Geometric Spatial and Temporal Constraints in Dynamical Systems and Their Relation to Causal Interactions between Time Series

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Abstract

This paper delves deeper into the definition of a causal interaction indicator between two time-series, as introduced by [1]. The indicator is based on the system's equivalence to a physical model involving a Coulomb gas confined to a line and influenced by forces exerted by two walls. The aim is to demonstrate the necessary and sufficient conditions for a system to permit causal interactions, including space-time non-negativity, and, as a consequence, time and logic ordering. This is done through an analogy to the Coulomb gas model, in which the gas is confined to a line and subject to pulling and pushing forces from two walls. First, by highlighting the geometric foundations of spatial and temporal constraints in dynamical systems, an analogy is established between the non-negative distance constraint in the Coulomb gas model and the non-reverting time-ordering causality conditions in temporal interaction models. The likelihood of an interaction—which can be causal or non-causal, depending on the aforementioned conditions—within a given time interval is then explained, and a method for computing it is shown. The indicator for causal interactions from [1], which is based on the standard deviation of the largest eigenvalue's explanatory power in lagged correlation matrices for a system of two time-series, provides a measure of the probability of causal interactions. Finally, the geometric properties presented in this paper enable new connections to be drawn between correlation and causal interactions.

1 Introduction

This paper explores the interplay between physical constraints and temporal dynamics within two analogous systems: a physical model involving a Coulomb gas confined in a line and being pulled and pushed by two walls, and a conceptual model describing interactions between random variables over time-series. Both systems involve two primary agents—either as physical walls or as abstract points (or dots)—whose relative positions are subjected to continuous changes, constrained by specific rules that prevent certain configurations. Specifically, it is shown that the constraint of non-negative distances in the Coulomb gas model can be geometrically and conceptually mapped to the preservation of causality in a temporal interaction model involving crossing timelines.

In the first system, two random variables represent walls that dynamically confine a Coulomb gas within a one-dimensional space or line. The critical constraint here is that the distance between the walls cannot become negative, ensuring the physical viability of the gas. Geometrically, this system is represented by a line segment whose length is strictly non-negative, and any reduction in this length has direct implications on the system's state (see Figure 1).

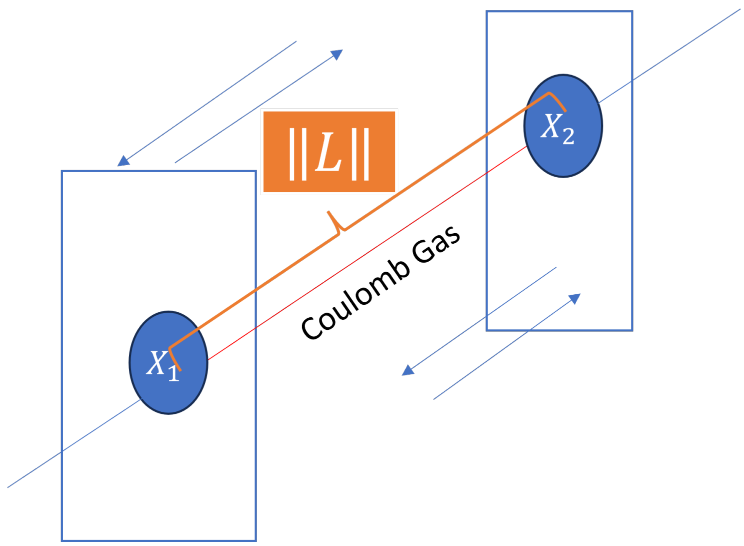

Figure 1: Geometric representation of a Coulomb gas confined in a line in red and being pulled and pushed by walls with coordinates X_1 and X_2, with a non-negative distance $||L||$ in between in orange.

The second system considers two random variables, A and B, traversing distinct timelines. These timelines intersect at a point, suggesting a potential interaction between the variables. The core analysis focuses on the straight line segment that can be drawn between A and B, connecting them as they move along their respective timelines. It is always assumed that the random variable A causally interacts with the effect variable B. This system is depicted in Figure 2, which includes two subfigures. In the upper subfigure, A passes through the intersection of the timelines. It is important to note that the intersection of the timelines is defined prior to A passing through it, as explained later. In the lower subfigure, A and B continue moving along their respective timelines, and the distance of the segment remains positive at all times, $||L|| > 0$.

