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Featured review of books on optimal control

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"*Because the shape of the whole universe is most perfect and, in fact, designed by the wisest Creator, nothing in all of the world will occur in which no maximum or minimum rule is somehow shining forth*". *Leohard Euler, 1744.*

Some of the books that are published in or after 1995 and considered for this short featured review on optimal control are (in chronological order)

1. Bertsekas DP. *Dyanamic Programming and Optimal Control*: *vol. I*. Athena Scientific: Belmont, MA, 1995.
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14. Burl JB. *Linear Optimal Control*: *H_2 and H_∞ Methods*. Addison-Wesley Longman: Menlo Park, CA, 1999.
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1. HISTORICAL TOUR

We will basically consider two stages of the tour: of the development of, first, calculus of variations, and, secondly, optimal control theory [1, 2].

1.1. Calculus of variations

A legend about the founding of Carthage shows how a Tyrian princess Dido used the shape of a circular arc to maximize the area of land enclosed by a rope of knotted cowhide. Although the story is legendary, it is probably the first account of a problem of the kind that inspired an entire mathematical discipline, the *calculus of variations* and its extensions such as optimal control theory [2].

The calculus of variations is that branch of mathematics which deals with finding a function which is an extremum (maximum or minimum) of a functional. A functional is loosely defined as a function of a function. The theory of finding maxima and minima of functions is quite old and can be traced back to the isoperimetric problems considered by Greek mathematicians such as Zenodorus (495–435 BC) and by Poppus (ca. 300 AD). But we will start with the works of Fermat in 1662 who discovered that light ray travels from one optical medium to another in the *minimum* possible time. In 1699 Johannes Bernoulli (1667–1748) challenged the mathematical world to solve the brachistochrone problem: *the problem of finding the path of quickest descent between two points not in the same horizontal or vertical line*. This problem which was first posed by Galileo (1564–1642) in 1638, was solved by John, his brother Jacob (1654–1705), by Gottfried Leibniz (1646–1716), and anonymously by Isaac Newton (1642–1727). Leonard Euler (1707–1783) joined John Bernoulli and made some remarkable contributions, which influenced Joseph–Louis Lagrange (1736–1813), who finally gave an elegant way of solving these type of problems by using the method of (*first*) variations. This led Euler to coin the phrase *calculus of variations*. Later this *necessary* condition for extrema of a functional was called Euler–Lagrange equation. Lagrange went on to treat variable end-point problems introducing multiplier method, which later became one of the most powerful tools-Lagrange (or Euler–Lagrange) multiplier method-in optimization.

The *sufficient* conditions for finding the extrema of functionals in calculus of variations was given by Andrien–Marie Legendre (1752–1833) in 1786, by considering additionally the *second* variation. His analysis drew harsh criticism from Lagrange. Then it was for Carl Gustav Jacob Jacobi (1804–1851) in 1836 to come up with a more rigorous analysis of the sufficient conditions, where he showed the partial derivatives with respect to each parameter of a family of extremals satisfy the Jacobi differential equation. This sufficient condition was later on called the Legendre–Jacobi condition. At about the same time Sir William Rowan Hamilton (1788–1856) did some remarkable work on mechanics, by showing that the motion of a particle in space, acted upon by various external forces, could be represented by a single function which satisfies *two* first-order partial differential equations. In 1838 Jacobi had some objections to this work and showed that the need for only *one* partial differential equation. This equation, called Hamilton–Jacobi equation, later had profound influence on calculus of variations, and as well as on mechanics.

At this point of time, there were two directions. First, although the problem of extrema of functionals seemed to have been solved, the distinction between *strong* and *weak* extrema created problems for the researchers. It was for Karl Weierstrass (1815–1897) to come up with the idea of the field of extremals and give the Weierstrass condition, and sufficient conditions for weak and strong extrema.

In the second direction, Rudolph Clebsch (1833–1872) and Adolph Mayer proceeded with establishing conditions for the more general class of problems. Clebsch formulated a problem in the calculus of variations by adjoining the constraint conditions in the form of differential equations. Recasting earlier problem of Legendre to fit into his framework of a general problem, Clebsch gave the Clebsch condition based on second variation. In 1868 Mayer reconsidered Clebsch's work and gave some elegant results for the general problem in the calculus of variations. This happened more or less at the same time of Weierstrass work. Later Mayer described in detail the problems: the problem of Lagrange in 1878, and the problem of Mayer in 1895.

