# Notes on Causality, Prediction and Search by Peter Spirtes

D. Gueorguiev, 9/13/23

## Notation and Basic Definitions:

### Graphs:

Notation for Edges:

**undirected edge** :

**directed edge** :

**non-directed edge** :

**partially directed edge** :

Note on Inducing Path Graph: contains both directed edges (e.g., ) , bi-directed edges (e.g., ), non-directed edges (e.g., ), and partially directed edges (e.g., ).

**Graph** (*traditional definition*): ordered pair where is a set of vertices and is a set of edges. The members of are pairs of vertices (an ordered pair in a directed graph and an unordered pair in an undirected graph). For example, the edge is represented by the ordered pair . We need to specify variables and **marks** at each end. In general, we will allow that the end of an edge can be unmarked, can be marked with an arrowhead , or can be marked with an . For example, the left end of can be represented as ordered pair , while the right end can be represented as the ordered pair . The entire edge is a set of ordered pairs representing the endpoints . The edge is the same as .

Note that a directed edge such as has no mark at the endpoint; we consider the mark at the A endpoint to be empty, but when we write out the ordered pair, we will use the notation to stand for the empty mark e.g.,

**Graph** (our definition): an ordered triple where is a non-empty set of vertices, is a non-empty set of marks, and is a set of sets of ordered pairs of the form , where and are in , , and and are in . Except in our discussion of systems with feedback we will always assume that in any graph, any pair of vertices and occur in at most one set in , or, in other words, that there is at most one edge between any two vertices. If we say that is over .

Figure 1: Example of directed graph

For example, the directed graph on Figure 1 can be represented as:

**edge**: any member of .

**edge-end**: each ordered pair

**endpoint**: each vertex in an edge

**adjacent endpoints:** vertices are adjacent iff there is an edge with endpoints

**undirected graph**: a graph in which the set of marks .

**directed graph**: a graph in which the set of marks .

**directed edge** from to : an edge

**edge into** : any edge

**edge out** of **:** any edge

**parent/child**: is parent of which is child of if there is a directed edge from to

**indegree** of vertex : the number of the parents of

**outdegree** of vertex : the number of children of

**undirected path** between and in graph : a sequence of vertices beginning with and ending with such that for every pair of vertices and that are adjacent in the sequence there is an edge in .

**edge is in path**: is in the path iff and are adjacent to each other (in either order) in .

vertices **adjacent on path**: if an edge between and is in the path we say that and are adjacent on .

**path is out of vertex**: if the edge containing in **an undirected** **path** between and **is** **out of** then we say that **path is out** of .

**path is into vertex**: if the edge containing in a path between and is into we say that **the path is into** .

**empty path**: sequence which consists of a single vertex.

**acyclic path**: a path which contains no vertex more than once; otherwise, it is **cyclic path**.

//TODO: finish basic definitions

## Introductory Notes

*Fundamental questions in this thesis*: understanding the systematic connections between causal dependency and stochastic dependency. What are the limits to reliable causal inference given the presence of such connections? Under what conditions the causal inference will produce reliable results? What are the limitations for causal inference given our theoretical understanding between causality and probability? Within these limits we can investigate rigorously the shortcomings of the algorithms that search for causal structures from statistical properties of the datasets.

The asymptotic reliabilities of well-defined procedures can be determined mathematically, while short run behavior can be estimated/bounded through well-chosen simulation experiments. This thesis investigates into the theory of model specification search adopting results from the theory of estimation.

*Additional investigations*:

Understanding latent variables and their impact on causality.

This work investigates various asymptotic methods and their reliability to obtain information about the presence or absence of unmeasured common causes, and about their causal relations. *The Markov Condition* and *The Faithfulness Conditions* are widely assumed throughout this work. Informative sufficient conditions for the presence of unmeasured common causes are investigated under the assumption of Markov and Faithfulness conditions. Theorems about causal conclusions and predictions will be drawn whether or not latent variables are present. Tetrad Representation Theorem is more powerful theorem valid under the assumption of linearity – this theorem can be used to identify the presence of unmeasured common causes.

Conditions sufficient and/or necessary to correctly predict the effects of a policy applied to a population that has been studied through a sample, and whose causal structure is not known a priori.

*Related question*: when will the conditional distribution of on when is *forced* to have a certain value, equal the probability of conditional on , in an observational or experimental population?

One can find an answer assuming that certain counterfactual claims are known. That is, knowing aspects of the causal structure of the system under study.

*Follow-up question*: when can the relevant causal knowledge necessary to answer the question above be obtained from the sample data?

In this work results are described that answer the follow-up question and give information from sample data about when the conditional probability of on and is invariant under a manipulation of .