A geometric equivalence is proved between this line segment and the one between the walls in the Coulomb gas model (see Figures 3 and 4) for the existence of causal interactions in the system of two random variables A and B. The timelines representation is referred to as the temporal model (Figure 2), while the Coulomb gas is referred to as the spatial model (Figure 1). The introduction of a geometric representation allows us to explore this analogy rigorously, proposing that the non-negativity of the line segment in the temporal model—similar to that in the spatial model—is essential for maintaining the causal integrity of the interactions. Non-negativity in the spatial model is essential for the existence of the Coulomb gas, and in the temporal model, it is crucial for time ordering and the non-invertibility of time, allowing causal interactions to exist.

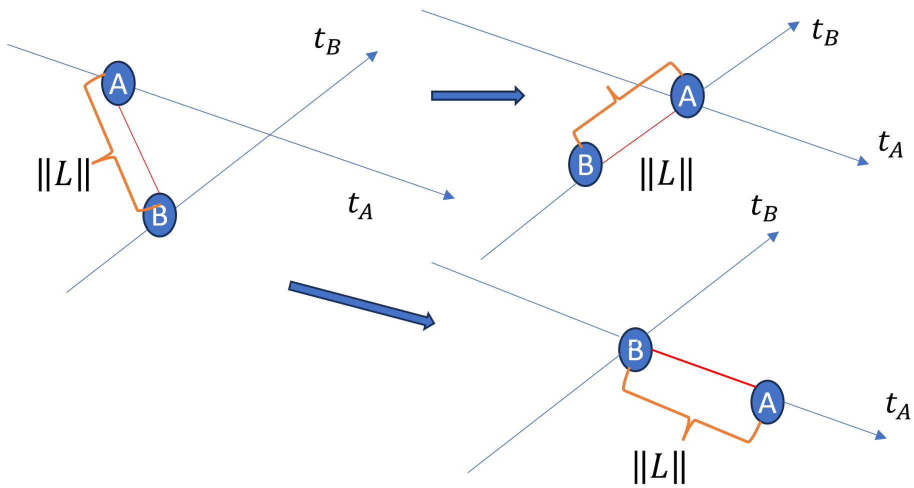

Figure 2: Temporal interaction model with non-negative line segment (part of transversal line) between random variables.

It is argued that if the distance between the walls—or equivalently, the length of the line segment between the moving random variables—remains non-negative, it preserves a form of temporal ordering necessary for causality, as shown in Figures 3 and 4. In Figure 3, the limiting case where the distance is zero is depicted. In this case, A and B coincide at a particular moment in time, yet causal interaction is still possible. In Figure 4, it is shown on the left that the length of the red segment can become negative if the time ordering is inverted. In this scenario, the interaction is non-causal, as it occurs after any possible effect of B. In other words, A intersects the timeline of B when B is further along its timeline, making the interaction non-causal.

This geometric perspective sheds light on the foundational properties that govern both the physical and the temporal domains, providing insights into how spatial constraints influence temporal dynamics and interactions. The analysis not only bridges concepts across physics and theoretical mathematics but also enhances our understanding of causality in dynamical systems.

Finally, the ideas in this article form the basis of the indicator presented in [1], which are further extended here. For details on the experiments, algorithm,

