Notes on Reinforcement Learning and Deep Reinforcement Learning

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# Introductory Notes

What is Reinforcement Learning: branch of machine learning concerned with making decisions and taking sequences of actions based on some current state thereby maximizing some reward objective over time.

action

Environment

Agent

state, reward

Figure 1: Feedback loop between the Agent and the Environment in RL

The Agent and the Environment interact with each other on discrete timesteps creating a feedback loop depicted in Figure 1. The Agent has a goal of maximizing the cumulative reward while interacting with the Environment.

Observations in RL:

Robotics: camera images, joint angles

Actions in RL:

Robotics: joint torques

Rewards in RL:

Robotics: stay balanced, navigate to target locations

Approaches to RL

Dynamic Programming

Policy Optimization

Value Iteration

Policy Iteration

Policy Gradients

DFO / Evolution

Q Learning

Actor-Critic Methods

Two approaches to RL – the first approach is to optimize policy and the second one is dynamic programming.

Policy is the function which takes the observations with the state of the system and outputs actions. The Policy Optimization approach looks at the RL problem as an optimization problem trying to optimize the expected reward , there are parameters in the policy, and we want to find such set of parameters which maximizes the expectation of the stochastic reward. Posing the problem as an optimization problem ignores all of the structure of the problem conveyed through the Bellman’s equations. We are getting a noisy estimate of how good each parameter is and try to move toward that part of the parameter space where we are getting better performance – that is, higher expected reward. So, this is how the Derivative Free Optimization (DFO) methods and Evolutionary algorithms work – they work as a black box which takes a policy parameter vector and outputs a noisy performance number. These methods are very simple to implement, and they work surprisingly well. The other approach for Policy Optimization is by using Policy Gradient methods trying to measure the gradient of the performance with respect to the parameter vector of the policy. These second type of Policy Optimization methods scale better with respect to the number of parameters. Dynamic Programming / Approximate Dynamic Programming is a very different approach to solving RL problems. In certain cases, we can solve control problems exactly. What if we have slightly different parameter settings, will these algorithms still work? It turns out we can modify these algorithms in certain ways which can keep them valid. Policy Iteration and Value Iteration are the two algorithms which will exactly solve the MDP formulation of the RL problem and finding the optimal policy, but they only work if you have discrete state space and action space. If these spaces are finite sets, we can solve exactly the RL problem. In many real world problems, we need to do approximate versions of these algorithms. There is a dedicated field developing approximate dynamic programming algorithms for those real world problems which cannot be solved exactly. Q-Learning is one quite popular and successful method in this category. We can do Q Learning with function approximation performed by Neural network. Lastly, there are Actor-Critic methods – they are policy gradient methods, but they also use value functions helping the policy gradient method. These methods can scale well and be used to solve large / hard problems.

What is Deep RL?

It is RL using nonlinear function approximators, which do not make a lot of assumptions about the form of the approximated function. At any given time, the algorithm is solving optimization problem with gradient descent.

Markov Decision Process (MDP)

MDP is defined by the triplet where

is the state space

is the action space

is the transition probability distribution

tells us the probability of the reward (scalar) , the next state (vector) given the current state (vector) and the action (vector).

Extra objects can be defined depending on the problem setting:

- initial state distribution

– discount factor

In each episode, the initial state is sampled from , and the process proceeds until terminal state is reached.

In the episodic setting the agent experiences are broken up into a sequence of episodes. In each episode a reward and a new state are generated from the old state after action is chosen. And this process continuous for each episode sequentially until we reach a terminal state. The terminal state is a special state with 0 reward from which there is no continuation. We can have different termination semantics – termination can indicate a good outcome (example: taxi robot reaches its destination), termination is neither good nor bad and always occurs (waiter robot finishes its shift after fixed amount of time), termination indicates bad outcome (walking robot fails over).

We want to maximize the expected reward per episode.

We consider two kinds of policies:

deterministic policies

stochastic policies – in this case the policy defines a conditional probability distribution over actions.

The policy will be the optimized function in our optimization problem.

Parametrized policy

there is some parameter vector which indexes over the policy.

So based on this discussion we can write:

– terminal state

Objective:

maximize , where

//TODO: finish this section (Lecture 1 of John Schulman)

# Intro to Policy Optimization

We consider the case of stochastic parametrized policy . We want to maximize the expected return . For the purposes of this derivation, we will take to give the finite horizon undiscounted return, but the derivation for the infinite horizon discounted return setting is almost identical.