*Fundamental issue about prediction*:

Let us assume that the distribution of is to be directly manipulated. Let us denote the manipulated distribution with . *Question*: when can the resulting distribution of a set of variables conditional on a set be calculated from the distributions , of , , and other variables in an observational or experimental population in which were not manipulated for each unit in the sample?

In order to answer this question, we need to find out more about the formal connections between probability and causal structure.

*Another topic of discussion*: the commonly used statistical search procedures are sub-optimal for causal inference.

Automated model search procedures (especially linear and logistic regression) are asymptotically unreliable against alternative causal hypotheses that are often consistent with prior knowledge. If the procedures give the right answers in the ideal case of perfect information about the population distribution, one can look around for better tests and more computationally efficient algorithms. But if, as in the case of regression and many other automated techniques, probability relations and causal relations are incorrectly matched there is no way to produce good inferences and there is no point to investigate more performant algorithms based on flawed causal models.

## Chapter 1 The Issue

Claim is made that existing methods for function fitting through some minimization technique *cannot* be reliably used to infer causal dependencies from random uncontrolled samples or predict the effects of manipulation. Linear regression is taught as means of fitting a line to sample data and also as means of predicting new values of a variable. Many of the real applications of regression are to predict values of a variable when the regressors are *manipulated*, that is, when an action or policy forces some new distribution to the regressors. Lots of claims are circulated that regression cannot be used to infer causes from non-experimental samples, or to make predictions about manipulated systems. Bolder claim is made : it is not possible for any statistical method to be used to reliably infer causes or to make prediction about manipulated systems.

*The Arguments against the possibility of reliable inference of causal structure from statistical samples*

* Many examples in which regression yields erroneous causal inferences
* Underdetermination of causal structure by statistical dependencies

Claim: Statistical dependency is possible to be observed between sample values of variables X and Y for the following:



# Bibliography

Eells, E. (1991). *Probabilistic Causality.* Cambridge UK: Cambridge University Press.

G.E. Hughes, M. C. (1996). *A New Introduction To Modal Logic.* London: Routledge.

Good, I. J. (1961). A Causal Calculus (I and II). *The British Journal for the Philosophy of Science*, 305-318, 43-51.

Hume, D. (1748). *Philosophical Essays Concerning Human Understanding.* London: Printed for A. Millar, opposite Katharine-Street in the Strand. MDCCXLVII.

Lewis, D. (1973). *Counterfactuals.* Malden, Massachusetts: Blackwell Publishers.

Lewis, D. (1974). Causation. *Journal of Philosophy*, 556-567.

Mackie, J. (1980). *The Cement of The Universe: A Study of Causation.* New York: Oxford University Press, New York, United States.

Otte, R. E. (1982). *Probability and Causality, PhD Thesis.* Ann Arbor , MI, 48106: University of Arizona Graduate College, University Microfilm International.

Reichenbach, H. (1956). *The Direction of Time.* Berkeley and Los Angeles, California: University of California Press.

Salmon, W. C. (1980). Probabilistic Causality. *Pacific Philosophical Quarterly*, pp. 137-153.

Salmon, W. C. (1998). *Causality and Explanation.* Pittsburgh, Pennsylvania: Oxford University Press.

Spirtes, P., Glymour, C., & Sheines, R. (1993). *Causation, Prediction and Search.* New York: Springer Verlag.

# Downloadable Links for the Bibliography

(Eells, 1991): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/eells_probabilistic_causality_1991.pdf)

(Reichenbach, 1956): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/the-direction-of-time-hans-reichenbach-ucal-press-1971.pdf)

(Spirtes, Glymour, & Sheines, 1993): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/CausationPredictionandSearch_Spirtes_CMU_2000.pdf)

(Otte, 1982): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/Probability_and_Causality_PhD_Thesis_Otte_1982.pdf)

(G.E. Hughes, 1996): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/ModelChecking/huges_cresswell_modal_logic.pdf)

(Salmon, Causality and Explanation, 1998): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/Causality_and_Explanation_Wesley_Salmon_1997.pdf)

(Salmon, Probabilistic Causality, 1980): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/Probabilistic_Causality_Salmon_1980.pdf)

(Good, 1961): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/CausalCalculus_part_I_and_II_Good_1960.pdf)

(Mackie, 1980): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/cement-of-the-universe-a-study-of-causation-JL-Mackie-1980.pdf)

(Lewis, Counterfactuals, 1973): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/books/Counterfactuals-lewis-1973.pdf)

(Lewis, Causation, 1974): [here](https://github.com/dimitarpg13/root_cause_analysis_and_model_checking/blob/main/literature/Lewis-Causation_1974.pdf)