Notes on Stochastic Optimization Problems Formulation, Warren Powell

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# Formulating a Stochastic Optimization Problem

We will consider the ubiquitous inventory problem as an example and will study deterministic and stochastic formulations of the problem.

This requires the introduction of the notion of *policy* (or *control law*) which is a method for making decisions.

## Deterministic inventory problem

We want to solve a simple inventory in which we have to decide how much to order, , at time . We are going to assume that when we order , the items cannot be used until time . Let be the cost of items ordered in period (which can vary from one time period to the next), and assume we are paid a price when we satisfy the demand given by . Let be the sales at time , which is limited by the demand , and the available product which is our inventory plus our orders , so we can write

(1)

(2)

(3)

(4)

We assume that unsatisfied demand is lost. The left-over inventory is:

(5)

We set .

Now we formulate our deterministic optimization problem as

(6)

subject to the constraints (1) – (6). The solution is in the form of the vector of production and sales decisions . It becomes clear that we need to solve the inventory problem over the entire horizon to make the best decision now. For example, if we specified in advance , then this could easily change what we do now, , at time .

# The transition to a stochastic formulation

Let us consider random demand and random inventory . This means that the order quantity is random, as is the sales . Given this, the optimization problem (6) simply does not make sense as performing optimization over a set of random variables is not meaningful concept.

We fix this deficiency by replacing the decisions with a function, known as *policy* (or *control law*)

where is our state variable,

# References

[1] [Reinforcement Learning and Stochastic Optimization: A Unified Framework for Sequential Decisions, Warren Powel, 2019](https://github.com/dimitarpg13/reinforcement_learning_and_game_theory/blob/main/books/Powell-Reinforcement-Learning-and-Stochastic-Optimization.pdf)