In 1898, Adolf Kneser gave a new approach to the calculus of variations by using the result of Karl Gauss (1777–1855) on geodesics. For variable end-point problems, he established the transversality condition which includes orthogonality as a special case. He along with Bolza gave sufficiently proofs for these problems. In 1900, at an international mathematical congress, David Hilbert (1862–1943) gave a profound discussion on the calculus of variations. One of his greatest contributions is his perception of the second variation as a quadratic functional with eigenvalues and eigenfunctions. Between 1908 and 1910, Gilbert Bliss and Max Mason looked in depth the results of Kneser.

In 1911, Hahn gave an important theorem establishing the sufficient conditions for variable end-point problems. Caratheodory in 1904 reconsidered the brachistochrone problem, introduced the idea of direction of steepest descent and gave some elegant look at the usual results of the calculus of variations. In 1913 Bolza formulated what Bliss called the problem of Bolza as a generalization of the problems of Lagrange and Mayer.

1.2. Optimal control theory

There is no way we can have a tour of optimal control theory without ploughing through that of classical control theory. Without going into complete development of control theory, we may safely mention that J. C. Maxwell was the first to give a mathematical treatment of a feedback control system in 1868. According to Friedland [3], we may label the period before 1868 as *prehistoric* period of automatic control; from 1868 to early 1900s the *primitive* period. Usually, the control theory developed between 1900 and 1960s is the *classical* control, and from 1960s to the present times is the so-called *modern* control era. Again, people divide *modern* control into two periods: *state-variable* or time-domain period (1960–1975), and *frequency-domain* period (1975 to present) [4].

Briefly, we account for the development of control as follows [5]. An indirect way of determining the stability of a characteristic equation was provided by Routh [6]. The stability of nonlinear systems was greatly influenced by the work of Lyapunov [7]. The *proportional-integral-derivative* (*PID*) controller was first used in steering ships by N. Minorsky in 1922. The idea of negative feedback was introduced by Black in 1927 [8]. The stability of amplifiers using regenerative theory was discussed by Nyquist [9], leading to his famous *Nyquist stability criterion* based on mapping principle of complex variables. Bode in 1938 introduced the magnitude and phase plots in frequency domain [10]. During 1940s (World War II), the MIT Radiation Laboratory contributed vastly to the development of control theory. Here in 1947, Nichols was responsible for developing the Nichols chart for the design of feedback control systems [11]. The root locus was invented by Evans in 1948 [12].

Let us now get back to optimization and optimal control theory [4, 5]. The linear quadratic control problem has its genesis in the celebrated work of Wiener on mean-square filtering for weapon fire control during World War II (1940–1945) and the results were published soon after [13, 14]. Using spectral factorization approach, Wiener solved the problem of designing filters that minimize a mean-square-error criterion (performance measure). For a deterministic case, the error criterion is generalized as an integral quadratic term. In 1957, Bellman [15] introduced the technique of *dynamic programming* to solve discrete-time optimal control problems. But, the most important contribution to optimal control is made in 1958 by Pontryagin (of formerly USSR) by the development of his celebrated *maximum principle* [16]. At this time in United States, Kalman [17] provided *linear quadratic regulator* (*LQR*) and *linear quadratic Gaussian* (*LQG*) theory to design optimal feedback controls. He went on to present optimal filtering and estimation theory leading to his famous *discrete Kalman filter* [18]. The *continuous Kalman filter* was developed by Kalman in association with Bucy [19]. Kalman had a profound effect on optimal control theory and the Kalman filter is one of the most widely used technique in applications of control theory to real-world problems in a variety of fields.

At this point we have to mention about the *matrix Riccati equation* that appears in all the Kalman filtering techniques, and other fields. Riccati [20, 21] published his result in 1724 on the solution for some type of nonlinear differential equations, without ever knowing that the Riccati equation would become so famous after nearly two centuries! Thus, it is evident that the optimal control was born in 1697, more than 300 years ago, in Groningen, The Netherlands, when Johann Bernoulli, published his solution of the *brachistochrone problem* [22]. For additional details about the historical perspectives on calculus of variations and optimal control, the reader is referred to some excellent publications [1, 2, 22–25].

In the so-called *linear quadratic control*, the term 'linear' refers to the plant being *linear* and the term 'quadratic' refers to the performance index that involve the *square* or *quadratic* of an error, and/or control. Originally, this problem was called *mean-square* control problem and the term 'linear quadratic' did not appear in the literature until late 1950s.

