# Reinforcement Learning in the Blackwell’s Game of Addition

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## The Game *Addition*

Let us consider the game described in *Blackwell’s Theory of Games and Statistical Decisions* (Blackwell & Girshik, 1978, p. 14)[[1]](#footnote-1):

and alternatively choose integers, each choice being one of the integers and each choice made with the knowledge of all preceding choices. As soon as the sum of the chosen integers exceeds , the last player to choose pays his opponent one unit.

The situation at which player finds himself at his th move is described by a sequence with each being one of the integers and

Denote by the set of possible sequences where and denotes the closest integer which does not exceed . A strategy for consists of a set of functions , where is a function defined on assuming only values : specifies ’s th move when the previous history of the play is . Similarly, a strategy for is a set of functions , where is defined for the set of all sequences with each being one of the integers and

Define and inductively for ,

(this induction describes the manner in which a referee would carry out the instructions of the players) and let be the largest for which is defined. Then

## Reinforcement Learning formulation for the game *Addition*

The state of the game before ’s next move is represented by the sequence s.t. . The state of the game before ’s next move is represented by the sequence s.t. .

Each actor in this game can be in

Let us suppose that an agents select all possible choices with equiprobable (or random, hence the superscript ) policy which we will denote with .

# References

[1] [Theory of Games and Statistical Decisions, David Blackwell, 1954](https://github.com/dimitarpg13/reinforcement_learning_and_game_theory/blob/main/books/TheoryofGamesAndStatisticalDecisionsBlackwell1954.pdf)

[2] [Reinforcement Learning - An Introduction, Second Edition, Richard Sutton, Andrew Barto, 2020](https://github.com/dimitarpg13/reinforcement_learning_and_game_theory/blob/main/books/ReinforcementLearningSuttonSecondEdition2020.pdf)

[3]

1. This game was also denoted as in Blackwell’s book [↑](#footnote-ref-1)