# Discrete Time Markov Chains

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Introduction

**Definition**: *Discrete Time Markov Chain (DTMC)*

For a countable set , a discrete valued random sequence is called if for all , all states and any historical event the process satisfies the Markov property

The probability of a DTMC being in state at time from a state at time , is determined by the *transition probability* denoted by

The set is denoted as *the state space* of the Markov chain. The *transition probability matrix* at time is denoted by , such that . We observe that each row is the conditional distribution of given .

For all states , a matrix with non-negative entries is called *sub-stochastic* if the row-sum for all rows . If the above property holds with equality for all rows, then it is denoted as *stochastic matrix*. If matrices and are both stochastic, then the matrix A is called *doubly stochastic*.

i ) all of the entries of a sub-stochastic matrix lie in .

ii ) Each row of the stochastic matrix is probability mass function over the state space .

iii ) Every finite stochastic matrix has a right eigenvector with unit eigenvalue. This can be observed by taking to be an all-one vector of length . Then we see that , since for each

iv ) Every probability transition matrix is a stochastic matrix.

v ) Every finite doubly stochastic matrix has a left and right eigenvector with unit eigenvalue. This follows from the fact that finite stochastic matrices and have a common right eigenvector . It follows that A has a left eigenvector.