As we have seen that basically the *classical* control theory using *frequency* domain deals with single-input and single-output (SISO) systems, whereas *modern* control theory works with *time* domain for SISO and multi-input and multi-output (MIMO) systems. Although modern control and hence optimal control appeared to be very attractive, it lacked a very important feature of *robustness*. That is, controllers designed based on LQR theory failed to be robust to disturbances, unmodelled dynamics, and measurement noise. Whereas, frequency-domain techniques using the ideas of gain margin and phase margin offer robustness in a natural way. Thus, some researchers [26, 27], especially in United Kingdom, continued to work on developing frequency-domain approaches to MIMO systems.

One important and relevant field that has been developed recently around 1980s is the $\mathscr{H}_\infty$-optimal control theory. In this framework the work developed in 1960s and 1970s is labeled as $\mathscr{H}_2$-optimal control theory. The seeds for $\mathscr{H}_\infty$-optimal control theory were laid by Zames [28], who formulated the optimal $\mathscr{H}_\infty$-sensitivity design problem for SISO systems and solved using optimal Nevanilina–Pick interpolation theory. An important publication in this field came from a group of four active researchers Doyle *et al.* [29] who won the 1991 W. R. G. Baker Award as the best IEEE Transactions paper. There are other books in the related field [30–42].

2. BOOKS UNDER REVIEW

There were a good number of books published around 1960s and 1970s during the era of the 'glory of modern control' (listed in chronological order) (Athans and Falb (1966) [43], Lee and Markus (1967) [44], Sage (1968) [45], Pierre (1969) [46], Kirk (1970) [47], Anderson and Moore (1971) [48], Kwakernaak and Sivan (1972) [49], Bryson and Ho (1975) [50]), Jacobson (1977) [51], Sage and White (1977) [52]), Anderson and Moore (1979) [53]. There has been renewed interest with the second wave of books published during 1980s and 1990s (Lewis (1986) [54, 55], Stengal (1986) [56], Grimble and Johnson (1988) [57, 58] Anderson and Moore (1990) [59], Hocking (1991) [60], Teo *et al.* (1991) [61], Gregory and Lin (1992) [62], Lewis (1992) [5], Pinch (1993) [63], Dorato *et al.* (1995) [4], Lewis and Syrmos (1995) [64]), Saberi *et al.* (1995) [65], Siouris (1996) [66], Vincent and Grantham (1997) [67] and Bryson (1999) [68]), Burl (1999) [69], Kolosov (1999) [70].

However, the present featured review focuses on some of the books published in or after 1995 and listed at the beginning of this review. It is to be noted that this is not intended to be an exhaust review of each book.

2.1. Bertsekas, volumes I and II: 1995

This two-volume set on the *dynamic programming* and *optimal control* provides a modern flavor to this twin areas of optimization. The first volume has the following chapters.

1. The Dynamic Programming Algorithm.
2. Deterministic Systems and the Shortest Path Problem.
3. Deterministic Continuous-Time Optimal Control.
4. Problems with Perfect State Information.
5. Problems with Imperfect State Information.
6. Suboptimal and Adaptive Control.
7. Introduction to Infinite Horizon Problems.

with the following appendices on:

(a) Mathematical Review.
(b) On Optimization Theory.
(c) On Probability Theory.
(d) On Finite-State Markov Chains.
(e) Kalman Filtering.
(f) Modeling of Stochastic Linear Systems.

The second volume consists of the following topics:

1. Infinite Horizon—Discounted Problems.
2. Stochastic Shortest Path Problems.
3. Undiscounted Problems.
4. Average Cost per Stage Problems.
5. Continuous-Time Problems.

A distinguishing feature of this set is 'a fairly extensive exposition of simulation-based approximation techniques for dynamic programming'. These techniques, which are often referred to as 'neuro-dynamic programming' or 'reinforcement learning', represent a break-through in the practical application of dynamic programming to complex problems that involve the dual curse of large dimension and lack of an accurate mathematical model'. This set is best suitable for a first-year graduate course for students from engineering, operations research, economics and applied mathematics. This set also comes with solutions manuals which make the set ideal for class room adaptation.