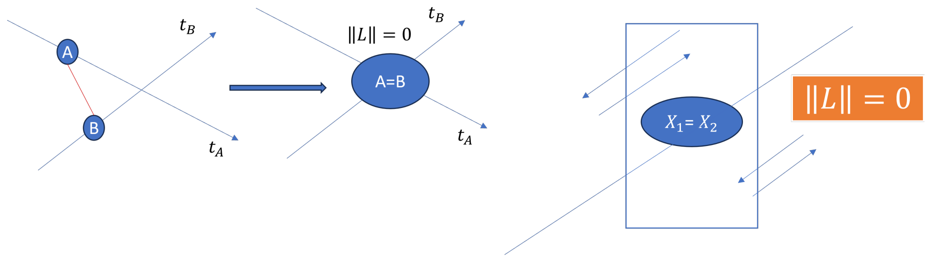

Figure 3: Limiting case in which the distance $\|L\| = 0$

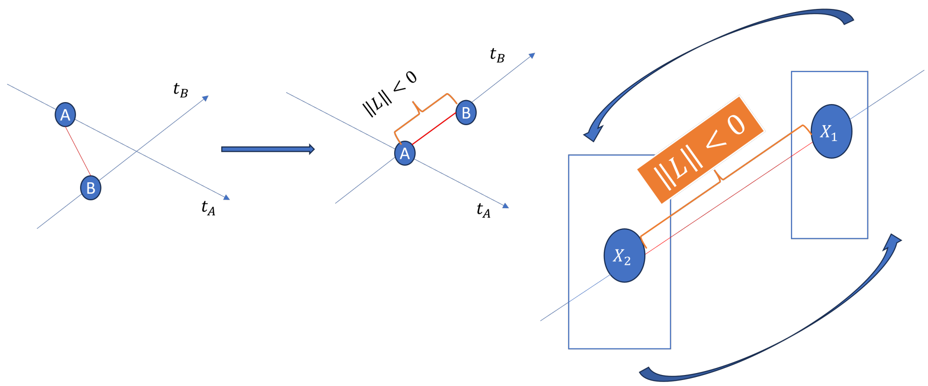

Figure 4: Visual representation of the consequence of not respecting the non-negative distance in a spatial model and its equivalence in failing to respect time ordering in a temporal interaction model. This illustrates how a negative distance metaphorically translates into a violation of causality in temporal dynamics.

literature and original work, the reader is referred to that paper.

2 Geometric Proof of Equivalence Between Spatial Non-Negativity and Logical Causality

This section provides a geometric proof of the equivalence between the physical constraint of maintaining a non-negative distance in a Coulomb gas model and the temporal ordering of events in intersecting timeline scenarios. In the Coulomb gas model, two walls at positions $X_1(t)$ and $X_2(t)$, confine a gas, with $X_1(t) \leq X_2(t)$. The interval $[X_1(t), X_2(t)]$ represents the permissible space for the gas, enforced by $d(t) = X_2(t) - X_1(t) \geq 0$. Similarly, two random variables, $A(t_A)$ and $B(t_B)$, move along their respective timelines, which intersect at $t = t_A = t_B$ for $t_A, t_B \in t$ (see Figure 3 for illustration). A line segment connecting $A(t_A)$ and $B(t_B)$ at any given time t represents potential interactions or states between these variables (see segment in red in Figure 2).

The dynamics of these line segments are crucial for both models. In the physical model, the non-negative distance $d(t) = |X_2(t) - X_1(t)|$ ensures that the physical space does not collapse. In the timelines, the distance $\|L\|$ between $A(t_A)$ and $B(t_B)$ is given by:

$$\|L\| = \sqrt{(B(t_B) - A(t_A))^2},$$

where $\|L\| > 0$ at all times to ensure a valid interaction space. The maintenance of this positive length $\|L\|$ validates causality by ensuring that interactions or event sequences follow a logical and uninterrupted order.

However, an interaction (causal or not) between $A(t_A)$ and $B(t_B)$ is not inherently guaranteed by this model. The probability of an interaction occurring is influenced by the variance in the positions of the walls or the standard deviation of the distance $\|L\|$ between the random variables. If the standard deviation of $\|L\|$, denoted as $\sigma(\|L\|)$, is large, it implies greater fluctuations in the distance between $A(t_A)$ and $B(t_B)$, thereby increasing the likelihood of these variables coming into proximity, which could facilitate an interaction. The increased standard deviation provides:

$$\text{Probability of Interaction} \propto \sigma(\|L\|),$$

where a higher $\sigma(\|L\|)$ reflects more significant deviations from the mean distance, enhancing interaction opportunities. The non-negativity property will then guarantee causality in that interaction as stated above.

This analysis establishes a foundational equivalence between spatial and temporal constraints and illustrates how variations in system components can dynamically influence the probabilities of interaction, effectively linking physical configurations to temporal event sequences.

3 Collinearity and Concurrency

- **Two Dots on a Line:** As in the Coulomb Gas model.
  - When two dots (points) are on a single straight line, they are described as *collinear*. This indicates that they lie along the same geometric line. In a two-dimensional plane, there is exactly one line that can pass through both points, defining their linear relationship.
- **Two Dots on Two Lines and an Intersecting Line:** As in the system of timelines for A and B.
  - If one dot is placed on one line and another dot on a different line, and these two lines intersect (assuming they are not parallel), the intersection occurs at a point where they meet.

