# Preliminaries needed to understand Proximal Policy Optimization Algorithms

Notes on discussion and derivations from Sutton’s book and John Schulman’s articles

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## A bit of theory on Policy Gradient Reinforcement Learning Methods

Assumptions:

The environment can be represented by a finite MDP

This is equivalent of saying that its state , action and reward sets are finite, and its dynamics is given by a set of probabilities , for all and ( is plus a terminal state if the problem is episodic).

We would like to compute value functions to organize and search for good policies.

The optimal value functions satisfying the Bellman’s optimality equations were derived and discussed in (Gueorguiev, 2023) (see Eq. (22) and (23)).

(1)

(2)

### Policy evaluation (Prediction)

#### Policy evaluation for the state values

**Definition**: *policy evaluation* *for state values* - computation of the state-value function for a given policy . This is also known as the *prediction problem for state values*.

Question: How to compute the state-value function for an arbitrary policy .

From (11) and (12) in (Gueorguiev, 2023) we can write:

(3)

(4)

(5)

where is the probability of taking action in state under policy , and the expectations are subscribed by to indicate that they are conditional on being followed.

The existence and uniqueness of are guaranteed as long as either or eventual termination is guaranteed from all states under the policy .

If the environment’s dynamics are completely known, then (5) is a system of simultaneous linear equations in unknowns (the ).

(6)

Clearly, is fixed point for (6) because the Bellman equation for assures equality in this case. We are going to be looking into iterative solution of (5). Indeed, the sequence can be shown to converge to under the same conditions which guarantee the existence of . This algorithm is known as *iterative policy evaluation*.

To produce each successive approximation , the iterative policy evaluation applies the same operation to each state : it replaces the old value of with a new value obtained from the old values of the successor states of , along all the one-step transitions possible under the policy being evaluated. We call this kind of operation an *expected update*. Each iteration of the iterative policy evaluation updates the value of every state once to produce the new approximate value function .

Note: There are several different kinds of expected updates, depending on whether a **state** (as in (5)) or a **state-action pair** is being updated, and, depending on the precise way the estimated values of the successor states are combined.

Note2: All the updates done in the algorithms based on Bellman equations are ***expected updates*** because they are based on the expectation over ***all possible next states*** rather than on a sample next state.

Note3: the nature of the update can be expressed by using backup diagram (backup diagrams were discussed in (Gueorguiev, 2023) and in Chapter 3 of (Richard D. Sutton, 2020)).

In-place Algorithm for iterative policy evaluation:

Input: Policy to be evaluated, the environment dynamics

Output:

Algorithm Parameter: , small threshold determining accuracy of estimation

Initialize , for all , arbitrarily except that .

Loop:

Loop for each :

Until

Figure 1: Backup diagram for the update relation (5)

Note: this is the same backup diagram on Figure 1 in (Gueorguiev, 2023).

*Short explanation on the backup diagram above taken from* (Gueorguiev, 2023)*:*

Each open circle, which will be denoted as *reward-state node* so forth, colored in red represents a state and the reward, which is associated with this state For instance, the root node shown on Figure 1 has associated reward and state . Each solid circle, colored in blue represents a state-action pair and will be denoted as *state-action node* so forth. The specific state on the rightmost state-action node is shown as . Each directed blue edge connects state node with state-action node and represents application of the policy to the root reward-state node . Each directed red edge emanating from a state-action node ends in a possible reward-state node corresponding to specific probable pair of reward and new state . Thus, each directed red edge represents the application of the function of the MDP dynamics. The Bellman equation (5) averages over all of the possibilities weighing each possibility represented by a path from the root of the Backup diagram on Figure 1 to a leaf by its probability of occurring. It states that the value of the start state must equal the discounted value of the expected next state plus the reward expected along the way.

#### Policy evaluation for the action values

An algorithm on iterative policy evaluation for the state-action values can be constructed using the Bellman equations for state-action values.

Starting from Bellman equation for state-action values we write:

In Chapter 3 of (Richard D. Sutton, 2020) is defined as:

(7)

(also see Eq (7) in (Gueorguiev, 2023))

Using the recursive relation for the total return in the right-hand side of (7) gives:

(8)

The expectation in the right-hand side of (7) is explicitly expressed in terms of the environment dynamics

(9)

Thus we arrive at the Bellman equation for the state-action values:

(10)

Figure 2: Backup diagram for the update relation (10)

//TODO: finish the algorithm of policy-evaluation for action values

### Policy Improvement

We are computing the (state/state-action) value function for a policy to help find better policies.

Let us assume that we have determined the value function for arbitrary deterministic policy . For some state we would like to know whether or not we should change the policy to deterministically choose an action .

We know how good it is to follow the current policy from – that is, , is the expected return when starting in and following thereafter.

Question: would be for a better or worse to change to a new policy from the current moment on (that is, state ) ?

One way to answer this question is to consider selecting in and thereafter following the existing policy .

The value of this way of behaving is:

(10)

The key criterion is whether computed by (10) is greater or less than . If it is greater this means it is better to select once in , and thereafter follow . In such case one would expect that it would be better still to select every time is encountered than it would be to follow all the time. So one would expect that the new policy constructed in such way by amending would be better one overall. This is a special case of the *policy improvement theorem*.

**Policy improvement theorem**

Let and be any pair of deterministic policies such as that, for all ,

(11)

Then the policy must be as good as, or better than, . That is, it must obtain greater or equal expected return from all states :

(12)

Moreover, if there is strict inequality of (11) at any state , then there must be strict inequality of (12) at the same state .

The policy improvement theorem applies to the two policies which we considered in the earlier paragraph – the original deterministic policy and a changed policy that is identical to except that . For states other than , (11) holds because the two sides are equal. Thus, if then is better than .

Idea of proof:

From (11) we have:

(by the definition of )

(by (11))

. (13)

Notice that in the latter we have used the following expression for

(14)

In (14) notice that the RHS is not subscripted with as there is no dependency on in the RHS because for

So far we have seen how,

## Appendix

### Solution of the Bellman system of equations for state values

We notice that the Bellman system of equations with respect to (5) can be rewritten as:

(A1)

hence

(A2)

The left-hand side of (A2) can be rewritten as:

(A3)

The right-hand side of (A2) are rearranged as :

(A4)

we denote with and the following expressions:

(A5)

(A6)

(A7)

Using (A3)-(A7) in (A2) leads to :

(A8)

(A8) represents a linear system of equations with respect to the unknowns .

(A8) in matrix form:

(A9)

For convenience we abbreviate:

, , (A10)

Thus,

, (A11)

//TODO: derive degeneracy conditions on the function of the environment dynamics

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