2.2. *Dorato et al., 1995*

This books provides an introduction to linear quadratic theory in terms of classical LQR and LQG topics and also provides a bridge to loop transfer recovery (LTR) and a bried survey of H_∞ methods. The book is composed of the following topics after an introduction:

1. The LQR Problem.
2. Tracking and Disturbance Rejection.
3. LQR Robustness Properties.
4. Stochastic Control.
5. The LQG Problem.
6. Robustness Design.
7. Digital Control.

The writing style is particularly easy without lots of theorems and proofs. The interleaving of MATLAB® throughout the book is a very attractive feature for both graduate students and practicing engineers.

2.3. *Lewis and Syrmos, 1995*

This is perhaps the most popular book being presently used in many universities for a first-year graduate course on optimal control. This is a second edition of the earlier book by the first author. This book has the following chapters.

1. Static Optimization.
2. Optimal Control of Discrete-Time Systems.
3. Optimal Control of Continuous-Time Systems.
4. The Tracking Problem and other LQR Extensions.
5. Final-Time-Free and Constrained Input Control.
6. Dynamic Programming.
7. Optimal Control for Polynomial Systems.
8. Output Feedback and Structured Control.
9. Robustness and Multivariable Frequency-Domain Techniques.

This book has good collection of problems at the end of each chapter with focus on applications to aerospace and robotics areas.

2.4. *Li and Yong, 1995*

While most of the books on optimal control focus on *finite-dimensional* systems described by ordinary differential and/or difference equations, this book focuses on *infinite-dimensional* systems. The optimal control of infinite-dimensional systems is important because it has a wide range of applications in different disciplines of engineering, economics and other fields, and many physical phenomena in the real-world situations such as heat conduction, vibration of elastic material, diffusion-reaction processes, population systems and many others are described by infinite-dimensional systems. In summary the book provides the existence theory for optimal control, the Pontryagin maximum principle, the Bellman dynamic programming and the linear quadratic optimal control problems for infinite-dimensional systems. The book has the following chapters:

1. Problems in Infinite Dimensions.
2. Mathematical Preliminaries dealing with the topics of functional analysis, Banach spaces, theory of C_0 semigroups, evolution equations and elliptic partial differential equations.
3. Existence Theory of Optimal Control.
4. Necessary Conditions for Optimal Controls—Abstract Evolution Equations.
5. Necessary Conditions for Optimal Controls—Elliptic Partial Differential Equations.
6. Dynamic Programming Method for Evolution Systems.
7. Controllability and Time Optimal Control.
8. Optimal Switching and Impulse Controls.
9. Linear Quadratic Optimal Control Problems.

This book, although does not have typical chapter-end-problems feature, is well suited for a second-level graduate course in optimal control for graduate students and as a reference text for other researchers.

2.5. Saberi, Sannuti and Chen, 1995

This book 'presents a state-of-the-art view of H_2-optimal control theory and various design methods'. The book treats simultaneously both continuous- and discrete-time cases and aims at a higher level of rigor focusing on multivariable control theory. The books is composed of the following chapters:

1. Introduction.
2. Statements of H_2-Optimal and Suboptimal Control Problems.
3. A Special Coordinate Basis (SCB) of Linear Multivariable Systems.
4. Algebraic Riccati Equations, Linear Matrix Inequalities, and Quadratic Matrix Inequalities.
5. Infima, Existence, and Uniqueness Conditions—Continuous-time Systems.
6. Infima, Existence, and Uniqueness Conditions—Discrete-time Systems.
7. H_2-Optimal State Feedback Controllers—Continuous-time Systems.
8. H_2-Optimal State Feedback Controllers—Discrete-time Systems.
9. H_2-Optimal Measurement Feedback Controllers—Continuous-time Systems.
10. H_2-Optimal Measurement Feedback Controllers—Discrete-time Systems.
11. H_2-Suboptimal State and Measurement Feedback Control for Continuous-time and Discrete-time Systems.

This book is suitable for students in graduate courses in disciplines of electrical engineering, mechanical engineering, chemical engineering, aerospace engineering and applied mathematics.

2.6. Siouris, 1996

In the field of optimal control, most of the books focus on theorem and proof mode, the author of this book states that this book ‘provides a practical and accessible guide, focusing on applications and implementation, and answering real-world questions faced by control engineers’. This book is aimed at graduate level students in electrical and mechanical engineering interested in aerospace and many other applications. The various topics addressed in the book are:

1. Linear Regression: Least-Squares and Maximum-Likelihood Estimation.
2. The Kalman Filter.
3. Linear Regulators.
4. Covariance Analysis and Suboptimal Filtering.
5. The α–β–γ Tracking Filters.
6. Decentralized Kalman Filters.