- If there are two lines, each with a dot, and a third line intersects both lines at the points where the dots are located, this configuration involves the concept of *concurrency.* The third line, intersecting the other two at their respective dots, acts as a *transversal.*

Definition (Concurrence):

Let L_1 and L_2 be two distinct lines in a Euclidean plane. If L_1 and L_2 are not parallel, there exists a unique point P such that P lies on both L_1 and L_2. This property is known as *concurrence*, and the point P is called the *point of intersection* of the two lines.

In other words, two distinct non-parallel lines are said to be concurrent if they intersect at exactly one point.

3.1 Geometric and Mathematical Relevance

The equivalence between the two scenarios can be understood through concepts of intersection and alignment:

- In the first case, the two dots are directly aligned on a single straight line, establishing a straightforward linear connection.
- In the second scenario, although the dots do not initially share the same line, the introduction of an intersecting line creates a connection point that indirectly aligns the two dots through their intersection points.

This geometric setup has various applications in mathematical proofs or constructions, such as:

- Proving properties related to angles formed by a transversal.
- Establishing relationships in coordinate geometry using equations of lines.
- Exploring geometric concepts like the concurrency points of a triangle (e.g., centroid, circumcenter), which involve the intersection of lines constructed from points.

4 Proof of Equivalence of Geometric Configurations

The goal is to prove the equivalence of two geometric configurations:

1. Two points on a single line (representing the Coulomb gas system).
2. Two points on two different lines intersected by a third line (representing the timeline system of A and B) .

Scenario 1: Two Dots on a Line:
Assume points P_1 and P_2 lie on line L. The line equation passing through these points is given by:

$$ax + by + c = 0$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of P_1 and P_2, respectively.

Scenario 2: Two Dots on Two Lines with a Transversal:
Assume:

- Line L_1 passes through P_1 with equation $a_1x + b_1y + c_1 = 0$.
- Line L_2 passes through P_2 with equation $a_2x + b_2y + c_2 = 0$.
- Line L_3 is a transversal intersecting L_1 at P_1 and L_2 at P_2, with equation $a_3x + b_3y + c_3 = 0$.

4.1 Proof of Equivalence

- In Scenario 1, points P_1 and P_2 are collinear by definition.
- In Scenario 2, the existence of line L_3 as a transversal creates a geometrical setup where P_1 and P_2 satisfy the line equation of L_3, thereby making them collinear with respect to L_3.

Both points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ satisfy the equation of L_3:

$$a_3x_1 + b_3y_1 + c_3 = 0 \quad \text{and} \quad a_3x_2 + b_3y_2 + c_3 = 0$$

This implies P_1 and P_2 are collinear with respect to L_3.

Thus, the connection of points via a direct line or through a transversal line in different scenarios demonstrates their geometric equivalence, supporting the underlying principle of linear connectivity and alignment in both configurations.

5 The Line Segment Model: Spatial Non-Negativity and Logical Causality

This section demonstrates how principles of geometry, which inherently apply to both spatial and conceptual constructs, can establish the equivalence between the non-negativity of spatial configurations and the causal ordering of logical sequences. The concept of ordering on a line, translated between physical constraints and logical orderings without directly referencing temporal dimensions, is applied.

Spatial Representation of Coulomb Gas
Consider a line segment on the x-axis within a 2D plane, representing the space confined between two walls of a Coulomb gas. Let the positions of these walls be denoted by x_1 and x_2, where $x_1 < x_2$. This arrangement ensures the non-negativity constraint, defined by the distance $d = x_2 - x_1 \geq 0$.

Logical Representation with Two Lines and a Transversal
Each point on the line segment from x_1 to x_2 can be mapped onto corresponding states in a logical process or argument. Rather than viewing this mapping as a function of time, it is considered as a series of dependent states—S_1 leading to S_2—which directly mirrors the physical arrangement from x_1 to x_2.

5.1 Maintaining Physical and Logical Order

Geometric Order (Non-negativity) The stipulation that x_2 must always be greater than or equal to x_1 translates into an inherent order. This is equivalent to positing that the process commencing at $S_1(x_1)$ and advancing to $S_2(x_2)$ must progress in one direction along the line segment, precluding any backward movement.