We would like to optimize the policy by gradient ascent as

The gradient of policy performance, , is called policy gradient, and algorithms that optimize the policy this way are called *policy gradient algorithms*. Examples of such algorithms include Vanilla Policy Gradient and TRPO. PPO is often referred to as a policy gradient algorithm, though this is slightly inaccurate as PPO uses value functions to obtain better policy approximation.

To actually use this algorithm, we need an expression for the policy gradient which we can numerically compute. This involves two steps : 1) deriving the analytical gradient of policy performance, which turns out to have the form of an expected value, which can be computed with data from a finite number of agent-environment interaction steps.

We begin with few definitions and statements used in the derivation of policy gradient

**Definition** *Probability of a Trajectory*

The probability of a trajectory given that actions come from is

**Definition** *The Log-Derivative Trick*

**Definition** *Log-Probability of a Trajectory*

The log-prob of a trajectory is just

**Statement**: *The gradients of environment functions are zero*. This is because the environment has no dependence on , so gradients of , and are zero.

Then the expression of the Grad-Log-Prob of a trajectory is given with

.

Putting it all together, we derive the following:

Derivation for Basic Policy Gradient

(via the Log derivative trick)

(rewritten as expectation)

(expression for grad-log-prob)

This is an expectation, which means that we can estimate it with a sample mean. If we collect a set of trajectories where each trajectory is obtained by letting the agent act in the environment using the policy , the policy gradient can be estimated with

,

where is the number of trajectories in (here, ).

This last expression is the simplest version of the computable expression we desired. Assuming that we have represented our policy in a way which allows us to calculate , and if we are able to run the policy in the environment to collect the trajectory dataset, we can compute the policy gradient and take an update step.

**Lemma** *EGLP Lemma*

Expected Grad-Log-Prob is zero.

Suppose that is a parametrized probability distribution over a random variable, . Then:

Proof:

Since is a distribution, we have:

Applying gradient on both sides gives us:

Use the log derivative trick to get:

Let us examine the most recent expression for the policy gradient:

//TODO: finish the section on Policy Optimization (OpenAI Spinning Up Intro to RL part 3)

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# Appendix

## The Reparametrization Trick

## On The Expectation of The Grad Log Prob

The statement "expectation of grad log prob is equal to zero" is generally true for a parameter with respect to which a probability distribution is defined, but not for the sample itself. This is a consequence of the reparameterization trick and the log-derivative trick, which are important tools in probabilistic modeling and reinforcement learning.

Here's a more detailed explanation:

1. The Log-Derivative Trick

The log-derivative trick, also known as the score function, is a fundamental tool in probabilistic modeling.

It states that the gradient of the log probability of a sample with respect to a parameter is equal to the derivative of the log of the probability density function (or mass function).

Mathematically, this can be expressed as: ∇θ log p(x; θ) = ∇θ log p(x | θ).

The expected value of this gradient is zero.

2. Reparameterization Trick

The reparameterization trick is a technique used to make it easier to calculate gradients of stochastic models.

It involves expressing a random variable as a function of a deterministic parameter and a random variable with a known distribution.

When backpropagating through the log probability of a sample, the reparameterization trick can lead to gradients that are zero with respect to the mean parameter, as the mean is canceled out.

3. Example

Consider a normal distribution with mean μ and standard deviation σ.

The log probability of a sample x is: log p(x; μ, σ) = -0.5 \* log(2πσ^2) - 0.5 \* ((x - μ)^2 / σ^2).

The gradient of this log probability with respect to μ is: ∇μ log p(x; μ, σ) = (x - μ) / σ^2.

The expected value of this gradient is: E[∇μ log p(x; μ, σ)] = E[(x - μ) / σ^2] = 0.

4. Implications

The fact that the expected value of the gradient of the log probability is zero is important for training probabilistic models using gradient-based optimization methods.

It means that we can use the log-derivative trick to calculate gradients of stochastic models without having to calculate the expectation directly.

This is particularly useful in reinforcement learning, where we often need to calculate gradients of the log probability of an action given a state, which can be done using the log-derivative trick and the reparameterization trick.

[OpenAI's Spinning Up in Deep RL](https://spinningup.openai.com/en/latest/spinningup/rl_intro3.html) [32] provides a good overview of the policy gradient algorithm, which relies on the concept of the expected value of the gradient of the log probability being zero.