2.7. Troutman, 1996

This second edition of the book presents the closely related twin areas of calculus of variations and optimal control with a high dose of *math* flavour. This book ‘combines rudiments of analysis in (normed) linear spaces with simpler aspects of convexity to offer a multilevel strategy for handling such problems. It also employs convexity considerations to broaden the discussion of Hamiltonian’s principle in mechanics and to introduce Pontryagin’s principle in optimal control’. The book has three parts dealing with basic theory, advanced topics and optimal control.

Part 1. *Basic Theory*:

1. Standard Optimization Problems.
2. Linear Spaces and Gâteaux Variations.
3. Minimization of Convex Functions.
4. The Lemmas of Lagrange and Du Bois–Reymond Problems.
5. Local Extrema in Normed Linear Spaces.
6. The Euler–Lagrange Equations.

Part 2. *Advanced Topics*:

7. Piecewise C^1 Extremal Functions.
8. Variational Principles in Mechanics.
9. Sufficient Conditions for a Minimum.

Part 3. *Optimal Control*:

10. Control Problems with Sufficiency Considerations.
11. Necessary Conditions for Optimality.

This book is aimed at upper level undergraduates mathematics and graduates in engineering.

2.8. Whittle, 1996

This book covers some *basic* topics and in optimal control such as the optimal control of deterministic and stochastic systems and then goes *beyond* in presenting risk-sensitive and

H_∞ criteria, time-integral methods, optimal stationary policies and near determinism and large deviation theory. Essential topics in this book are

1. Deterministic Models covering linear quadratic (LQ) case and Pontryagin maximum principle.
2. Stochastic Models including dynamic programming, linear quadratic Gaussian (LQG) method and imperfect state observation.
3. Risk-Sensitive and H_∞ Criteria.
4. Time Integral Methods and Optimal Stationary Policies.
5. Near-Determinism and Large Deviation Theory.

Thus this book provides a refreshing flavour to optimal control area. This book provides an essential reading for students and researchers in the area of optimal control theory. Interested readers should also consult a related book by the same author [71].

2.9. Polak, 1997

This book essentially 'covers algorithms and discretization procedures for the solution of nonlinear programming, semi-infinite optimization, and optimal control problems and presents first rigorous treatment of implementation optimization algorithms for optimal control problems with state-trajectory and control constraints'. This book has the following topics:

1. Unconstrained Optimization covering optimality conditions, algorithm models and convergence conditions, gradient methods, Newton's method, method of conjugate directions, quasi-Newton methods, one-dimensional optimization and Newton's method for equations and inequalities.
2. Finite Min–Max and Constrained Optimization including optimality conditions for mini–max and unconstrained optimization, first-order mini–max algorithms, Newton's method, penalty function algorithms, augmented Lagrangian methods and sequential quadratic programming.
3. Semi-Infinite Optimization focusing on optimality conditions, theory of consistent approximations, and algorithms.
4. Optimal Control focusing on optimality conditions, unconstrained optimal control, mini–max algorithms, and algorithms for state constraints.
5. Mathematical Background summarizing results from functional analysis, convex sets and convex functions, set-valued functions, max functions, minimax theorems and differential equations.

This book is useful to graduate students in engineering, mathematics and economics and other scientists and engineers working in the optimization field.

2.10. Vincent and Grantham, 1997

This book 'provides a solid introduction to the analysis techniques used in the design of nonlinear and optimal feedback control systems'. The various topics discussed in this book are

1. Nonlinear Dynamical Systems focusing on numerical solution algorithms, nonlinear system phenomena and classical analysis techniques.

2. Nonlinear Control Systems covering controllability, domain of attraction boundaries and chaotic systems.
3. Nonlinear Optimization with constrained function minimization, necessary conditions for a local minimum, mini–max parametric games, and calculus of variations.
4. Lyapunov Stability including first and second methods and construction of Lyapunov functions and asymptotic stability.
5. Lyapunov Control Systems Design.
6. Controllability of Nonlinear Systems including controllability minimum principle.
7. Optimal Control Systems focusing on Pontryagin's minimum principle and its applications, dynamic programming and state constraints.
8. Optimal Control Design describing linear quadratic regulator problem, and nonlinear optimal control systems.
9. Differential Games both qualitative and quantitative and missile guidance.