Logical Order (Causality) Mapping these physical positions to states in a logical sequence implies that the non-reversal property of the line segment necessitates a forward progression in the logical process. This condition directly aligns with the causal order where S_1 precedes S_2, and retroactive influences are logically impossible.

5.1.1 Line Segment as a Universal Model

The line segment serves as a universal model for order and progression, with any movement along this segment representing a progression in a physical or logical system. The inherent properties of the line segment (its directionality and non-negative length) guarantee that any mapping onto another line or space retains this order, whether the dimension be spatial, temporal, or logical.

Projecting the line segment onto another space representing a sequence of events or states ensures that the geometric properties of the segment (directionality and boundedness) preserve the order. Thus, maintaining non-negativity in spatial terms is equivalent to ensuring a forward progression in logical terms. Through geometric interpretation, it can be demonstrated that maintaining a non-negative distance between two points on a line is equivalent to upholding a forward logical progression between two states or events represented by these points. This equivalence leverages the inherent properties of geometrical structures to bridge physical constraints with logical sequencing, highlighting the interconnected nature of spatial and logical dimensions in system modeling.

5.2 Equivalence of Spatial-Time Non-Negativity and Logical Causality

In this section, linear algebra and geometric principles are used to demonstrate that the constraint of non-negativity in space-time configurations, such as in the Coulomb gas model, is equivalent to maintaining causal ordering in logical or abstract representations.

Define a line segment on the real number line as $[x_1(t), x_2(t)]$, where $x_1(t)$ and $x_2(t)$ are real numbers and $x_1(t) < x_2(t)$. This segment is used to model the space between two confining walls of a Coulomb gas. The distance between these points is $d = x_2(t) - x_1(t)$, satisfying the non-negativity constraint:

$$d \geq 0$$

A mapping is made from this physical scenario onto an abstract logical sequence, where $x_1(t)$ corresponds to an initial state S_1 and $x_2(t)$ to a subsequent state S_2, $\forall t$. The transformation $f : [x_1(t), x_2(t)] \to [S_1, S_2]$ is defined as:

$$f(x(t)) = ax(t) + b$$

where $a > 0$ and b are constants to ensure that f is a monotonically increasing function, preserving the order. The transformation f must maintain the order from $x_1(t)$ to $x_2(t)$. Given that $x_1(t) < x_2(t)$:

$$ax_1(t) + b \leq ax_2(t) + b$$

This implies:

$$a(x_1(t) - x_2(t)) \leq 0$$

which holds true for $a > 0$, ensuring that $f(x_1(t)) < f(x_2(t))$. The non-negativity $d = x_2(t) - x_1(t) \geq 0$ translates to an order-preserving mapping in logical terms through f. Therefore, maintaining space-time order implies maintaining logical order:

$$x_1(t) < x_2(t) \implies S_1 < S_2$$

This direct correlation shows that spatial non-negativity is inherently linked to causal logical ordering. It has been shown that preserving space-time non-negativity equates to maintaining a logical order in a corresponding causal, abstract or logical, representation. This result highlights the interconnected nature of spatial and temporal dimensions in dynamic system modeling, establishing a fundamental link between physical constraints and causal logical sequencing.

6 Causal Interactions Indicator [1], in Greater Detail

This section provides further intuition for the causal interaction indicator presented by Rodriguez Dominguez A. and Hari Yadav O. [1]. The first objective is to find a statistical measure that can be used to indicate the likelihood of an interaction between the two timelines within a given time period. The second objective is to apply the properties described in this article to ensure that the interaction is causal. The setup follows Figure 2, where the timelines and a transversal line intersect the random variables. The system lies in a Euclidean

2D plane, and in the previous section, the properties of concurrence in Euclidean spaces were discussed. The ideas in this section will serve as a foundation for the development of the causal interaction indicator in [1], in conjunction with the work from previous sections.

It is essential to first understand what happens when the time to concurrence for these two timelines, which have different directions, is minimized.

Minimizing Time to Concurrency with Different Directions:

In cases where two points move along lines with random velocity fluctuations, the eigenvalues of the covariance matrix help understand the relationship between their movements. A large first eigenvalue indicates significant motion in a principal direction, suggesting that the points are moving in a correlated way along that axis.

