# Reinforcement Learning and Optimal Control, Dimitri Bertsekas

## Deterministic Dynamic Programming

All DP problems involve a discrete time dynamic system that generates a sequence of states under the influence of control. In finite horizon problems the system evolves over a finite number of time steps (also called stages). The state and control at time are denoted by and , respectively. In deterministic systems, is generated non-randomly, i.e., it is determined solely by and .

### Deterministic Problems

A deterministic DP problem involves a discrete-time dynamic system of the form:

(1)

where

is the time index

is the state of the system, an element of some space

is the control or decision variable, to be selected at time k from some given set that depends on

A diagram of a diagram

Description automatically generated

Figure 1: A deterministic N-stage optimal control problem. Starting from state , the next state under control is generated non-randomly according to

and a stage cost is incurred.

is a function of that describes the mechanism by which the state is updated from time to time .

is the horizon or number of times control is applied,

The set of all possible is called the ***state space*** at time . Similarly, the set of all possible is called the ***control space*** at time . Both the state space and the control space can depend on .

The problem also involves a cost function that is additive in the sense that the cost incurred at time , denoted by , accumulates over time. Formally, is a function of that takes real number values, and may depend on . For a given initial state , the total cost of a control sequence is

(2)

where is a terminal cost incurred at the end of the process. This cost is a well-defined number, since the control sequence together with determines exactly the state sequence via the system equation (1). We want to minimize the cost (2) over the sequences that satisfy the control constraints, thereby obtaining the optimal value

as a function of . Figure 1 illustrates the main elements of the problem.

We will next illustrate deterministic problems with some examples.

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Figure 2: Transition graph for a deterministic finite-state system. Nodes correspond to states . Arcs correspond to state-control pairs . An arc has start and end nodes and , respectively.