generated

Figure 1: Visualization of 2D Gaussian density. The major and minor axes of the ellipse are defined by the first two eigenvectors of the covariance matrix, namely and . See Chapter 2 of [2] for details.

## Appendix

### Mahalanobis distance

Mahalanobis distance is a measure of the distance between point and a distribution . It was introduced by P.C. Mahalanobis in 1936 (see [4]).

The Mahalanobis distance is a multivariate generalization of the *standard score* : how many standard deviations away is from the mean of . This distance is zero for at the mean of and grows as moves away from the mean along each *principal component axis*. If each of these axes is rescaled to have unit variance, then the Mahalanobis distance corresponds to standard Euclidean distance in the transformed space. Thus, the Mahalanobis distance is unitless, scale-invariant and accounts for correlations in the data set.

**Definition** *Mahalanobis distance*

Let us have a probability distribution on , generating random vectors of the form where each is a random variable. Let us denote the mean of this distribution with . Also associated with Q is the positive definite covariance matrix , where . Then the Mahalanobis distance of a point from is

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Given two points and in , the Mahalanobis distance between them with respect to is:

## References

[1] [Chapter 4 of Machine Learning - A Probabilistic Perspective, K. Murphy, 2012](https://github.com/dimitarpg13/probabilistic_machine_learning/blob/main/books/MachineLearningProbabilisticPerspective.pdf)

[2] [Pattern Recognition and Machine Learning, Christopher M. Bishop, 2006](https://github.com/dimitarpg13/probabilistic_machine_learning/blob/main/books/Bishop-Pattern-Recognition-and-Machine-Learning-2006.pdf)

[3] [Mahalanobis distance, Wikipedia](https://en.wikipedia.org/wiki/Mahalanobis_distance)

[4] [On the Generalized Distance in Statistics, P.C. Mahalanobis, 1936](https://github.com/dimitarpg13/probabilistic_machine_learning/blob/main/articles/On_The_Generalized_Distance_In_Statistics_Mahalanobis_vol02_1936.pdf)

[5] [Introduction to Gaussian Processes and Gaussian Process Regression, Nando de Freitas, CBSC 540, UBC Jan 31, 2013](https://youtu.be/4vGiHC35j9s)

[6] [Active Learning with Gaussian Processes, Nando de Freitas, BCSC 540, UBC Feb 05, 2013](https://youtu.be/MfHKW5z-OOA)