If this direction aligns with their velocity vectors, the likelihood of intersection increases, as their trajectories are more likely to converge. Smaller or similar eigenvalues suggest less correlation and weaker directional movement, reducing the probability of intersection or extending the time it would take for the paths to cross. Thus, larger eigenvalues point to a higher chance of faster convergence.

6.1 Problem Description

Let consider two moving dots, each following a linear trajectory in space. The lines along which these dots move may intersect, and we want to determine if the probability of these lines intersecting is maximized over a given time interval.

Key challenge: The dots themselves may not meet at the same point in space and time, but we are interested in whether their trajectories (the lines they trace) are likely to intersect (See Figure 2).

Goal: Use the covariance between the dots' movements, along with eigenvalue analysis, to estimate whether the probability of the lines intersecting is maximized.

The positions are random variables given by:

$$\mathbf{r}_1(t) = \mathbf{p}_1 + t\mathbf{v}_1$$

represents the position of dot 1 at time t, where $\mathbf{p}_1$ is the initial position and $\mathbf{v}_1$ is the velocity vector. Similarly,

$$\mathbf{r}_2(t) = \mathbf{p}_2 + t\mathbf{v}_2$$

for the position of dot 2 at time t. The velocities are assumed random. The covariance matrix $\mathbf{C}$ describes the variance in the positions and the covariance between the motions of the two dots:

$$\mathbf{C} = \begin{pmatrix} \mathrm{Var}(\mathbf{r}_1) & \mathrm{Cov}(\mathbf{r}_1, \mathbf{r}_2) \\ \mathrm{Cov}(\mathbf{r}_1, \mathbf{r}_2) & \mathrm{Var}(\mathbf{r}_2) \end{pmatrix}$$

To understand how the dots are moving in relation to each other, eigenvalue decomposition of the covariance matrix is performed:

$$\mathbf{C} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T$$

where $\mathbf{V}$ is the matrix of *eigenvectors*, representing the principal directions in which the two dots move, $\mathbf{\Lambda}$ is a diagonal matrix containing the *eigenvalues* λ_1, λ_2, which measure the variance along the principal directions. The eigenvalues λ_1 and λ_2 provide crucial information about how the two dots move in relation to each other:

- **Large Eigenvalue** λ_1: A large eigenvalue indicates that there is significant variance in the motion of the two dots in a particular direction. This corresponds to a situation where the dots' movements are correlated in this principal direction (either moving toward or away from each other in that direction).
- **Small Eigenvalue** λ_2: A small eigenvalue indicates little variance in the motion in that direction, suggesting that the dots are moving relatively independently or are moving in a direction that does not bring them closer together.

Key Observation: If the two dots are moving along trajectories that intersect, the covariance between their movements should be relatively large. This is reflected in the covariance matrix by having a large eigenvalue. Here's the procedure for assessing the probability of intersection:

1. **Dominant Eigenvalue**: The largest eigenvalue λ_1 represents the direction of maximum variance (or maximum correlation) between the two dots' movements. If this eigenvalue is significantly larger than the other eigenvalue, it means that there is a strong directional correlation between the motions of the two dots. This large correlation increases the probability that the two lines will intersect because the dots are more likely to be moving in ways that bring their trajectories closer together.
2. **Eigenvectors and Direction of Motion**: The eigenvector corresponding to the largest eigenvalue tells us the direction in which the two dots are moving most similarly. If this eigenvector aligns with the relative velocity vectors of the two dots, the probability of their lines intersecting is maximized.

7 Monitoring the Standard Deviation of the Largest Eigenvalue a.k.a the Causal Interactions Indicator [1]

To further monitor the likelihood of intersection (or interaction, but not necessarily causal), the *standard deviation of the largest eigenvalue* can be used. The standard deviation of the largest eigenvalue λ_1 is given by $\sigma_1 = \sqrt{\lambda_1}$, which provides insight into how much the dots' motions are varying along the principal direction of their movement. **Large Standard Deviation**: If λ_1 is large, then σ_1 will also be large, meaning there is significant movement along this principal direction. This suggests that the dots are more likely to be moving in ways that could result in their trajectories intersecting. **Small Standard Deviation**: If λ_1 is small, then the standard deviation will be small, indicating little movement along the principal direction, reducing the probability of intersection.