This book features examples and exercises from a wide range of disciplines in biological, economic and other systems.

2.11. Aliev and Larin, 1998

The authors of this monograph present 'analytical methods for synthesis of linear stationary and periodical optimal control systems and create effective computational algorithms for synthesis of optimal regulators and filters' using Youla–Jabr–Bongiorni and Desoer–Liu–Murray–Saeks procedures. This book consists of the following topics:

1. Linear Quadratic Problem covering both regular and singular linear quadratic Gaussian problems, linear quadratic problem for periodic system.
2. Method of H_2-Optimization (Frequency Method of Analysis) including parameterization of transfer function, synthesis and optimization of systems with several degrees of freedom.
3. Computing Algorithms of the State Space Model describing the construction of the solution of the Lyapunov equation, numerical methods for solving algebraic Riccati equation (ARE) and Gramian block diagonalization.
4. Computing Algorithms of the Frequency Method of Synthesis focusing on matrix polynomial factorization and the related algorithms.

This book is aimed at graduate students in engineering and applied mathematics interested in optimal control and its algorithms.

2.12. Bryson, 1999

This book, *dynamic optimization*, 'the process of determining control and state histories for a dynamic system over a finite time period to minimize a performance index' using calculus of variations and dynamic programming, updates and extends the first half of the classic text *Applied Optimal Control* by Bryson and Ho (1975) [50].

The book is composed of the following topics:

1. Static Optimization.
2. Dynamic Optimization.
3. Dynamic Optimization with Terminal Constraints.
4. Dynamic Optimization with Open Final Time.

5. Linear Quadratic Terminal Constraints.
6. Linear Quadratic Regulators.
7. Dynamic Programming.
8. Neighbouring Optimal Control.
9. Inequality Constraints.
10. Singular Optimal Control Problems.

A special feature of this book is the presentation of computational algorithms which are coded in MATLAB®. This book with more examples and problems than its predecessor [50], has focused on aerospace applications, and is ideally suitable for a graduate course on optimal control for students in various disciplines of engineering and mathematics.

2.13. Burl, 1999

This is a 'reader-friendly book that features recent research results on robustness, H_∞-control, and μ-synthesis and covers the analysis of control systems, H_2 (linear quadratic Gaussian), and H_∞ …'. The book consists of three parts: analysis, H_2- and H_∞-control. The composition of the book is as follows.

Part 1. *Analysis of Control Systems*:

1. Multivariable Linear Systems covering frequency response, stability and controllability and observability.
2. Vector Random Processes including white noise and coloured noise.
3. Performance in terms of transient, tracking, disturbance rejection and cost functions.
4. Robustness covering Nyquist stability, structured and unstructured uncertainty, and structured singular values.

Part 2. *H_2-Control*:

5. The Linear Quadratic Regulator.
6. The Kalman Filter.
7. Linear Quadratic Gaussian Control.

Part 3. *H_∞-Control*:

8. Full Information Control and Estimation.
9. H_∞ Output Feedback.
10. Controller Order Reduction.

One of the special features of this book is a good collection of exercises at the end of each chapter including a special section on *computer exercises* to be solved using the industry standard MATLAB® and hence its suitability for course adaptation. This book is suitable for graduate students in electrical, mechanical, chemical and aerospace engineering.

2.14. Kolosov, 1999

This book covers the 'design methods for optimal (or quasioptimal) control algorithms in the form of synthesis for deterministic and stochastic dynamical systems—with applications to biological, radio engineering, mechanical and servomechanical technologies' and discusses analytical, approximate, and numerical techniques for solving optimal control problems. The book discusses the following topics:

1. Synthesis Problems for Control Systems and the Dynamic Programming Approach.
2. Exact Methods for Synthesis Problems such as linear quadratic and optimal tracking problems.
3. Approximate Synthesis of Stochastic Control Systems With Small Control Actions.
4. Synthesis of Quasioptimal Systems in the Case of Small Diffusion Terms in the Bellman Equation.
5. Control of Oscillatory Systems.
6. Some Special Applications of Asymptotic Synthesis Methods.
7. Numerical Synthesis Methods.

This book is suitable as a reference book for applied mathematicians; analysts; control, automation, electrical, and mechanical engineers; physists and biologists.

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