By performing eigenvalue decomposition of the covariance matrix, the principal directions of motion for the two dots can be determined, allowing an assessment of whether their lines are likely to intersect. A large eigenvalue indicates a strong correlation between the dots' motions, suggesting a higher probability of their lines intersecting. Additionally, by monitoring the standard deviation of the largest eigenvalue, it is possible to continuously evaluate whether the probability of interaction is increasing or decreasing over time (in an indicator sense). A large standard deviation implies a greater likelihood of intersection.

How to Monitor Interactions that are Causal?

By using a time-lag in one of the variables it enforces the method to focus only on interactions in which the space-time non-negativity and the order of time and causal logic is preserved. Hence, the focus of the indicator is only on the likelihood of causal interactions in a given interval in time and the possibility of monitoring this continuously. The algorithm from the original paper [1] is shown in Algorithm 1 for two candidates random variables time-series X and Y.

Algorithm 1: Causal interactions time-series indicator for 2-variables set ($\boldsymbol{\sigma_\Gamma}$)

Data: Causal time-series candidate $\boldsymbol{X} \in \mathbb{R}^{L\times 1}$ and effect candidate time-series $\boldsymbol{Y} \in \mathbb{R}^{L\times 1}$, both of length L; lag values, $\tau = 1, \ldots \max\tau$, for a given $\max\tau$; $k = t, \ldots, T$ are the timestamps for the time-series of the indicator.

Result: The time-series $\boldsymbol{\sigma_\Gamma(t \to T)} = \{\sigma_\Gamma(t), \ldots, \sigma_\Gamma(T)\}$, with $k = t, \ldots, T$.

for $t \leq k \leq T$ **do**

  $\boldsymbol{X}(k) = \boldsymbol{X}[k-L\colon k]$; $\boldsymbol{X}(k) = (\boldsymbol{X}(k) - mean(\boldsymbol{X}(k)))/std(\boldsymbol{X}(k))$

  $\boldsymbol{Y}(k) = \boldsymbol{Y}[k-L\colon k]$; $\boldsymbol{Y}(k) = (\boldsymbol{Y}(k) - mean(\boldsymbol{Y}(k)))/std(\boldsymbol{Y}(k))$

  for $1 \leq \tau \leq \max\tau$ **do**

    $\boldsymbol{X}(k,\tau) = \boldsymbol{X}(k).shift(\tau)$

    $PCA(k,\tau) = PCA([\boldsymbol{Y}(k), \boldsymbol{X}(k,\tau)]^T)$

    $V(k,\tau) = \frac{\lambda_1(k,\tau)}{\lambda_1(k,\tau)+\lambda_2(k,\tau)}$

  end

  $\sigma_\Gamma(k) = Std(V(k,1), \ldots, V(k, \max\tau))$

  $\boldsymbol{\sigma_\Gamma(t \to T)}.append(\sigma_\Gamma(k))$

end

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8 Conclusion

In this article, the foundation and main ideas surrounding the development of the causal interaction indicator [1] have been extended, incorporating new developments and ideas that were assumed to be understood in the original paper. These ideas include demonstrating the equivalence between the Coulomb gas model and a system of two time-series, the non-negativity property of space-time in the Coulomb gas system, and how it is connected to time-ordering and causal logic in the system of two random variables. Additionally, the geometric equivalence between the transversal line in the time-series system that connects both random variables and the segment of the gas supporting the rest of the proof is discussed. Finally, the probabilistic inference of interactions, methods to isolate causality using lag, and its connection to spectral theory are explained, as well as the rationale for using the standard deviation as a metric instead of the value of the first eigenvalue itself.

To conclude, this work not only reinforces and motivates the use of the causal interaction indicator for time-series, but also sheds light on how geometry can help explain the distinction between correlation and causality. Correlation can be considered a geometric property of the system's representation, which, in future work, may be related to the transversal line that connects the two random variables on their respective timelines (the red segment in Figure 2), with potential for proving properties related to angles formed by a transversal and the timelines. For extensive work on angle representations of correlations, see [2]. Causality, on the other hand, requires the existence of a logical order and a time-ordering, which arises from certain space-time constraints, as seen in this work.

References

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